Deterministic 7/8-Approximation for the Metric Maximum TSP

(Extended Abstract)

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Abstract. We present the first 7/8-approximation algorithm for the maximum traveling salesman problem with triangle inequality. Our algorithm is deterministic. This improves over both the randomized algorithm of Hassin and Rubinstein [2] with expected approximation ratio of $7/8 - O(n^{-1/2})$ and the deterministic $(7/8 - O(n^{-1/3}))$-approximation algorithm of Chen and Nagoya [1].

In the new algorithm, we extend the approach of processing local configurations using so-called loose-ends, which we introduced in [4].

1 Introduction

The Traveling Salesman Problem and its variants are among the most intensively researched problems in computer science and arise in a variety of applications. In its classical version, given a set of vertices V and a symmetric weight function $w : V^2 \to \mathbb{R}_{\geq 0}$ satisfying the triangle inequality one has to find a Hamiltonian cycle of minimum weight.

There are several variants of TSP, e.g. one can look for a Hamiltonian cycle of minimum or maximum weight (MAX-TSP), the weight function can be symmetric or asymmetric, it can satisfy the triangle inequality or not, etc.

In this paper, we are concerned with the MAX-TSP variant, where the weight function is symmetric and satisfies the triangle inequality. This variant is often called *the metric MAX-TSP*.

MAX-TSP (not necessarily metric) was first considered by Serdyukov in [5], where he gives a $\frac{3}{4}$-approximation. Next, a $\frac{5}{6}$-approximation algorithm for the metric case was given by Kostochka and Serdyukov [3]. Hassin and Rubinstein [2] used these two algorithms together with new ideas to achieve a randomized approximation algorithm with expected approximation ratio of $(\frac{7}{8} - O(n^{-1/2}))$. This algorithm has later been derandomized by Chen and Nagoya [1], at a cost of a slightly worse approximation factor of $(\frac{7}{8} - O(n^{-1/3}))$.

In this paper, we give a deterministic $\frac{7}{8}$-approximation algorithm for metric MAX-TSP. Our algorithm builds on the ideas of Serdyukov and Kostochka, but

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is completely different from that of Hassin and Rubinstein. We apply techniques similar to those used earlier in [4] for the directed version of MAX-TSP with triangle inequality.

1.1 Closer Look at Previous Results

Classic undirected MAX-TSP algorithm of Serdyukov [5] starts by constructing two sets of edges of the input graph G: a maximum weight cycle cover $\mathcal{C}$ and a maximum weight matching M, and then removing a single edge from each cycle of $\mathcal{C}$ and adding it to M. It can be shown that we can avoid creating cycles in M, so in the end we get two sets of paths: $\mathcal{C}'$ and M'. These sets can be extended to Hamiltonian cycles arbitrarily. Since we started with a maximum weight cycle cover and a maximum weight matching, we have $w(\mathcal{C}') + w(M') \geq w(\mathcal{C}) + w(M) \geq \frac{3}{2}\mathrm{OPT}$. It follows that the better of the two cycles has weight at least $\frac{3}{4}\mathrm{OPT}$. Here, we used two standard inequalities: $w(\mathcal{C}) \geq \mathrm{OPT}$ and $w(M) \geq \frac{1}{2}\mathrm{OPT}$. The latter only holds for graphs with even number of vertices. The case of odd number of vertices needs separate treatment.

Serdyukov's algorithm works for any undirected graph, with weight function not necessarily satisfying the triangle inequality. However, if this inequality is satisfied, we can get a much better algorithm. Kostochka and Serdykov observed the following useful fact (see e.g. [2] for a proof).

Lemma 1 (Kostochka, Serdyukov [3]). *Let $G = (V, E)$ be a weighted complete graph with a weight function $w : E \to \mathbb{R}_{\geq 0}$ satisfying the triangle inequality. Let $\mathcal{C}$ be a cycle cover in G and let $Q = \{e_1, \ldots, e_{|\mathcal{C}|}\}$ be a set of edges with exactly one edge from each cycle of $\mathcal{C}$. Then the collection of paths $\mathcal{C} \setminus Q$ can be extended in polynomial time to a Hamiltonian cycle H with*

$$w(H) \geq w(\mathcal{C}) - \sum_{i=1}^{|\mathcal{C}|} w(e_i)/2.$$

Kostochka and Serdyukov [3] propose an algorithm which starts by finding a maximum weight cycle cover $\mathcal{C}$ and then applies the above lemma with Q consisting of the lightest edges of cycles in $\mathcal{C}$. Since all cycles have length at least 3, the weight of the removed edges amounts to at most $\frac{1}{3}w(\mathcal{C})$, so we regain at least $\frac{1}{6}w(\mathcal{C})$, which leads to $\frac{5}{6}$-approximation. (Note that if it happens that all the cycles in $\mathcal{C}$ have length at least 4 we get $\frac{7}{8}$-approximation).

2 Our Approach

Similarly to Serdykov's algorithm (as well as that of Hassin and Rubinstein), our algorithm starts by constructing a maximum weight cycle cover $\mathcal{C}$ and maximum weight matching M. In our reasoning we need the inequality $w(M) \geq \frac{1}{2}\mathrm{OPT}$, which holds only for graphs with even number of vertices. In the remainder of this paper we only consider such graphs. Our results can be extended to graphs with odd number of vertices, we defer the details to the full version of the paper.