Collaborative Optimization under a Control Framework for ATSP

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Abstract. A collaborative optimization algorithm under a control framework is developed for the asymmetric traveling salesman problem (ATSP). The collaborative approach is not just a simple combination of two methods, but a deep collaboration in a manner like the feedback control. A notable feature of the approach is to make use of the collaboration to reduce the search space while maintaining the optimality. Compared with the previous work of the reduction procedure by Carpaneto, Dell'Amico et al. (1995) we designed a tighter and more generalized reduction procedure to make the collaborative method more powerful. Computational experiments on benchmark problems are given to exemplify the approach.

Keywords: ATSP, Collaborative optimization, Control framework, Ant colony optimization, Branch and Bound.

1 Introduction

ATSP is one of the most well-known combinatorial optimization problems due both to its practical relevance and to its considerable difficulty. **ATSP** and its variations are commonly used models to formulate many practical applications, such as the scheduling of chemical process[3], the scheduling of steel production[17][18], and printed circuit board punching sequence problem [15], and so on. The problem is concerned with finding the shortest Hamiltonian cycle or tour in a weighted directed graph without loops and multiple arcs. Although simple to state, **ATSP** is very difficult to attack and much effort has been, and will continue to be, devoted to the design of good optimization algorithms. Roughly speaking, methods of solving **ATSP** problems can fall into three categories, i.e., rigorous, heuristic and hybridized. Rigorous method can guarantee the optimality of the solution obtained. Many algorithms have been developed for the exact solution of **ATSP**. The representative ones are the Branch-and-Bound (**B&B**) based on Assignment Problem (**AP**) relaxation[2][16]. Recently, Turkensteen, Ghosh et al.[19] reported a new branching rule in **B&B** algorithm which utilizes ***tolerances*** to indicate which arcs are preferred to save in the optimal **ATSP** tour. The Branch-and-Cut (**B&C**) algorithm is also explored by [8]. Since solving the **ATSP** optimally is **NP**-hard, especially in many real industrial problems, exact optimization algorithms require overlong execution

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time and huge memory. In most of cases they can't produce an acceptable or even feasible solution given limit time. Hence, heuristics were dominating later even though they can't provide any guarantee on the solution quality. Karp [13] presented the so-called Patching Algorithm **(PA)-**a convincible heuristic method, and showed that the heuristic solution asymptotically converges to the optimal solution as the size of **ATSP** tends to infinity. Glover, Gutin et al. [9] introduced several construction heuristics, and conducted a large number of computational experiments for several families of **ATSP** instances. In more recent years, metaheuristics are booming, such as genetic algorithms (**GA**) [3][10], simulated annealing **(SA)** [14], tabu search **(TS)** [7], ant colony optimization **(ACO)**[1][6][15], and so on. Rigorous and heuristic approaches have ever-conflicting advantages and disadvantages in terms of computational load and solution quality. Hence, growing attentions have been given to hybridized methods.

The rest of the paper is organized as follows. In the next section, the mathematical formulation of the **ATSP** is presented. In section 3, the collaborative optimization framework is described. The key components that constitute the framework are illustrated in detail. Furthermore, theoretical analysis with respect to the performance is also presented briefly. In section 4, a large number of computational experiments are given. Finally, concluding remarks are included in section 5.

2 Problem Description

The **ATSP** can be formulated as an Integer Linear Programming

$$z^* = \min \sum_{(i,j)\in \boldsymbol{A}} c_{(i,j)} \cdot x_{(i,j)} \tag{1}$$

subject to：

$$\sum_{(i,j)\in \boldsymbol{A}} x_{(i,j)} = 1 \quad i = 1,\cdots,n \tag{2}$$

$$\sum_{(i,j)\in \boldsymbol{A}} x_{(i,j)} = 1 \quad j = 1,\cdots,n \tag{3}$$

$$\sum_{i\in \boldsymbol{S}}\sum_{j\in \boldsymbol{S}} x_{(i,j)} \le |\boldsymbol{S}| - 1 \qquad \boldsymbol{S} \subset \{1,2,\cdots,n\}, 2 \le |\boldsymbol{S}| \le n-2 \tag{4}$$

$$x_{(i,j)} \in \boldsymbol{D}_{(i,j)} \triangleq \{0,1\} \qquad (i,j)\in \boldsymbol{A} \tag{5}$$

where $x_{(i,j)} = 1$ $(i,j)\in \boldsymbol{A}$, if the arc is selected in the optimal solution; and $x_{(i,j)} = 0$ otherwise. Apparently, equations (1)(2)(3) and (5) define the **AP** problem and constraints (4) forbid subtours. The number of constraints described in constraints (4) will be exponentially explosive as the size of the problem increases. This reason cause the computational difficult in solving large-scale **ATSP**.