Hardness of Approximating Independent Domination in Circle Graphs

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Abstract. A graph $G = (V, E)$ is called a *circle graph* if there is a one-to-one correspondence between vertices in V and a set C of chords in a circle such that two vertices in V are adjacent if and only if the corresponding chords in C intersect. A subset V' of V is a *dominating set* of G if for all $u \in V$ either $u \in V'$ or u has a neighbor in V'. In addition, if no two vertices in V' are adjacent, then V' is called an *independent dominating set*; if $G[V']$ is connected, then V' is called a *connected dominating set*. Keil (*Discrete Applied Mathematics*, 42 (1993), 51-63) shows that the minimum dominating set problem and the minimum connected dominating set problem are both NP-complete even for circle graphs. He leaves open the complexity of the minimum independent dominating set problem. In this paper we show that the minimum independent dominating set problem on circle graphs is NP-complete. Furthermore we show that for any ε, $0 \leq \varepsilon < 1$, there does not exist an n^{ε}-approximation algorithm for the minimum independent dominating set problem on n-vertex circle graphs, unless P = NP. Several other related domination problems on circle graphs are also shown to be as hard to approximate.

1 Introduction

For a graph $G = (V, E)$, a subset V' of V is a *dominating set* of G if for all $u \in V$ either $u \in V'$ or u has a neighbor in V'. In addition, if no two vertices in V' are adjacent, then V' is called an *independent dominating set*; if the subgraph of G induced by V', denoted $G[V']$, is connected, then V' is called a *connected dominating set*; if $G[V']$ has no isolated nodes, then V' is called a *total dominating set*; and if $G[V']$ is a clique, then V' is called a *dominating clique*. Garey and Johnson [18] mention that problems of finding a minimum cardinality dominating set (MDS), minimum cardinality independent dominating set (MIDS), minimum cardinality connected dominating set (MCDS), minimum cardinality total dominating set (MTDS), and minimum cardinality dominating clique (MDC) are all NP-complete for general graphs.

Restrictions of these problems to various classes of graphs have been studied extensively [11]. Table 1 shows the computational complexity of three of the problems mentioned above when restricted to different classes of graphs. P indicates

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the existence of a polynomial time algorithm and NPc indicates that the problem is NP-complete. Some of the references mentioned in the table are to original papers where the corresponding result first appeared, while some are to secondary sources. It should be noted that this list is far from being comprehensive and that these problems have been studied for several other classes of graphs. In this paper we focus on the only "question mark" in the above table, corresponding to MIDS on circle graphs. Not only do we show that MIDS is NP-complete for circle graphs, we also show that it is extremely hard to approximate.

Class of graphs	MDS	MIDS	MCDS
trees	P [18]	P [2]	P [11]
cographs	P [10]	P [10]	P [6]
interval graphs	P[4]	P [9]	P [17]
permutation graphs	P [10]	P [10]	P [5]
cocomparability graphs	P [11]	P [15]	P [15]
bipartite graphs	NPc [8]	NPc [6]	NPc [19]
comparability graphs	NPc [8]	NPc [6]	NPc [19]
chordal graphs	NPc [3]	P [9]	NPc [16]
circle graphs	NPc [14]	?	NPc [14]

Fig. 1. Complexity of three domination problems when restricted to different classes of graphs.

A graph $G = (V, E)$ is called a *circle graph* if there is a one-to-one correspondence between vertices in V and a set C of chords in a circle such that two vertices in V are adjacent if and only if the corresponding chords in C intersect. C is called the *chord intersection model* for G. Equivalently, the vertices of a circle graph can be placed in one-to-one correspondence with the elements of a set I of intervals such that two vertices are adjacent if and only if the corresponding intervals overlap, but neither contains the other. I is called the *interval model* of the corresponding circle graph. Representations of a circle graph as a graph or as a set of chords or as a set of intervals are equivalent via polynomial time transformations. So, without loss of generality, in specifying instances of problems, we assume the availability of the representation that is most convenient.

The complexity of MDS on circle graphs was first mentioned as being unknown in Johnson's NP-completeness column [13]. Little progress was made towards solving this problem until Keil [14] resolved the complexity of MDS on circle graphs by showing that it is NP-complete. In the same paper, Keil showed that for circle graphs MCDS is NP-complete, MTDS is NP-complete, and MDC has a polynomial algorithm. He left the status of MIDS for circle graphs open. In this paper we show that MIDS is also NP-complete on circle graphs. We also attack the approximability of MIDS on circle graphs and show that this problem is extremely hard to approximate. An *α-approximation* algorithm for a minimization problem is a polynomial time algorithm that guarantees that the