Properties of Planar Triangulation and Its Application*

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Abstract. Further research will be done on triangulation partitions. particularly, more careful analysis is made on even triangulation of simply connected domain, and a number of new properties are obtained. Using these new properties, some proof of the theorems on graph theory become easy and simple. For example, using the property an arbitrary planner even triangulation can be expressed as the union of a number of disjoint star domains, one can easily prove the equivalence of the three statement triangulation is even, triangulation is 3-vertex signed and triangulation is 2-triangle signed.

Keywords: even partition; triangulation; 3-vertex signed.

1 Introduction

Let $\Omega \in R^2$ be a simply connected domain, and let $\Delta = \{T_i\}_{i=1}^{N}$ be a triangulation of Ω satisfying

(1) $\Omega = \bigcup_{i=1}^{N} T_i$,
(2) Each T_i is a closed triangle,
(3) For all the $1 \leq i \neq j \leq N$,no vertex of T_i lies in the interior of T_j or in the interior of an edge of T_j.

The triangular domain of the partition is called a cell of Δ,N is the number of triangles in Δ. The edges of the triangles are called the partition lines, those fall within the domain Ω are called interior lines,or are called boundary lines. The vertex of triangle is called partition vertex,that lies within the domain is called interior vertex, or are called boundary vertex. Suppose v be a vertex of partition

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Δ, and $Star(v) = \{T_i \in \Delta | v \in T_i \ a \ cell \ of \ \Delta\}$ be the collection of cells in Δ sharing v as a common vertex, we call $Star(v)$ the star region of v. The degree, $d(v)$, of a vertex v is the number of edges with which it is incident. We call two vertices v_1 and v_2 are adjacent, if there is a line connect v_1 and v_2 ([9]).

Triangulation is widely used in various fields of scientific research. Especially in the applications such as computer-aided geometric design, graphics, image processing, surface reconstruction, finite element method, multi-spline methods([2,3,7]). Take data fitting and surface reconstruction for instance, in order to improve the accuracy, we often need to divide the region into many small regions which is grid division. Given any polygonal domain in R^2, it can be surely triangulated, but not necessarily can be subdivided by quadrilateral or other polygon mesh. In practice, if you are not satisfied on the accuracy obtained on a given triangulation, you can easily get an overall breakdown or partial breakdown ([2,8]). Triangulation is a very useful tool not only for the application but also for the theory perspective. The author has been working on the theory of spline, simply speaking, spline function is a piecewise polynomial function with a certain smoothness degree([7]). The properties of multivariate spline function on arbitrary triangulations has been a hot and difficult topic. So the research on nature properties of triangulation is of basic theoretical importance, and it is necessary to do more in-depth research on it.

In this paper, taking the triangulation as a graph, we consider its topology properties. As we all know, given a triangulation, denote p be the number of the vertices of the partition, l be the number of the lines (number of the triangle edges), n be the number of the triangles, then $e = p - l + n$ is a topological invariants, that is $e = p - l + n$ is unrelated to the geometry. e is called the Euler characteristic([1]). If the vertices of the partition Δ can be marked with -1 and 1 so that all the triangles in the Δ are marked with different symbols, we call such triangulation is 2-vertex signed. An interesting conjecture is proposed in reference [6]: any triangulation is 2-vertex signed. And this is proved in reference [9] using some definition and results from graph theory.

In section 2 of this paper, some new results is got based on in-depth study on the properties of triangulation especially even triangulation.

2 Properties of Triangulation

Let Δ be a triangulation of domain $\Omega \subset R^2$, then for any interior line in Δ, there are and only are two triangles sharing this line as an edge. The position relationship between two different star domain in triangulation has following three cases:

1. have common cells,
2. have common lines but none common cells,
3. have common vertices but none common lines.

In the rest of this paper, if not specified the discusses are limited on 2-dimensional simply connected region, and we say two star domain intersect means they having common cells.