## **Cooperative Transport of Planar Objects by Multiple Mobile Robots Using Object Closure**

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**Abstract.** This paper addresses the problem of transporting objects by multiple mobile robots using the concept of *object closure*. In contrast to other manipulation techniques that are typically derived from form or force closure constraints, object closure requires the less stringent condition that the object be trapped or caged by the robots. We present experimental results that show car-like robots controlled using visual feedback, transporting an object in an obstacle free environment toward a prescribed goal.

## 1 Introduction

Transport and object manipulation with mobile robots have been extensively discussed in the literature. Most approaches use the notions of force and form closure to perform the manipulation of relatively large objects [1][2][3]. *Force closure* is a condition that implies that the grasp can resist any external force applied to the object [4]. *Form closure* can be viewed as the condition guaranteeing force closure, without requiring the contacts to be frictional. In general, robots are the agents that induce contacts with the object, and are the only source of grasp forces. But, when external forces acting on the object, such as gravity and friction, are used together with the contact forces to produce force closure, we have a situation of *conditional force closure*. Several research groups use conditional closure to transport objects by pushing it from an initial position to a goal [5][6].

In contrast to these approaches, object closure requires the less stringent condition that the object be trapped or caged by the robots. In other words, although it may have some freedom to move, it cannot be completely removed [7][8]. Since this technique departs from approaches that require a relatively high degree of precision in relative positions and orientations for grasping it results in a more robust approach to manipulation. Caging, the basis for object closure, was first introduced in by Rimon and Blake in [9] for non-convex objects and two fingered gripers. Other papers addressing variation on this basic theme are [7][10][11][8].

We address the problem of transporting objects by multiple robots using object closure. This work is closest in spirit to the work by Sudsang and Ponce [11]. They

developed a centralized algorithm for moving three robots with circular geometry in an object manipulation task. In our approach, robots use decentralized controllers to move while achieving and maintaining object closure. Because the controllers are decentralized, the approach scales to multiple robots and large objects. Further we do not require the robots to be circular. However, it should be noted that our approach addresses the manipulation of the object in  $\mathbb{R}^2$  and does not address the object's orientation.

The remaining of this paper is organized as follows. Section 2 presents the proposed methodology for object closure. Sections 3 and 4 describe our approach to cooperative control and sensing and in Sect. 5 are the experiments illustrating our application. Finally, the main points of the paper and directions for future work are presented in Sect. 6.

## 2 Object Closure

Consider a planar world, W, and, for the sake of simplicity, assume  $W = \mathbb{R}^2$ . The entities of the world are the object  $\mathcal{O}$ , which is assumed for discussion to be convex and two-dimensional, and a group of n robots. The  $i^{th}$  robot  $R_i$  is described by the convex set  $\mathcal{A}_i(q_i) \in W$ , where  $q_i = (x_i, y_i, \theta_i)$  denotes the configuration of  $R_i$ . Convex robots and objects are represented by an intersection of m half planes derived from the equations for each edge. Let  $f_j(x, y) = a_j x + b_j y + c_j$  be the equation that corresponds to the edge from  $(x_j, y_j)$  to  $(x_{j+1}, y_{j+1})^1$  and  $f_j(x, y) < 0$  for all points in the interior of the polygon. The object,  $\mathcal{O}$  can be expressed as:

$$\mathcal{O}=H_1\cap H_2\cap\cdots\cap H_m\,,$$

where:  $H_j = \{(x, y) \in \mathbb{R}^2 \mid f_j(x, y) \le 0\}, \quad 1 \le j \le m$ .

The configuration space C for an object is the set of possible configurations,  $q = (x, y, \theta)$ , with each q corresponding to the pose of O [12]. Similarly, will use  $C_{R_i}$  to denote the configuration space of robot  $R_i$ . The region in the configuration space that corresponds to an inter-penetration between the object O and the robot i is:

$$\mathcal{C}_{obj\_i} = \{ q \in \mathcal{C} \mid \mathcal{A}_i(q_i) \cap \mathcal{O}(q) \neq \emptyset \},\$$

where  $\mathcal{O}(q)$  is the representation of  $\mathcal{O}$  in the configuration q. It is well known for a planar world,  $\mathcal{C}_{obj,i}$  can be represented as a three dimensional solid. There is an efficient method of computing each slice (a specific object orientation) of this solid in the case of convex polygonal objects and robots. The running time of the algorithm is O(l + m), where l is the number of edges of the robot and m is the number of edges of the object [12].

In order to introduce the concept of object closure, let the object motions be restricted to translations only and the robot be represented by a single point. In other words, let q = (x, y) and  $A_i(q_i) = q_i = (x_i, y_i)$ . Figure 1 shows the boundary of  $C_{obj,i}$  for such a situation and a 5-sided polygonal object.

 $j^{-1} j + 1 = 1$  for j = m and j - 1 = m for j = 1.