Oscillation Criteria for Nonlinear Neutral Perturbed Dynamic Equations on Time Scales

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Abstract. The paper is concerned with the oscillation for a class of second-order nonlinear neutral perturbed dynamic equations on time scales. By employing the generalized Riccati transformation and introducing a general class of parameter functions, some new sufficient conditions for oscillation of such dynamic equations are established. Our results extend and improve some known results in the literature and the results in particular are essentially new under the weak conditions for the parameter functions. An example to illustrate the main results is given.

Keywords: oscillation, nonlinear neutral perturbed dynamic equation, time scales, Riccati transformation.

1 Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Higher in his Ph.D. thesis [1] in 1988 in order to unify continuous and discrete analysis. Several authors have expounded on various aspects of this new theory, see the survey paper by Agarwal [2] and references cited therein. A time scale T is an arbitrary nonempty closed subset of the reals. A book on the subject of time scales by Bohner and Peterson [3] summarizes and organizes much of the time scale calculus. There are many interesting time scales and they give rise to plenty of applications, such as a time scale $\mathrm{P}_{a,b}\bigcup_{n=0}^{+\infty}[n(a+b),n(a+b)+a]$ is widely used to study population in biological communities, electric circuit and so on [3].

In recent years, there has been much research activity concerning the oscillation and nonoscillation of solutions of some dynamic equations on time scales, and we refer the reader to the papers [4-12] and references cited therein. Regarding neutral dynamic equations, Argarwal et al [6] considered the second order neutral delay dynamic equation

$$\{\alpha(t)[(x(t)+c(t)x(t-\tau))^{\Delta}]^{\gamma}\}^{\Delta}+f(t,x(t-\delta))=0\,. \tag{1}$$

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Where $\gamma > 0$ is an odd positive integer, τ and δ are positive constants, $\alpha^{\Delta}(t) > 0$, and proved that the oscillation of (1) is equivalent to the oscillation of a first order delay dynamic inequality. Saker [7] considered (1) where $\gamma \geq 1$ is an odd positive integer, the condition $\alpha^{\Delta}(t) > 0$ is abolished and established some new sufficient conditions for oscillation of (1).

Sahiner et al [8] considered the general equation

$$\{\alpha(t)[(x(t)+c(t)x(\tau(t))^{\Delta}]^{\gamma}\}^{\Delta} + f(t, x(\delta(t))) = 0\,. \tag{2}$$

On a time scale T, where $\gamma \geq 1$ and $\tau(t) \leq t$, $\delta(t) \leq t$, and followed the argument in [6-7] by reducing the oscillation of (2) to the oscillation of a first order delay dynamic inequality and established some sufficient conditions for the oscillation.

We note that in all the above results the condition $0 \leq c(t) < 1$, $\gamma \geq 1$ and $\delta(t) \leq t$ are required. Some authors utilized the kernel function $(t-s)^m$ or a general class of functions $H(t,s)$ and obtained some oscillation criteria, but $H^{\Delta_s}(t,s) \leq 0$ is required. This motivates us to study the oscillation for the second-order neutral nonlinear perturbed dynamic equations of the form

$$\{\alpha(t)((x(t)-c(t)x(\tau(t)))^{\Delta})^{\gamma}\}^{\Delta} + F(t, x(\delta(t))) = G(t, x(\delta(t)), x^{\Delta}) \tag{3}$$

On a time scales T, where γ is a quotient of positive odd integer, α, c is a positive real-valued rd-continuous function defined on a time scales T and the following conditions are satisfied:

(H1) $0 \leq c(t) \leq c_0 < 1$, and $\int_{t_0}^{+\infty} \Delta t / (\alpha(t))^{1/\gamma} = \infty$, for all $t \in$ T ;

(H2) τ, δ :T→T satisfies $\tau(t) \leq t$, for all $t \in$T, $\lim_{t\to\infty}\tau(t) = \lim_{t\to\infty}\delta(t) = \infty$, and either $\delta(t) \geq t$ or $\delta(t) \leq t$ for all sufficiently large t ;

(H3) $p, q : \mathrm{T} \to \mathbb{R}$ are rd-continuous function, such that $q(t) - p(t) > 0$, for all $t \in$ T ;

(H4) $F : \mathrm{T}\times\mathbb{R} \to \mathbb{R}$ and $G : \mathrm{T}\times\mathbb{R}^2 \to \mathbb{R}$ are functions such that $uF(t,u) > 0$ and $uG(t,u,v) > 0$, for all $u \in \mathbb{R} - \{0\}$, $v \in \mathbb{R}$, $t \in$ T ;

(H5) $F(t,u)/u^{\gamma} \geq q(t)$, and $G(t,u,v)/u^{\gamma} \leq p(t)$ for all $u, v \in \mathbb{R} - \{0\}$, $t \in$ T .

Since we are interested in the oscillatory and asymptotic behavior of solutions near infinity, we assume that $\sup \mathrm{T} = \infty$, let T=$[t_0, \infty)$. By a solution of (3), we mean a nontrivial real-valued function $x(t)$ satisfying (3) for $t \geq t_0$. A solution $x(t)$ of (3) is said to be oscillatory if it is neither eventually positive nor eventually negative, otherwise it is called nonoscillatory. Eq.(3) is said to be oscillatory if all its solutions are oscillatory. Our attention is restricted to those solutions of (3) which exist on some half line $[t_0, \infty)$ and satisfy $\sup\{|x(t)| : t \geq t_x\} > 0$, for any $t_x \geq t_0$.

The purpose of this paper is to derive some sufficient conditions for the solutions of (3) to be oscillatory or converge to zero by utilizing a general class of functions