Rectangle Visibility Graphs: Characterization, Construction, and Compaction

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Abstract. Non-overlapping axis-aligned rectangles in the plane define visibility graphs in which vertices are associated with rectangles and edges with visibility in either the horizontal or vertical direction. The recognition problem for such graphs is known to be NP-complete. This paper introduces the *topological rectangle visibility graph*. We give a polynomial time algorithm for recognizing such a graph and for constructing, when possible, a realizing set of rectangles on the unit grid. The bounding box of these rectangles has optimum length in each dimension. The algorithm provides a compaction tool: given a set of rectangles, one computes its associated graph, and runs the algorithm to get a compact set of rectangles with the same visibility properties.

1 Introduction

The problem of constructing visibility representations of graphs has received a lot of attention in the graph drawing literature, motivated, for example, by research in VLSI and visualization of relationships between database entities.

One particularly successful approach represents graphs with horizontal bars for the vertices and vertical, unobstructed visibility lines between bars for the edges. The class of graphs allowing such a representation (*bar visibility graphs*) has been studied by several authors ([26], [23], [27], [28], [20]), and its properties are well understood: it consists of planar st-graphs (i.e., those planar graphs having an embedding for which all cut vertices lie on the same face). Several authors have studied the properties of generalizations of such graphs to higher dimensions ([1], [4], [5], [6], [7], [8], [10], [11], [15], [16], [22], [24] and [25]), but so far, no simple characterizations have been found.

For *rectangle visibility graphs*, vertices correspond to non-overlapping isothetic rectangles, and edges correspond to unobstructed, axis-aligned lines of sight between rectangles (see section 2).

Any set of non-overlapping isothetic rectangles contains much more information of a topological, combinatorial nature than simply the visibility information

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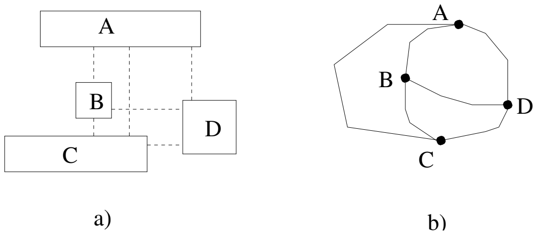

Fig. 1. a) A set $\mathcal{R}$ of rectangles. b) Its rectangle visibility graph.

in the horizontal and vertical directions. For example, walking the boundary of a rectangle in, say, the counterclockwise sense, gives rise to a cyclic ordering on those rectangles that are visible from the given one. In this paper, we make use of this kind of additional information to define the notion of a *topological rectangle visibility graph* (TRVG), defined in section 2. In brief, it is a graph arising from the horizontal and vertical visibilities among a set of non-overlapping isothetic rectangles in the plane, where the graph records ordering and direction information for these visibilities. Thus, every such set of rectangles in the plane gives rise to a TRVG.

Unlike bar visibility graphs, no combinatorial characterization of rectangle visibility graphs is known, and furthermore, it has been shown by Shermer ([24], [25]) that the problem of recognizing them is NP-complete.

The vertical visibilities among horizontal segments are parallel, so bar visibility graphs are planar. However, the axis-aligned visibilities of axis-aligned rectangles cross. Our characterization of visibility graphs of axis-aligned rectangles weaves together the properties of four planar graphs, namely, the planar vertical and horizontal visibility graphs arising from the rectangles, and the duals of these graphs.

Results. We give a complete combinatorial characterization of TRVG's. For any graph structure that satisfies the necessary and sufficient conditions to be a TRVG, we give an algorithm that constructs a set of rectangles on the unit grid realizing the graph. The bounding box of this grid realization is optimum.

Our construction provides a compaction method for axis-aligned rectangles drawn on a grid. A new set of such rectangles is produced that has the same visibility properties as the original set, such that both the width and the height of the bounding box of the new set is optimum for visibility preserving compactions of the original set. Compaction algorithms typically are heuristics that alternate between processing one direction, then the other. Our approach considers both directions simultaneously and produces an optimal result.

2 Preliminaries

We consider sets $\mathcal{R}$ of non-overlapping, isothetic (i.e., axis-aligned) rectangles in the plane and the horizontal and vertical visibilities among them. The rectangles