# On General Regression Neural Network in a Nonstationary Environment

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**Abstract.** In this paper we present the method for estimation of unknown function in a time-varying environment. We study the probabilistic neural network based on the Parzen kernels combined with the recursive least square method. We present the conditions for convergence in probability and we discuss the experimental results.

## 1 Introduction

In literature there are many problems requiring finding the unknown regression function in a time-varying environment. This article will describe a method of solving this problem in the following case. Let $(X_i, Y_i)$ for $i = 1, \ldots, n$ be a sequence of pairs of random variables, where $X_i \in \mathbb{R}^p$ and $Y_i \in \mathbb{R}$. We will study the system

$$Y_i = \phi(X_i) + ac_i + Z_i, \quad i = 1, \ldots, n, \tag{1}$$

where $\phi(x)$ is an unknown regression function, $c_i$ in a known sequence of numbers, $a$ is unknown constant, $Z_i$ are random variables representing noise and $X_i$ are equally distributed random variables with density function $f(x)$. We consider the case when the noise $Z_i$ satisfies the following conditions

$$E[Z_i] = 0, \qquad Var[Z_i] = \sigma_i^2 < \sigma_Z^2, \qquad \text{for } i = 1 \ldots, n. \tag{2}$$

The problem is to find unknown value of parameter $a$ and to estimate unknown function $\phi$. It should be emphasized that such problem was never solved in literature. The method applied in this paper is based on the nonparametric estimates, named in the area of soft computing, probabilistic neural networks [40]. Nonparametric regression estimates in a stationary environment were studied in [6], [7], [10], [17]-[21], [27]-[29] and [32]-[35], whereas non-stationary environment was considered in [9], [22]-[26], [30] and [31], assuming stationary noise. For excelent overviews of those algorithms the reader is reffered to [8] and [11].

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## 2 Algorithm

The estimation of regression function $\phi(x)$ requires two steps. First we estimate the unknown value of parameter $a$ by estimate $\hat{a}_n$. Then we find the regression function of random variables $(X_i, Y_i')$ for $i \in 1, \dots, n$, where $Y_i'$ is in the form

$$Y_i' = Y_i - \hat{a}_n c_i = \phi(X_i) + (a - \hat{a}_n)c_i + Z_i, \quad i = 1, \dots, n. \tag{3}$$

To estimate the value of parameter $a$ we use the recursive least squares error method [1]. Therefore we use the formula

$$\hat{a}_i = \hat{a}_{i-1} + \frac{c_i}{\sum_{j=1}^{i} c_i}(Y_i - \hat{a}_{i-1}c_i). \tag{4}$$

After computing the value of $\hat{a}_n$ we can estimate the regression function $\phi(x)$ under assumption $a := \hat{a}_n$. In literature there are known methods of finding the unknown regression function. We propose to use the Parzen kernel in the form

$$K_n'(x,u) = h_n^{'-p} K(\frac{x-u}{h_n'}), \tag{5}$$

$$K_n(x,u) = h_n^{-p} K(\frac{x-u}{h_n}), \tag{6}$$

where $K$ is an appropriately selected function such that

$$||K||_\infty < \infty, \tag{7}$$

$$\int |K(y)|dy < \infty, \tag{8}$$

$$\lim_{y \longrightarrow \infty} |yK(y)| = 0, \tag{9}$$

$$\int_{R^p} K(y)dy = 1 \tag{10}$$

and $h_n$, $h_n'$ are certain sequences of numbers. Let us denote the estimator of density function $f(x)$ by $\hat{f}(x)$ and the estimator of function $R(x) = f(x)\phi(x)$ by $\hat{R}(x)$. Then function

$$\phi(x) = \frac{R(x)}{f(x)} \tag{11}$$

can be estimated by

$$\hat{\phi}_n(x) = \frac{\hat{R}_n(x)}{\hat{f}_n(x)}. \tag{12}$$

By using the formulas 5 and 6 we obtain

$$\hat{f}_n(x) = \frac{1}{n}\sum_{i=1}^{n} K_n'(x, X_i) \tag{13}$$