

---

## Contents

<b>1</b>	<b>Introduction</b>	1
<b>2</b>	<b>Coordinate and Time Systems</b>	3
2.1	Geocentric Earth-Fixed Coordinate Systems	3
2.2	Coordinate System Transformations	6
2.3	Local Coordinate System	7
2.4	Earth-Centred Inertial Coordinate System	9
2.5	Geocentric Ecliptic Inertial Coordinate System	13
2.6	Time Systems	13
<b>3</b>	<b>Satellite Orbits</b>	17
3.1	Keplerian Motion	17
3.1.1	Satellite Motion in the Orbital Plane	20
3.1.2	Keplerian Equation	23
3.1.3	State Vector of the Satellite	25
3.2	Disturbed Satellite Motion	27
3.3	GPS Broadcast Ephemerides	28
3.4	IGS Precise Ephemerides	30
<b>4</b>	<b>GPS Observables</b>	33
4.1	Code Pseudoranges	33
4.2	Carrier Phases	35
4.3	Doppler Measurements	37
<b>5</b>	<b>Physical Influences of GPS Surveying</b>	39
5.1	Ionospheric Effects	39
5.1.1	Code Delay and Phase Advance	39
5.1.2	Elimination of the Ionospheric Effects	41
5.1.3	Ionospheric Models	44
5.1.4	Mapping Functions	47
5.2	Tropospheric Effects	50
5.2.1	Tropospheric Models	51
5.2.2	Mapping Functions and Parameterisation	55
5.3	Relativistic Effects	57
5.3.1	Special Relativity and General Relativity	57
5.3.2	Relativistic Effects on GPS	60

---

5.4	Earth Tide and Ocean Loading Tide Corrections .....	62
5.4.1	Earth Tide Displacements of the GPS Station .....	62
5.4.2	Simplified Model of the Earth Tide Displacements .....	64
5.4.3	Numerical Examples of the Earth Tide Effects .....	65
5.4.4	Ocean Loading Tide Displacement .....	68
5.4.5	Computation of the Ocean Loading Tide Displacement .....	70
5.4.6	Numerical Examples of Loading Tide Effects .....	71
5.5	Clock Errors .....	72
5.6	Multipath Effects .....	73
5.6.1	GPS-Altimetry, Signals Reflected from the Earth-Surface .....	74
5.6.2	Reflecting Point Positioning .....	75
5.6.3	Image Point and Reflecting Surface Determination .....	77
5.7	Anti-Spoofing and Selective Availability Effects .....	77
5.8	Antenna Phase Centre Offset and Variation .....	78
5.9	Instrumental Biases .....	81
<b>6</b>	<b>GPS Observation Equations</b> .....	83
6.1	General Mathematical Models of GPS Observations .....	83
6.2	Linearisation of the Observational Model .....	85
6.3	Partial Derivatives of Observational Function .....	86
6.4	Linear Transformation and Covariance Propagation .....	90
6.5	Data Combinations .....	91
6.5.1	Ionosphere-Free Combinations .....	93
6.5.2	Geometry-Free Combinations .....	94
6.5.3	Standard Phase-Code Combination .....	96
6.5.4	Ionospheric Residuals .....	97
6.5.5	Differential Doppler and Doppler Integration .....	98
6.6	Data Differentiations .....	100
6.6.1	Single Differences .....	101
6.6.2	Double Differences .....	103
6.6.3	Triple Differences .....	106
6.7	A Unified GPS Data Processing Method .....	107
6.7.1	Introduction .....	108
6.7.2	Formation of Equivalent Observation Equations .....	109
6.7.3	Equivalent Equations of Single Differences .....	110
6.7.4	Equivalent Equations of Double Differences .....	113
6.7.5	Equivalent Equations of Triple Differences .....	116
6.7.6	Method of Dealing with the Reference Parameters .....	116
6.7.7	Summary of the Unified Equivalent Algorithm .....	117
<b>7</b>	<b>Adjustment and Filtering Methods</b> .....	119
7.1	Introduction .....	119
7.2	Least Squares Adjustment .....	119
7.2.1	Least Squares Adjustment with Sequential Observation Groups .....	121
7.3	Sequential Least Squares Adjustment .....	123
7.4	Conditional Least Squares Adjustment .....	124
7.4.1	Sequential Application of Conditional Least Squares Adjustment ....	126

---

7.5	Block-Wise Least Squares Adjustment .....	127
7.5.1	Sequential Solution of Block-Wise Least Squares Adjustment .....	129
7.5.2	Block-Wise Least Squares for Code-Phase Combination .....	131
7.6	Equivalently Eliminated Observation Equation System .....	132
7.7	Kalman Filter .....	135
7.7.1	Classic Kalman Filter .....	135
7.7.2	Kalman Filter – A General Form of Sequential Least Squares Adjustment .....	137
7.7.3	Robust Kalman Filter .....	138
7.7.4	Adaptively Robust Kalman Filtering .....	140
7.8	A Priori Constrained Least Squares Adjustment .....	144
7.8.1	A Priori Parameter Constraints .....	144
7.8.2	A Priori Datum .....	146
7.8.3	Quasi-Stable Datum .....	147
7.9	Summary .....	149
<b>8</b>	<b>Cycle Slip Detection and Ambiguity Resolution</b> .....	151
8.1	Cycle Slip Detection .....	151
8.2	Method of Dealing with Cycle Slips .....	153
8.3	A General Criterion of Integer Ambiguity Search .....	153
8.3.1	Introduction .....	153
8.3.2	Summary of Conditional Least Squares Adjustment .....	154
8.3.3	Integer Ambiguity Search in Coordinate Domain .....	156
8.3.4	Integer Ambiguity Search in Ambiguity Domain .....	158
8.3.5	Integer Ambiguity Search in Coordinate and Ambiguity Domains .....	159
8.3.6	Properties of the General Criterion .....	161
8.3.7	Case of Three Groups of Unknowns .....	162
8.3.8	Diagonalised Normal Equation and the Equivalent Observation Equation .....	164
8.3.9	An Equivalent Ambiguity Search Criterion and its Properties .....	167
8.3.10	Numerical Examples of the Equivalent Criterion .....	168
8.3.11	Conclusions and Comments .....	171
8.4	A Diagonalisation Algorithm for Ambiguity Search .....	172
8.4.1	Further Diagonalised Normal Equation and Observation Equation .....	172
8.4.2	Related Equivalent Ambiguity Search Criteria .....	174
8.4.3	Summary of Diagonal Ambiguity Search .....	174
8.5	Ambiguity Function .....	175
8.6	Ionosphere-Free Ambiguity Fixing .....	176
8.6.1	Introduction .....	176
8.6.2	Concept of Ionospheric Ambiguity Correction .....	177
8.6.3	Determination of the Ionospheric Ambiguity Correction .....	180
8.6.4	Integer Ambiguity Fixing Through Ambiguity-Ionospheric Equations .....	180
8.6.5	Float Ambiguity Fixing .....	181
8.7	Conditions of Integer Ambiguity Ionosphere-Free Combinations .....	181

---

<b>9    Kinematic and Static GPS Data Processing .....</b>	183
9.1 Preparation of GPS Data Processing .....	183
9.2 Solving Ambiguity-Ionospheric Equations .....	184
9.3 Single Point Positioning .....	186
9.4 Standard Un-Differential GPS Data Processing .....	191
9.5 Equivalent Method of GPS Data Processing .....	193
9.6 Relative Positioning .....	194
9.7 Velocity Determination .....	195
9.8 Kalman Filtering Using Velocity Information .....	197
9.9 Accuracy of the Observational Geometry .....	199
<b>10    Concept of Precise Kinematic Positioning and Flight-State Monitoring ...</b>	201
10.1 Introduction .....	201
10.2 Concept of Precise Kinematic Positioning .....	204
10.2.1 Combining the Static References with IGS Station .....	204
10.2.2 Earth Tide and Loading Tide Corrections .....	205
10.2.3 Multiple Static References for Kinematic Positioning .....	205
10.2.4 Introducing Height Information as a Condition .....	207
10.2.5 Creation of a Kinematic Tropospheric Model .....	207
10.2.6 Higher Order Ionospheric Effect Correction .....	207
10.2.7 A General Method of Integer Ambiguity Fixing .....	208
10.3 Concept of Flight-State Monitoring .....	208
10.4 Results, Precision Estimation and Comparisons .....	210
10.4.1 Multiple Static References for Kinematic Positioning .....	211
10.4.2 Ambiguity of Multiple Static References as a Condition for Kinematic Positioning .....	212
10.4.3 Multiple Kinematic GPS for Flight-State Monitoring and its Comparison with INS .....	214
10.4.4 Static GPS Data Kinematic Processing .....	214
10.4.5 Doppler Velocity Comparisons .....	215
10.5 Conclusions .....	216
<b>11    Perturbed Orbit and its Determination .....</b>	217
11.1 Perturbed Equation of Satellite Motion .....	217
11.1.1 Lagrangian Perturbed Equation of Satellite Motion .....	218
11.1.2 Gaussian Perturbed Equation of Satellite Motion .....	220
11.2 Perturbation Forces of Satellite Motion .....	223
11.2.1 Perturbation of the Earth's Gravitational Field .....	223
11.2.2 Perturbation of the Sun and the Moon as well as Planets .....	227
11.2.3 Earth Tide and Ocean Tide Perturbations .....	228
11.2.4 Solar Radiation Pressure .....	232
11.2.5 Atmospheric Drag .....	235
11.2.6 Additional Perturbations .....	238
11.2.7 Order Estimations of Perturbations .....	240
11.2.8 Ephemerides of the Moon, the Sun and Planets .....	240
11.3 Analysis Solution of the $C_{20}$ Perturbed Orbit .....	244
11.4 Orbit Correction .....	250

---

11.5 GPS Precise Orbit Determination .....	254
11.5.1 Principle of Precise Orbit Determination .....	254
11.5.2 Numerical Integration and Interpolation Algorithms .....	257
11.5.3 Orbit-Related Partial Derivatives .....	267
11.6 Quasi-Real-Time Orbit Determination .....	276
<b>12 Discussions .....</b>	<b>279</b>
12.1 Clock Errors, Instrumental Biases and Ambiguities .....	279
12.2 Maximum Property of Ambiguity Function .....	280
<b>Appendix 1</b>	
<b>IAU 1980 Theory of Nutation .....</b>	<b>285</b>
<b>Appendix 2</b>	
<b>Numerical Examples of the Diagonalisation of the Equations .....</b>	<b>287</b>
<b>References .....</b>	<b>293</b>
<b>Subject Index .....</b>	<b>311</b>