Observation of Crises and Bifurcations in the Hodgkin-Huxley Neuron Model

Wuyin Jin[1], Qian Lin[2], Yaobing Wei[1], and Ying Wu[3]

[1] College of Mechano-Electronic Engineering, Lanzhou University of Technology, Lanzhou 730050, China
jinwuyin@263.net

[2] College of Petrochemical Technology, Lanzhou University of Technology, Lanzhou 730050, China

[3] College of Science, Lanzhou University of Technology, Lanzhou 730050, China

Abstract. With the changing of the stimulus frequency, there are a lot of firing dynamics behaviors of interspike intervals (ISIs), such as quasi-periodic, bursting, period-chaotic, chaotic, periodic and the bifurcations of the chaotic attractor appear alternatively in Hodgkin-Huxley (H-H) neuron model. The chaotic behavior is realized over a wide range of frequency and is visualized by using ISIs, and many kinds of abrupt undergoing changes of the ISIs are observed in deferent frequency regions, such as boundary crisis, interior crisis and merging crisis displaying alternately along with the changes changes of external signal frequency, too. And there are many periodic windows and fractal structures in ISIs dynamics behaviors. The saddle node bifurcation resulted collapses of chaos to period-12 orbit in dynamics of ISIs is identified.

1 Introduction

The bifurcation and crisis of neural system have been an object of major attention since the beginning of the study of chaos theory. As we all known, the neural systems have strong nonlinear characters, and are usually able to display different dynamics according to system parameters or external inputs in ISIs sequences. When these parameters are slightly modified, the system's dynamics usually experience also little modification, except when these changes occur in the vicinity of a critical point, in which case an abrupt qualitative change or transition in the dynamics occurs [1-3]. These transitions, for example, may be from periodic to chaotic, from chaotic to chaotic, and their inverse transitions [4].

And, the numerical evidence and theoretical reasoning has proved that there is a chaos-chaos transition in the neuron, in which the change of the attractor size is sudden but continuous, different from general discontinues chaos-chaos transitions, and which occurs in the Hindmarsh-Rose model of a neuron. This transition corresponds to different neural dynamics, i.e. the chaotic dynamics of

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bursting and spiking dynamics [3]. The crisis resulted from homoclinic bifurcation and the chaos collapsing to a period-3 orbit in the dynamics of a quadratic Logistic map neuron have also been studied [5,6]. Xie et al introduced periodic orbit theory to characterize the dynamical behavior of aperiodic firing neurons, and considered that bifurcations, crises and sensitive dependence of chaotic motions on control parameters can be the underlying mechanisms [7], and there are many chaotic activities have been observed in experimental studies of electroencephalogram(EEG) signals and neuronal ISIs sequence [8-10].

The transitions between different dynamic behaviors of ISIs sequence of H-H neuron model under external periodic stimulus and the saddle-node bifurcation are studied in this work, which is relevant both to the theory of nonlinear dynamics and to biophysics.

2 The Hodgkin-Huxley (H-H) Neuron Model

The equations that describe the H-H neuron model have been derived from a squid giant axon. These equations can describe the spiking behavior and refractoriness of real neuron very well, so that this kind of model is employed in this work. The H-H model for the action potential of a space clamped squid axon is defined by the four-dimensional vector field [11]

$$\begin{cases} \dot{u} = I_{ext} - [120m^3h(u+115) + 36n^4(u-12) + 0.3(u+10.6)] \\ \dot{m} = (1-m)\Psi(\frac{u+25}{10}) - m(4exp(\frac{u}{18})) \\ \dot{n} = 0.1(1-n)\Psi(\frac{u+10}{10}) - n(0.125exp(\frac{u}{80})) \\ \dot{h} = 0.07(1-h)\Psi(\frac{u}{20}) - h(\frac{1}{1+exp(\frac{u+30}{10})}) \end{cases}, \quad (1)$$

where

$$\Psi(x) = \frac{x}{exp(x)-1} \quad (2)$$

and variables u, m, n, and h represent membrane potential, activation of a sodium current, activation of a potassium current, and inactivation of the sodium current. There is also a current parameter I_{ext} that is an external periodic signal current into the space-clamped axon in this work, i.e.

$$I_{ext} = I_{shift} + sin(2\pi f_0 t), \quad (3)$$

where $I_{shift} = 10\mu A/cm^2$, being the amplitude of current shift, and $f_0 = 1/3$ Hz being the basic stimulus frequency in this work.

Recalling that the H-H convention for membrane potential reverses the sign from modern conventions, and so the voltage spikes of action potentials are negative in the H-H model. When improved models for the membrane potential of the squid axon have been formulated, the H-H model remains the paradigm for conductance-based models of neural systems. From a mathematical viewpoint, varied properties of the dynamics of the H-H vector field have been studied. Nonetheless, we remain far from a comprehensive understanding of the dynamics displayed by this vector field.