Nonlinear Dynamical Analysis on Coupled Modified Fitzhugh-Nagumo Neuron Model

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Abstract. In this work, we studied the dynamics of modified FitzHugh-Nagumo (MFHN) neuron model. This model shows how the potential difference between spine head and its surrounding medium vacillates between a relatively constant period called the silent phase and large scale oscillation reffered to as the active phase or bursting. We investigated bifurcation in the dynamics of two MFHN neurons coupled to each other through an electrical coupling. It is found that the variation in coupling strength between the neurons leads to different types of bifurcations and the system exhibits the existence of fixed point, periodic and chaotic attractor.

1 Introduction

Determining the dynamical behavior of an ensemble of coupled neurons is an important problem in computational neuroscience. The primary step for understanding this complex problem is to understand the dynamical behavior of individual neurons. Commonly used models for the study of individual neurons which display spiking/bursting behavior include (a) Integrate-and-fire models and their variants [1, 2] (b) FitzHugh-Nagumo model [3], (c) Hindmarsh-Rose model [13], (d) Hodgkin-Huxley model [4, 7] and (e) Morris-Lecar model [5]. A short review of models in neurobiology is provided by Rinzel in [6, 9, 12]. The study of Type I neuronal models is more important as pyramidal cells in the brain [11] exhibits this type of behavior. Biophysical models such as the Hodgkin-Huxley(HH) model and Morris-Lecar(ML) model have been observed to display Type I neural excitability [2]. Mathematical techniques to study Type I neurons were developed by Ermentrout and Kopell [1] and the individual behavior of Type I neurons was fairly well understood. Bifurcation phenomena in individual neuron models including the Hodgkin-Huxley, Morris-Lecar and FitzHugh-Nagumo have been investigated in the literature [9–11, 13, 14]. Rinzel and Ermentrout [9] studied bifurcations in the Morris-Lecar (ML) model by treating the externally applied direct current as a bifurcation parameter. It is also important to note that the choice of system parameters in these neuron models can influence the type of excitability [9]. Rinzel, proposed a neuron model which produces a Type III burst is studied here [12]. The study of coupled neuron models is one of the fundamental problem in computational neuroscience that

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helps in understanding the behavior of neurons in a network. From dynamical point of view it is of crucial importance to investigate the effect of variation in coupling strength to its dynamical behavior. In this work, we presented general conditions under which a system of two coupled neurons shows different types of dynamical behavior like converging, oscillatory and chaotic. For our analysis we considered coupling strength as a parameter. Our studies are based on modified FitzHugh-Nagumo system [12] which is an extension of FitzHugh-Nagumo system.

In section 2, we discuss the three dimensional mathematical model of modified FitzHugh-Nagumo type neurons. Nonlinear dynamical analysis of this model is presented in section 2. In section 3, we study a mathematical model of coupled neural oscillators and its bifurcation diagram. Finaly, in section 4, we concluded our work.

2 The Modified FitzHugh-Nagumo Neuron Model

The modified FitzHugh-Nagumo equations are a set of three simple ordinary differential equations which exhibit the qualitative behavior observed in neurons, viz quiescence, excitability and periodic behavior [12]. The system can be represented as

$$\dot{v} = -v - \frac{v^3}{3} - w + y + F(t) \tag{1}$$

$$\dot{w} = \phi(v + a - bw) \tag{2}$$

$$\dot{y} = \epsilon(-v + c - dy) \tag{3}$$

The function $F(t)$ represents the external stimulus. From biological point of view, variable v represents the potential difference between the dendritic spine head and its surrounding medium, w is recovery variable and y represents the slowly moving current in the dendrite. In this model, v and w together make up a fast subsystem relative to y. The equilibrium point (v^*, w^*, y^*) is calculated by substituting $\dot{v} = \dot{w} = \dot{y} = 0$ in equations (1), (2) & (3). The jacobian matrix at this point is found to be

$$J = \begin{bmatrix} -1 - v^{*2} & -1 & 1 \\ \phi & -b\phi & 0 \\ -\epsilon & 0 & -\epsilon d \end{bmatrix} \tag{4}$$

The three eigenvalues λ_1, λ_2 and λ_3 are the roots of equation $det(J - \lambda I) = 0$. If at a neighborhood of a particular value μ_0 of the parameter μ, there exists a pair of eigenvalues of $J(\mu)$ of the form $\alpha(\mu) \pm i\beta(\mu)$ such that $\alpha(\mu) = 0$, $\beta(\mu) \neq 0$, then no other eigenvalue of $A(\mu_0)$ will be an integral multiple of $i\beta(\mu_0)$. Thus $A(\mu_0)$ has a pair of pure imaginary eigenvalues. This provides the information about the bifurcation in the system.