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AUG 26 1974

Professor Marlon Rayburn
Department of Mathematics
University of Manitoba
Winnipeg 19, Manitoba
CANADA

Dear Professor Rayburn:

In 1968 you published a paper in the Proceedings of A.M.S. giving the values $g(1) = 1$, $g(2) = 2$, $g(3) = 5$, $g(4) = 15$ for the number of Borel fields on a finite set. Could you please tell me if any more values of $g(n)$ are now known?

Many thanks.

Yours sincerely,

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N. J. A. Sloane

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The University of Manitoba

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16 ~~Augu~~ September 1974

Dr. Neil Sloane - 2C-363
Bell Laboratories
600 Mountain Avenue
Murray Hill, New Jersey 07974

Dear Dr. Sloane:

The base of a Borel field on a finite set is just a partition of that set. Hence the number $g(n)$ of Borel fields on a set of n points is better known as the Bell number B_n of partitions on such a set. An excellent survey is given by Gian-Carlo Rota, The number of partitions of a set, MAA Monthly 71 (1964) 498-504. Also a good one by Moser + Wyman, Trans. Roy. Soc. Canada, Sec III 49 (1955) 49-54. Recursively: $B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_n$. So $g(5) = B_5 = 52$, $B_6 = 203$, $B_7 = 877$, etc.

Louis Comtet, Recouvrements, bases de filtre et topologies d'un ensemble fini, C.R. Acad. Sci. Paris Sér A-B 262 (1966) A1091-A1094, M.R. 34 # 1209 shows the number of topologies $f(n)$ on a set of n points obeys the asymptotic law: $\log \log f(n) = 2 \log(n)$. Between this and the formula $\sum_{n=0}^{\infty} \frac{B_n}{n!} t^n = e^{(e^t - 1)}$ my conjecture $\lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$ is easy to prove.

My apologies for taking so long to answer, but we have just returned to Winnipeg after visiting some of my wife's relatives in Basking Ridge, New Jersey.

Yours,
Marlon Rayburn