An Estimation for the Average Error of the Chebyshev Interpolation in Wiener Space

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Abstract. In this paper, the first kind of Chebyshev interpolation in the Wiener space are discussed. under the L_p norm, the convergence properties of Chebyshev interpolation polynomials base on the zeros of the Chebyshev polynomials are proved. Furthermore, the estimation for the average error of the first kind of Chebyshev interpolation polynomials are weakly equivalent to the average errors of the corresponding best polynomial approximation. while $p=4$,the weakly asypmtotic order $e^4(H_n, G_4) \approx 1/\sqrt{n}$ of the average error in the Wiener space is obtained.

Keywords: Chebyshev interpolation polynomials, average error, L_2-norm, Wiener-space.

1 Introduction

Let F be a real separable Banach space equipped with a probability measures μ on the Borelsets of F. Let G be another normed space such that F is continuously embedded in G By $\|\bullet\|$ we denote the norm in G . Any $A: F \to G$ such that $f \mapsto \|f - A(f)\|$ is a measurable mapping called an approximation operator (or just approximation). The average error of A is defined as

$$e_p(A,H) = \left(\int_F \|f - A(f)\|^p \mu(df)\right)^{\frac{1}{p}} \tag{1.1}$$

Since the target function in practical problems is usually given by its(exact or noisy) values at finitely many points, the approximation operator $A(f)$ is often considered depending on some function values about f only. Many papers such as [1], [2] studied the complexity of computing an ε-approximation in average case setting.

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Papers [3], [4] obtained the weak asymptotic order of the average error of Lagrange interpolation and Hermite-Fej´er interpolation in the Wiener space.

In this paper, we will show an estimation of the average error (in the $L_2 - norm$) of Chebyshev polynomial of the first kind in the Wiener space when $p = 4$. Now we turn to show the result.

Let X be the space of continuous function f defined on $[0,1]$ such that $f(0) = 0$. The space X is equipped with the sup norm. The Wiener measure ω is uniquely defined by the following property

$$\omega(f \in C[-1,1] : (f(t_1), \cdots f(t_n)) \in B) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi(t_j - t_{j-1})}} \bullet$$
$$\int_B \exp\left[\sum_{j=1}^{n} \frac{-(u_j - u_{j-1})^2}{2(t_j - t_{j-1})}\right] du_1 du_2 \cdots du_n \tag{1.2}$$

For every $0 = t_0 < t_1 < t_2 < \cdots < t_n \le 1$ with $u_0 = 0$ and $n \ge 1, B \in \mathrm{B}(\Re^n)$, in which $\mathrm{B}(\Re^n)$ is the class of all Borel subsets of $\Re^n$. It follows from [1] that:

$$\int_X f(x_1) f(x_2) ud(f) = \min\{x_1, x_2\}, \forall x_1, x_2 \in [0,1] \tag{1.3}$$

Let $F = \{f \in C[-1,1] : g(t) = f(2t-1) \in X\}$ and for every measurable subset $A \subset F$, we define

$$\mu(A) = \omega(\{g(t) = f(2t-1) : f \in A\}) \tag{1.4}$$

Where $1 \le p \le \infty$, let $G_p[-1,1]$ be the linear normed space of all $L_p - \mathrm{int}\, egrable$ functions f on $[-1,1]$ with the following finite norm

$$\|f\|_p = \left(\int_{-1}^{1} |f(x)|^p \frac{d(x)}{\sqrt{1-x^2}} \right)^{\frac{1}{p}} \tag{1.5}$$

Let $t_k = t_{nk} = \cos\frac{k\pi}{n+1}, k = 1, 2, \cdots n$ be the zeros of $T_n(x) = \cos n\theta,$ $x = \cos\theta$ which is the n-th degree Chebyshev polynomial of the first kind. The Chebyshev interpolation polynomial based on the zeros above is as follows