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K-Theory for Operator Algebras

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to Martha

PREFACE

K-Theory has revolutionized the study of operator algebras in the last few years. As the primary component of the subject of "noncommutative topology," K-theory has opened vast new vistas within the structure theory of C*-algebras, as well as leading to profound and unexpected applications of operator algebras to problems in geometry and topology. As a result, many topologists and operator algebraists have feverishly begun trying to learn each others' subjects, and it appears certain that these two branches of mathematics have become deeply and permanently intertwined.

Despite the fact that the whole subject is only about a decade old, operator K-theory has now reached a state of relative stability. While there will undoubtedly be many more revolutionary developments and applications in the future, it appears the basic theory has more or less reached a "final form." But because of the newness of the theory, there has so far been no comprehensive treatment of the subject.

It is the ambitious goal of these notes to fill this gap. We will develop the K-theory of Banach algebras, the theory of extensions of C*-algebras, and the operator K-theory of Kasparov from scratch to its most advanced aspects. We will not treat applications in detail; however, we will outline the most striking of the applications to date in a section at the end, as well as mentioning others at suitable points in the text.

There is little in these notes which is new. They represent mainly a consolidation and integration of previous work. I have borrowed freely from the ideas and writings of others, and I hope I have been sufficiently conscientious in acknowledging the sources of my presentation within the text and in the notes at the end of sections. There are some places where I have presented new arguments or points of view to (hopefully) make the exposition cleaner or more complete.

These notes are an expanded and refined version of the lecture notes from a course I gave at the Mathematisches Institut, Universität Tübingen, West Germany, while on sabbatical leave during the 1982-83 academic year. I taught the course in an effort to learn the material of the later sections. I am grateful to the participants in the course, who provided an enthusiastic and critical audience: A. Kumjian, B. Kümmerer, M. Mathieu, R. Nagel, W. Schröder, J. Vazquez, M. Wolff, and L. Zsido; and to all the others in Tübingen who made my stay pleasant and worthwhile. I am also grateful to the Alexander von Humboldt-Stiftung for their financial support through a Forschungsstipendium.

I have benefited greatly from numerous lectures and discussions at the Mathematical Sciences Research Institute, Berkeley, during the 1984-85 academic year. I am particularly indebted to P. Baum, L. Brown, A. Connes, J. Cuntz, R. Douglas, N. Higson, J. Kaminker, C. Phillips, M. Rieffel, J. Rosenberg, and C. Schochet for sharing their knowledge and insights.

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I am also grateful to Ed Wishart, Jana Dunn, Bill Rainey, Mark Schank, and Ron Sheen for patiently helping me master the UNIX† system and managing to keep the UNR system operating long enough to produce the manuscript.

Since these notes are primarily written for specialists in operator algebras, we will assume familiarity with the rudiments of the theory of Banach algebras and C*-algebras, such as can be found in the first part of **[Dx 1]**, **[Pd 1]**, or **[Tk]**. Some of the sections, particularly later in the book, require more detailed knowledge of certain aspects of C*-algebra theory.

Most of the notation we use will be standard, and will be explained as needed. Some basic notation used throughout: **N, Z, Q, R, C** will denote the natural numbers, integers, and rational, real, and complex numbers respectively; $\mathbf{M}_n$ will denote the $n \times n$ matrices over **C**; **H** will denote a Hilbert space, separable and infinite-dimensional unless otherwise specified; and **B(H)** and **K(H)**, or often just **B** and **K**, will respectively denote the bounded operators and compact operators on **H**. $diag(x_1,...,x_n)$ will denote the diagonal matrix with diagonal elements $x_1, \ldots, x_n$. If A and B are C*-algebras, $A \otimes B$ will always denote the minimal (spatial) C*-tensor product of A and B.

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CONTENTS