Modelling Line and Edge Features Using Higher-Order Riesz Transforms*

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Abstract. The 2D-complex Riesz transform is an extension of the Hilbert transform to images. It can be used to model local image structure as a superposition of sinusoids, and to construct 2D steerable wavelets. In this paper we propose to model local image structure as the superposition of a 2D steerable wavelet at multiple amplitudes and orientations. These parameters are estimated by applying recent developments in super-resolution theory. Using 2D steerable wavelets corresponding to line or edge segments then allows for the underlying structure of image features such as junctions and edges to be determined.

Keywords: Riesz transform, 2D steerable filter, super-resolution, semi-definite program, local feature analysis.

1 Introduction

Low-level image features arise from interesting structures such as lines, edges, corners and junctions. Detection, classification and parametrisation of such features is a useful first step in the image analysis pipeline, providing input for higher-level pattern recognition. One approach is to project the local image patch onto a particular signal model. The model may be as simple as edge detection, or a more complex description with geometric information, e.g. [1]. The monogenic signal [2] locally models an image as an oriented sinusoid with a certain amplitude, phase and orientation, using the 1st order Riesz transform (RT). This representation is useful as amplitude encodes feature strength and phase encodes feature type, e.g. line or edge. Higher-order RTs are used for more complex signal models in [3] [4] and [5], and recent research has shown RTs can be used to construct 2D steerable wavelets [6], [7].

In this paper we propose to extend this work to model local signal structure as the superposition of a particular 2D steerable wavelet at multiple amplitudes and orientations. This is an inverse problem where these parameters must be optimised such that the model describes as much of the signal as possible. Our novel approach is to use the theory of super-resolution of spike trains via semi-definite programming [8], [9] to solve for the wavelet parameters. The choice of wavelet

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depends on the feature of interest. For example, if a wavelet corresponding to a line segment or wedge segment is used, then the amplitude and orientation of individual line or wedge segments that make up more complex features, such as junctions or edges, can be determined.

2 Signal Model

2.1 Riesz Transform

The RT is an extension of the Hilbert transform to two or more dimensions [2]. The n-th order 2D complex RT, $\mathcal{R}^n$, is a complex-valued operator that can be expressed as either a convolution in the spatial domain, or multiplication in the Fourier domain. We shall use the following definition from [7], which differs from [4], [10] only in sign changes and the position of $\mathrm{i} = \sqrt{-1}$,

$$\mathcal{R}^n f(\mathbf{z}) \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad \left(\frac{u_x + \mathrm{i}u_y}{\|\mathbf{u}\|}\right)^n \hat{f}(\mathbf{u}) \tag{1}$$

where $f(\mathbf{z}) \in L_2(\mathbb{R}^2)$, $\hat{f}(\mathbf{u}) = \mathcal{F}f(\mathbf{u})$ is its Fourier transform, and $\mathbf{u} = [u_x, u_y]$. Also $\mathcal{R}^{n*} = \mathcal{R}^{-n}$ and $\mathcal{R}^0$ is the identity operator. The vector valued multi-order RT, $\boldsymbol{\mathcal{R}}^N$, maps the $-N$ to N-th order RT responses to a vector valued signal [7],

$$\boldsymbol{\mathcal{R}}^N f(\mathbf{z}) = \{\mathcal{R}^n f(\mathbf{z}) \,|\, n \in \mathbb{Z}, |n| \leq N\} \tag{2}$$

We shall refer to the response at one point as an RT vector in the rest of this paper. The higher-order RT is a unitary operator that is translation and scale invariant [2], [7], and appears as a spherical harmonic in the Fourier domain. Local signal structure with a particular rotational symmetry gives the greatest response for an RT of the same order, e.g. an 'X' junction responds most to the 4-th order RT. Therefore describing local signal structure using larger order RTs allows for more complex structures to be analysed.

2.2 2D Steerable Wavelet

The RT is also used to construct 2D steerable wavelets that are self-reversible and polar separable in the Fourier domain, [6], [7]. A 2D steerable wavelet, $s(\mathbf{z};\theta)$, given by the weighted summation of different order RTs of an isotropic bandpass filter kernel, $p(\mathbf{z})$, is

$$s(\mathbf{z};\theta) = \sum_{n=-N}^{N} e^{\mathrm{i}n\theta} u_n \mathcal{R}^n p(\mathbf{z}) \tag{3}$$

where θ is the orientation of the wavelet, and $\mathbf{u}$ is the weights. Let $\mathbf{f}^N(\mathbf{z}) = \boldsymbol{\mathcal{R}}^N(p * f)(\mathbf{z})$. The 2D steerable wavelet, $s_f(\mathbf{z})$, whose RT vector is the same as