

# Revisiting Optimal Power Control: Dual Effects of SNR and Contention

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**Abstract**—In this paper we study a transmission power-tune/control problem in the context of 802.11 Wireless Local Area Networks (WLANs) with multiple (and possibly densely deployed) access points (APs). Previous studies on power control tend to focus on one aspect of the control, either its effect on transmission capacity (PHY layer) assuming simultaneous transmissions, or its effect on contention order (MAC layer) by maximizing spatial reuse. We observe that power control has a *dual effect*: it affects both spatial reuse and capacity of active transmission; moreover, maximizing the two separately is not always aligned in maximizing system throughput and can even point in opposite directions. In this paper we introduce an optimization formulation that takes into account this dual effect, by measuring the impact of transmit power on system performance from both PHY and MAC layers. We show that such an optimization problem is intractable and develop an analytical framework to construct simple yet efficient solutions. Through numerical results, we observe clear benefits of this dual-effect model compared to solutions by trying to maximize spatial reuse and transmission capacity separately. This problem does not invoke cross-layer design, as the only degree of freedom in design resides with transmission power. It however highlights the complexity in tuning certain design parameters, as the change may manifest itself differently at different layers which may be at odds.

## I. INTRODUCTION

Power-tune has emerged as an important issue in an IEEE 802.11 WLAN network of multiple interacting users (Access Points, or APs). Earlier results in this area may be classified into the following two independent approaches.

The first relies on a PHY-layer framework in interference-bounded networks, i.e., the optimal power-tune problem is defined with respect to the Signal-Noise-Ratio (SNR) of each AP or the entire network. For example, Chiang et al. looked into the transmit power control problem through management of interference, energy and connectivity in [2]. In [5], Phan et al. investigate distributed power control problem on physical layer; a distributed algorithm is given and critical performance criteria, such as convergence are analyzed. In [11], Tan et al. analyze several multi-user spectrum management problems with focus on power control. Within this framework, each AP's transmission power has two contradicting roles: The first is that a higher transmission power will improve the noise resistance capability for its own communication and thus potentially the network capacity. The other role is the unavoidable interaction

with other APs. A higher transmission power will contribute higher noise/interference to other APs using the same channel (we assume Orthogonal Frequency Division Multiplexing, OFDM, at PHY layer and thus we will not consider inter-channel interference). Many results have been established in this framework with different techniques focusing on either centralized or distributed solutions.

The second class of results stems from MAC layer techniques by trying to reduce the level of contention within a network, or improving *spatial reuse*, as more generally referred to. For instance in [10], Wan et al. consider a power control problem w.r.t. reducing contention order on the link layer while keeping the physical layer interference under certain levels. Kim et al. investigated a transmit power and carrier sensing threshold tuning problem for improving spatial reuse in [3]. Specifically, when users fall into each other's audible range, transmission back-off under CSMA/CA is triggered to resolve contention and enable sharing. Therefore, decreasing users' transmission range helps improve spatial reuse of a given channel. It follows that they are often modeled as congestion games or other similar graph problems. Specifically Sharma et al. proposed a game theoretical approach for decentralized power allocation in [6]. In [9], a congestion game model is proposed to analyze power control problem as a form of resource allocation. Equilibrium strategies have been given under certain assumptions. In [12], a power control problem is modeled as repeated games with strategic users and intervention theory is proposed to induce target strategy from users.

Even though conceptually both frameworks aim at optimizing the same system performance, e.g., overall throughput, the technical objectives and thus the net impact under the two are clearly not always aligned, and in fact can be quite different and even point in opposite directions. To illustrate, consider maximizing users' achievable throughput or capacity without considering the induced spatial contention relationship; the resulting power-tune can create areas of very high contention order. Thus, even though a user's (or the network's) transmission capacity/rate may be maximized on a *per transmission basis*, significant amount of air time may be spent in the back-off process instead of data transmission, leading to wasted spectrum resources. The opposite may also be true. If we simply control the contention topology of the network, the transmission

power settings may be such that users do not have sufficient noise resistance capability and thus fall short of the theoretically achievable capacity. In this case, even though we may have successfully reduced the contention and saved a lot of air time, the quality of active transmissions (or on a per-transmission basis) may be low.

In short, reducing transmission power has a dual effect on the MAC and PHY layers: it can help increase spatial reuse order under CSMA/CA, but can at the same time decrease noise resistance capacity and therefore the transmission capacity. A desirable solution should thus take both effects into consideration in determining the optimal power control. This is strictly speaking not a cross-layer problem, as the only degree of freedom in design resides with transmission power, i.e., there is no joint design or feedback between different layers. This problem simply highlights the complexity in tuning certain design parameters, as the change may manifest itself differently at different layers which could be self-defeating as illustrated above.

In this study we approach this problem by introducing a performance measure (or utility function) based on the power-tune impact on both PHY and MAC layers simultaneously. An interesting technical aspect of this formulation is the combination of both continuous (SNR and PHY) and discrete (MAC or graph-based) elements in a single optimization problem. Not surprisingly, this problem is intractable; its properties and structures we then investigate to help construct efficient solution techniques. Extensive simulation is conducted to verify the effectiveness and performance of our solution approach. An equally important aspect of our study, besides solving the above optimization problem, is to obtain insight into how the resulting power-tune differs from the two approaches outlined earlier, each focusing on the effect on a single layer, respectively.

The remainder of the paper is organized as follows. Section II gives the system model and problem formulation, while Section III characterizes the optimal solution. Section V provides extensive numerical results to evaluate our approach. Section VI concludes the paper.

## II. SYSTEM MODEL

### A. Preliminaries

Consider a WLAN network with  $N$  APs denoted by the set  $\Omega = \{1, 2, \dots, N\}$ . Each AP is associated with a number of stations with whom it communicates. Denote an AP's transmission power space (i.e., the set of power levels it may employ) by  $\mathcal{Q}_i, i \in \Omega$ . Different from many prior works, here we do *not* assume any finiteness of  $\mathcal{Q}_i$ ; instead, we will show that the finiteness of the *optimal* power profile follows naturally from our formulation. We will assume  $\mathcal{Q}_i, i \in \Omega$  are all compact (closed and bounded) and use  $\bar{\mathcal{P}}_i$  and  $\underline{\mathcal{P}}_i$  to denote the maximum and minimum value in  $\mathcal{Q}_i$ , respectively. The transmission power profile of all users is denoted by  $\mathcal{P} = [\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_N]$ .

Each AP also has a certain attempt rate for channel access under IEEE 802.11, and these are denoted by the vector

$\mathbf{p} := [p_1, p_2, \dots, p_N]$ , also referred to as the attempt rate profile. Channel gain (or path loss) from user  $i$  to  $j$  is denoted by  $h_{ij}$ . We will assume  $h_{ij}, i, j \in \Omega$  stay unchanged during a single transmission; alternatively, we may view  $h_{ij}$  as the expectation of channel dynamics.  $\mathcal{N}_0$  denotes the average noise level, and  $\mathcal{P}_{cs}^i$  denotes the carrier sensing (CS) threshold of the  $i$ -th AP.

For the rest of the paper we will use the terms *AP* and *user* interchangeably.

### B. Contention domain

Due to the fact that many hardware/circuits put a requirement on CS signal's strength, some CS signals cannot be correctly decoded and the corresponding back-off actions will not be triggered; only those with strength higher than the CS threshold can be correctly identified. We thus define two notions of a *contention domain* for user/AP  $i$ . The first one  $\Delta_i^r$ , the receive contention domain, is the set of users/APs whose CS signals can be correctly decoded by user  $i$ ; while the other  $\Delta_i^t$ , the transmit contention domain, is the set of users who can decode user  $i$ 's CS signals correctly. Mathematically we have

$$\Delta_i^r = \{j : \mathcal{P}_j \cdot h_{ji} \geq \mathcal{P}_{cs}^i, j \in \Omega \setminus \{i\}\} \quad (1)$$

$$\Delta_i^t = \{j : \mathcal{P}_i \cdot h_{ij} \geq \mathcal{P}_{cs}^j, j \in \Omega \setminus \{i\}\} \quad (2)$$

By definition, contention domain is closely related to *spatial reuse*. With a larger contention domain, the degree of spatial reuse is potentially smaller around that user. Define  $n_i(\mathcal{P})$  to be the number of competing users of user  $i$  under power profile  $\mathcal{P}$ ; i.e.,  $n_i(\mathcal{P}) := |\Delta_i^r|$ . This will also be referred to as the contention order for user  $i$ .

### C. Neighbor-reaching threshold

Consider AP  $i$  and the maximum (resp. minimum) transmission power it can use without reaching (resp. so that it can still reach) another AP  $k$ , expressed as follows (assumed to exist):

$$\mathcal{P}_{ik}^- := \max\{\mathcal{P}_i : \mathcal{P}_i \cdot h_{ik} < \mathcal{P}_{cs}^k, \mathcal{P}_i \in \mathcal{Q}_i\} \quad (3)$$

$$\mathcal{P}_{ik}^+ := \min\{\mathcal{P}_i : \mathcal{P}_i \cdot h_{ik} \geq \mathcal{P}_{cs}^k, \mathcal{P}_i \in \mathcal{Q}_i\}. \quad (4)$$

To make it complete for  $k = i$  we have

$$\mathcal{P}_{ii}^- := \underline{\mathcal{P}}_i, \mathcal{P}_{ii}^+ := \bar{\mathcal{P}}_i \quad (5)$$

With a real WLAN, these values exist in general; when they do not exist (in the form of inf or sup) we can instead use approximated values around the limits.

Denote the set of these neighbor-reaching thresholds for AP  $i$  as

$$\tilde{\mathcal{Q}}_i = \cup_{k \in \Omega} \{\mathcal{P}_{ik}^-, \mathcal{P}_{ik}^+\}. \quad (6)$$

and denote the neighbor-reaching profile space for the whole network as

$$\tilde{\mathcal{Q}} := \tilde{\mathcal{Q}}_1 \times \tilde{\mathcal{Q}}_2 \times \dots \times \tilde{\mathcal{Q}}_N. \quad (7)$$

Since we are considering a finite size network (i.e., the number of APs,  $N$ , is a finite positive integer), this profile space is finite, i.e.,  $|\tilde{\mathcal{Q}}_i| < \infty, \forall i \in \Omega$ , and consequently  $|\tilde{\mathcal{Q}}| < \infty$ .

#### D. A performance measure/utility function

From AP  $i$ 's point of view, its transmission power setting has the following implications:

- I. Higher transmission power will increase AP  $i$ 's received SNR ( $\text{SNR}_i$ ) by its associated stations.
- II. Higher transmission power will cause higher interference to APs outside  $\Delta_i^r \cup \Delta_i^t$ .
- III. Higher transmission power will add  $i$  to some other AP  $j$ 's contention domain.

As a result, AP  $i$ 's perceived performance, or utility  $\mathcal{U}_i$ , as a function of the transmission power profile  $\mathcal{P}$  and attempt rate profile  $\mathbf{p}$  across all APs, may be captured by the following expression:

$$\mathcal{U}_i(\mathcal{P}, \mathbf{p}) = \mathcal{S}_i(\mathcal{P}, \mathbf{p}) \cdot \mathcal{C}_i(\mathcal{P}, \mathbf{p}), \quad i \in \Omega, \quad (8)$$

where  $\mathcal{S}_i$  is the ‘‘sharing’’ function representing AP  $i$ 's share of channel access under CSMA/CA-type of collision avoidance mechanism, and  $\mathcal{C}_i$  is the ‘‘capacity’’ function representing the rate/quality of active transmissions under  $\mathcal{P}$  and  $\mathbf{p}$ .

Under 802.11, we can approximate  $\mathcal{S}_i$  using the probability of successful channel reservation given by  $\prod_{j \in \Delta_i^r} (1 - p_j) p_i$ , where the dependence on the transmission power profile  $\mathcal{P}$  is implicit through the contention domain  $\Delta_i^r = \Delta_i^r(\mathcal{P})$ . Therefore, we have

$$\mathcal{S}_i(\mathcal{P}, \mathbf{p}) = \prod_{j \in \Delta_i^r} (1 - p_j) p_i \quad (9)$$

which approximates the air time share of AP  $i$  within its contention domain.

$\mathcal{C}_i$  is intended to capture the rate or capacity of active transmission attained by AP  $i$ . To make this concrete, we will focus on the downlink capacity from the AP to its associated stations. As the stations' locations can change dynamically and are often unknown, we will not explicitly model this level of detail and simply assume that the stations are sufficiently close to their associated AP, relative to the other APs. Consequently, their capacity may be approximated using the transmit power by the AP (rather than the received power at a station) and the interference at the AP (rather than at a station), in the standard Shannon formula: (similar formulation can also be found in [8]):

$$\mathcal{C}_i(\mathcal{P}, \mathbf{p}) = \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} \mathcal{S}_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}}\right), \quad (10)$$

where  $\bar{\Delta}_i^r$  ( $\bar{\Delta}_i^t$ ) denotes the complement of AP  $i$ 's contention domains, i.e.,  $\bar{\Delta}_i^r = \Omega_{-i} \setminus \Delta_i^r$ , reflecting the fact that the interference comes primarily from APs outside the contention domain as a result of the back-off mechanism of IEEE 802.11 collision avoidance.

### III. THE OPTIMAL POWER-TUNE PROBLEM AND ITS CHARACTERIZATION

In this section, we formally define our optimization problem and do so in comparison with its single-layer counterparts, i.e., that aim at only the PHY or MAC layer effect, respectively.

#### A. Considering only PHY layer effects

When we limit our attention to PHY layer effects of power control, typically no contention is considered and parallel transmissions are implicitly assumed. Therefore each single user's transmissions will contribute to other users' noise/interference. Problems along this line have been well investigated in the literature, see e.g., [2], [8]. Specifically these assumptions result in the following optimization problem

$$\begin{aligned} \text{(PPHY)} \quad & \max \quad \sum_{i \in \Omega} \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \neq i} \mathcal{P}_j \cdot h_{ji}}\right) \\ & \text{s.t.} \quad \mathcal{P}_j \in \mathcal{Q}_i, \forall i \in \Omega. \end{aligned}$$

This rate maximization problem is in general hard to solve. Previous work has focused on different approximation techniques, see e.g., [8]. In order to obtain comparable results in order to compare this with our optimization formulation, in our numerical experiments (Section V) we shall use the following simple approximation

$$\log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \neq i} \mathcal{P}_j \cdot h_{ji}}\right) \approx \log\left(\frac{\mathcal{P}_i}{\mathcal{N}_0}\right) - \frac{\sum_{j \neq i} \mathcal{P}_j \cdot h_{ji}}{\mathcal{N}_0} \quad (11)$$

Since both terms  $\log(\frac{\mathcal{P}_i}{\mathcal{N}_0})$  and  $-\frac{\sum_{j \neq i} \mathcal{P}_j \cdot h_{ji}}{\mathcal{N}_0}$  are concave, we now have an approximate/relaxed optimization problem which is convex:

$$\begin{aligned} \text{(R-PPHY)} \quad & \max \quad \sum_{i \in \Omega} \left\{ \log\left(\frac{\mathcal{P}_i}{\mathcal{N}_0}\right) - \frac{\sum_{j \neq i} \mathcal{P}_j \cdot h_{ji}}{\mathcal{N}_0} \right\} \\ & \text{s.t.} \quad \mathcal{P}_j \in \mathcal{Q}_i, \forall i \in \Omega. \end{aligned}$$

This problem can be efficiently solved using classical convex optimization techniques (assuming all  $\mathcal{Q}_i$  are convex or piecewise convex). These are used in our numerical results provided later, and the algorithmic details are omitted for brevity.

#### B. Considering only MAC layer effects

We next limit our attention to MAC layer effects of power control, in which case the objective is typically to minimize the sum of contention orders over all users, given as  $\min \sum_{i \in \Omega} n_i(\mathcal{P})$ , see, e.g., a similar objective used in [10]. However, without any constraint on  $\mathcal{P}$ , the above minimization could lead to somewhat pathologic solutions, i.e., with very low power, we can obtain  $\sum_{i \in \Omega} n_i(\mathcal{P}) \approx 0$  thereby achieving the minimum value. However, with very low transmission powers and relatively constant background noise, each AP's SNR is significantly impacted leading to poor transmission performance. Consequently, in order to make this model comparable with others considered in our numerical experiments, we will instead consider a similar optimization problem with an SNR constraint. Specifically, we will try to minimize the

total contention order within the network, subject to a minimum requirement on each AP's SNR, as shown below:

$$\begin{aligned} \text{(PMAC)} \quad & \min \sum_{i \in \Omega} n_i(\mathcal{P}) \\ \text{s.t.} \quad & \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \Delta_i^r \cap \Delta_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}} \geq \text{SNR}_0, \forall i \in \Omega. \end{aligned}$$

Here we use  $\text{SNR}_0$  to denote some baseline SNR that needs to be met at each AP's transmission.

The above problem has a mixture of a combinatorial term and continuous term in the following sense: while the SNR is continuous w.r.t. setting transmission power  $\mathcal{P}_i$ , the contention domains  $\Delta_i$ s are discrete. Thus the problem is hard to solve in general. We thus consider a relaxation of the above problem. Since we have

$$\mathcal{P}_j \cdot h_{ji} < \mathcal{P}_{cs}^i, \forall j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t, \quad (12)$$

the inequality below holds immediately

$$\sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji} < \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_{cs}^i \quad (13)$$

Moreover, we have

$$\frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} \mathcal{P}_j \cdot h_{ji}} > \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_{cs}^i}. \quad (14)$$

Thus, we have the following relaxed problem that are solvable:

$$\begin{aligned} \text{(R-PMAC)} \quad & \min \sum_{i \in \Omega} n_i(\mathcal{P}) \\ \text{s.t.} \quad & \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_{cs}^i} \geq \text{SNR}_0, \forall i \in \Omega. \end{aligned}$$

**Theorem III.1.** *In solving R-PMAC there is no loss of optimality to limit our attention to the space  $\tilde{\mathcal{Q}}$ , i.e., if an optimal solution exists, we can always find an optimal solution within the space  $\tilde{\mathcal{Q}}$ .*

*Proof.* Suppose there is an optimal power profile with some element, say  $\mathcal{P}_j^* \notin \tilde{\mathcal{Q}}$ . Then consider relaxing/increasing  $\mathcal{P}_j^*$  to the nearest  $\mathcal{P}_{jk}^-$ ,  $\forall k \in \Omega$ . Note that such a change would not modify the contention topology and thus all  $n_i$  values and  $\Delta_i^{r(t)}$  remain the same, without violating the corresponding SNR constraint. Thus we have found an optimal solution within  $\tilde{\mathcal{Q}}$ .  $\square$

### C. Considering dual effects

We now formalize the optimal power-tune problem outlined earlier that takes into account both the PHY and MAC layer effects. Specifically, we will seek centralized solutions to the following social welfare maximization problem:

$$\begin{aligned} \text{(P)} \quad & \max \quad \mathcal{U} = \sum_{i \in \Omega} \mathcal{U}_i(\mathcal{P}, \mathbf{p}) \\ \text{s.t.} \quad & \mathcal{P}_i \in \mathcal{Q}_i, i \in \Omega. \end{aligned}$$

As the power profile space  $\mathcal{Q} := \mathcal{Q}_1 \times \mathcal{Q}_2 \times \dots \times \mathcal{Q}_N$  is potentially infinite, and  $\mathcal{U}$  is in general a non-convex and non-differentiable function w.r.t.  $\mathcal{P}$ , the optimization problem is NP-hard. To illustrate, Fig. 1 shows three examples of the sum utility as a function of the power  $\mathcal{P}_i$  of AP  $i$ , under

different parameter settings. As can be seen, there lack properties commonly used to derive efficient solution techniques (e.g., differentiability, convexity). There are, however, some interesting features such as the prominent step shape shown in the result. This observation motivates key results in subsequent section.

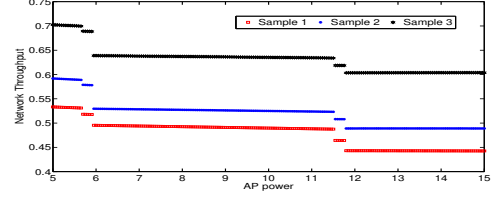


Fig. 1: Examples of  $\mathcal{U}$  w.r.t. a specific  $\mathcal{P}_i$

## IV. SOLUTION APPROACH

### A. A Lower bound problem

Recall that for  $j \in \bar{\Delta}_i^r$ , we have  $\mathcal{P}_j \cdot h_{ji} < \mathcal{P}_{cs}^i$ , i.e., the received signal strength is below the CS threshold. Thus

$$\begin{aligned} & \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}}\right) \\ & > \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_{cs}^i}\right). \quad (15) \end{aligned}$$

We use this relationship to form the following lower bound problem.

$$\begin{aligned} \text{(PL)} \quad & \max \quad \mathcal{U}_L = \sum_{i \in \Omega} S_i(\mathcal{P}, \mathbf{p}) \cdot \\ & \quad \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_{cs}^i}\right) \\ \text{s.t.} \quad & \mathcal{P}_i \in \mathcal{Q}_i, \forall i \in \Omega. \end{aligned}$$

**Lemma IV.1.** *For an optimal solution  $\mathcal{P}^*$  to (PL), we have  $\mathcal{P}^* \in \tilde{\mathcal{Q}}$ . That is, there is no loss of optimality in restricting the solution space to  $\tilde{\mathcal{Q}}$  in searching for an optimal solution.*

*Proof.* For AP  $i$ , suppose there exists a  $\mathcal{P}_i^* \notin \tilde{\mathcal{Q}}$ . This means one of the following must be true: (1)  $\mathcal{P}_{ik}^+ < \mathcal{P}_i^* < \mathcal{P}_{ij}^-$  for some  $(k, j)$ , (2)  $\mathcal{P}_i^* < \mathcal{P}_{ij}^-$  for all  $j$ , and (3)  $\mathcal{P}_{ik}^+ < \mathcal{P}_i^*$  for all  $k$ . For the first two cases, if we increase  $\mathcal{P}_i$  from  $\mathcal{P}_i^*$  to  $\mathcal{P}_{ij}^-$ , the resulting contention topology remain unchanged, i.e., the terms  $S_j, n_j(\mathcal{P}), \Delta_j^{r(t)}, \forall j \in \Omega$  stay unchanged, but  $\mathcal{P}_i$  is now bigger and  $\mathcal{U}_L$  is strictly increasing in  $\mathcal{P}_i$ . This contradicts the optimality of  $\mathcal{P}^*$ , so the first two cases cannot be true. If it's the third case and  $\mathcal{P}_{ik}^+ < \mathcal{P}_i^*$  for all  $k$ , then increasing  $\mathcal{P}_i$  from  $\mathcal{P}_i^*$  to  $\bar{\mathcal{P}}_i$  results in the same argument as above, so (3) also cannot be true, completing the proof.  $\square$

**Lemma IV.2.** *We can bound the performance of (PL) for a large dense network (i.e., with  $N$  being large) as follows*

$$\mathcal{U}^* - \mathcal{U}_L^* \leq \sum_{i \in \Omega} (1 - p_c) \left(\frac{\mathcal{P}_i}{\bar{\mathcal{P}}}\right)^{1/\gamma \cdot N} \frac{\left[1 - \left(\frac{\mathcal{P}_i}{\bar{\mathcal{P}}}\right)^{1/\gamma}\right] \cdot N \cdot \mathcal{P}_{cs}^i}{\mathcal{N}_0} \quad (16)$$

*Proof.* For simplicity of presentation we assume  $p_1 = p_2 = \dots = p_N = p_c$  for the analysis. Since our model applied to any topology, the bounding will become intractable when considering an arbitrary network. In order to give meaningful bounds, we consider a network with APs being placed randomly in a certain area. We first notice according to the wireless transmission model we have

$$n_i(\mathcal{P}_i) = \left(\frac{\mathcal{P}_i}{\bar{\mathcal{P}}}\right)^{1/\gamma} \cdot N \quad (17)$$

We take the difference between  $\mathcal{U}$  and  $\mathcal{U}_L$  for any power profile  $\mathcal{P}$ ,

$$\begin{aligned} & \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}}\right) \\ & - \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_{cs}^i}\right) \\ & \leq \log\left(\frac{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_{cs}^i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}}\right) \\ & \leq \log 2 \cdot \frac{\sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot (\mathcal{P}_{cs}^i - \mathcal{P}_j \cdot h_{ji})}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}} \\ & \leq \log 2 \cdot \frac{\left[1 - \left(\frac{\mathcal{P}_i}{\bar{\mathcal{P}}}\right)^{1/\gamma}\right] \cdot N \cdot \mathcal{P}_{cs}^i}{\mathcal{N}_0} \end{aligned} \quad (18)$$

Therefore we have

$$\begin{aligned} \mathcal{U}^* - \mathcal{U}_L^* & \leq \log 2 \cdot \sum_{i \in \Omega} (1 - p_c) \left(\frac{\mathcal{P}_i^*}{\bar{\mathcal{P}}}\right)^{1/\gamma} \cdot N \cdot \\ & \quad \frac{\left[1 - \left(\frac{\mathcal{P}_i^*}{\bar{\mathcal{P}}}\right)^{1/\gamma}\right] \cdot N \cdot \mathcal{P}_{cs}^i}{\mathcal{N}_0} \end{aligned} \quad (19)$$

□

**Remark IV.3.** When  $\mathcal{P}_{cs}^i$  is small or when the network is dense and large (i.e., with a large  $N$ ), the lower bound problem will provide a good approximation for the original problem.

### B. An Upper bound problem

We similarly form an upper bound to the original objective function:

$$\begin{aligned} & \log\left(1 + \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}}\right) \\ & \leq \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}} \cdot \log 2, \end{aligned} \quad (20)$$

and we have the following upper bound problem

$$\begin{aligned} \text{(PU)} \quad \max \quad & \mathcal{U}_U = \sum_{i \in \Omega} S_i(\mathcal{P}, \mathbf{p}) \\ & \cdot \frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}} \\ \text{s.t.} \quad & \mathcal{P}_i \in \mathcal{Q}_i, \forall i \in \Omega. \end{aligned}$$

**Lemma IV.4.**  $\mathcal{U}_U$  is piece-wise convex w.r.t. each  $\mathcal{P}_i, i \in \Omega$ .

*Proof.* Consider  $\mathcal{P}_i$  and fix the transmission power of all other APs. Suppose  $\mathcal{P}_i \in [\mathcal{P}_{ik}^+, \mathcal{P}_{ij}^-]$  for some  $k, j$ . Within this range,

the contention topology remains the same, i.e.,  $S_i(\mathcal{P}, \mathbf{p})$  is a constant for any value  $\mathcal{P}_i$  taking within this interval. Next consider the second term in  $\mathcal{U}_U$ .  $\mathcal{P}_i$  appears in this term in two forms: one as  $\frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{l \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_l(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_l \cdot h_{li}}$  which is convex w.r.t.  $\mathcal{P}_i$ , and the other as  $\frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{m \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_m(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_m \cdot h_{mi}}$  for some  $l$  such that  $i \in \bar{\Delta}_l^r \cap \bar{\Delta}_l^t$ , in the form of  $S_i(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_i \cdot h_{il}$ ; these terms are also convex w.r.t.  $\mathcal{P}_i$ . As the sum of convex functions is convex, we have established the convexity of  $\mathcal{U}_U$ .

When  $\mathcal{P}_{ik}^+ < \mathcal{P}_i$  for all  $k$ ,  $\mathcal{U}_U$  is convex w.r.t.  $\mathcal{P}_i$  over the interval  $[\max \mathcal{P}_{ik}^+, \bar{\mathcal{P}}_i]$  using the same argument as above. Similarly, when  $\mathcal{P}_i < \mathcal{P}_{ik}^-, \forall k \neq i$   $\mathcal{U}_U$  is convex w.r.t.  $\mathcal{P}_i$  over the interval  $[\mathcal{P}_i, \min \mathcal{P}_{ik}^-]$ . □

**Lemma IV.5.** Suppose  $\mathcal{P}_U^*$  is the optimal solution to (PU), we have  $\mathcal{P}_U^* \in \tilde{\mathcal{Q}}$ .

*Proof.* The result readily follows from Lemma IV.4 and the fact that optimal solution over a closed interval for a convex function is an end point. □

**Lemma IV.6.** We can bound the performance of (PU) for a large dense network (i.e., with  $N$  being large) as follows

$$\mathcal{U}^* - \mathcal{U}_U^* \leq \sum_{i \in \Omega} (1 - p_c) \left(\frac{\mathcal{P}_i^*}{\bar{\mathcal{P}}}\right)^{1/\gamma} \cdot N \cdot \frac{\mathcal{P}_i^*}{\mathcal{N}_0} \log\left(1 + \frac{\mathcal{P}_i^*}{\mathcal{N}_0}\right) \quad (21)$$

*Proof.* We follow similar assumptions as made in the proof for our lower bound problem. In our upper bound since we know

$$x - \ln(1 + x) \leq x \cdot \ln(1 + x) \quad (22)$$

and the R.H.S. is increasing ( $x \geq 0$ ) we know we only need to bound  $x$ . A trivial bound comes useful in our case

$$\frac{\mathcal{P}_i}{\mathcal{N}_0 + \sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}} \leq \frac{\mathcal{P}_i}{\mathcal{N}_0} \quad (23)$$

and therefore, for each term in  $\mathcal{U}_U$ , the difference is bounded by

$$(1 - p_c) \left(\frac{\mathcal{P}_i^*}{\bar{\mathcal{P}}}\right)^{1/\gamma} \cdot N \cdot \frac{\mathcal{P}_i^*}{\mathcal{N}_0} \log\left(1 + \frac{\mathcal{P}_i^*}{\mathcal{N}_0}\right) \quad (24)$$

□

**Remark IV.7.** The bound can be certainly made tighter by working more on the term  $\sum_{j \in \bar{\Delta}_i^r \cap \bar{\Delta}_i^t} S_j(\mathcal{P}, \mathbf{p}) \cdot \mathcal{P}_j \cdot h_{ji}$ . However dues to space limitation we omit the details.

**Remark IV.8.** By finding the optimal solution for problems (PL) and (PU), we have the bounds for the optimal solutions.

$$\mathcal{U}_L(\mathcal{P}_L^*) \leq \mathcal{U}(\mathcal{P}^*) \leq \mathcal{U}_U(\mathcal{P}_U^*) \quad (25)$$

Meanwhile we can use  $\mathcal{P}_L^*$  and  $\mathcal{P}_U^*$  as approximate strategies for our original problem (P) with

$$\mathcal{U}(\mathcal{P}_L^*) \geq \mathcal{U}_L(\mathcal{P}_L^*), \quad \mathcal{U}(\mathcal{P}_U^*) \leq \mathcal{U}_U(\mathcal{P}_U^*) \quad (26)$$

In next section we will focus on solving (PL) and (PU) instead of (P).

### C. Greedy search

In solving **(PL)** and **(PU)** instead of the original **(P)**, the problem reduces to searching over a finite strategy space which can be done within a finite number of steps dependent on the size of the network. For the lower bound problem **(PL)**, the strategy space is on the order of  $\mathcal{O}(N^N)$ , while for the upper bound problem **(PU)** the order is  $\mathcal{O}((2N-1)^N)$ . However with a large scale WLAN network (referring to the number of APs in the network), these could still be excessively large even though finite. This is the classical *rollout* problem in combinatorial optimization. In terms of computation, for standard integer optimization (or combinatorial optimization) problems researchers typically seek relaxation to convert the problem into a continuous problem in the hope it can be solved by standard LP or convex algorithms; in [4], [7], [14], efficient search algorithms have been proposed to tackle finite space optimization problems.

Following previous literatures, a heuristic greedy approach can be easily deployed, which is shown later through numerical experiment to provide a near-optimal solution efficiently. The basic idea of a greedy search method is to maximize the system's total throughput w.r.t. a single variable at each stage of the computation while keeping the others fixed.

### D. Optimal search

In this part, we present a randomized search algorithm that guarantees convergence to the optimal solution for **(PL)** and **(PU)**. The algorithm works in rounds starting from AP 1 and computes the power for one AP in each round. Denote the state of the system at round  $n$  as  $\mathcal{G}(n)$  and  $\mathcal{G}(n) := [\mathcal{P}_1(n), \mathcal{P}_2(n), \dots, \mathcal{P}_N(n)]$ . Suppose at round  $n$  AP  $i$ 's (i.e.,  $i = n \bmod N$ ) power is being computed. Then AP  $i$ 's next power level is updated using the following transition probability:

$$\begin{aligned} \mathbb{P}(\mathcal{G}(n+1) = (\mathcal{P}^i, \mathcal{G}_{-i}(n)) | \mathcal{G}(n)) \\ = \frac{e^{\mathcal{U}(\mathcal{P}^i, \mathcal{G}_{-i}(n))/\tau(n)}}{e^{\mathcal{U}(\mathcal{P}^i, \mathcal{G}_{-i}(n))/\tau(n)} + e^{\mathcal{U}(\mathcal{G}(n))/\tau(n)}} \cdot \frac{1}{\mathcal{L}_i}, \\ \forall \mathcal{P}^i \in \tilde{\mathcal{Q}}_i, \mathcal{P}^i \neq \mathcal{P}_i(n). \end{aligned} \quad (27)$$

with the probability of not changing the power level given as

$$\begin{aligned} \mathbb{P}(\mathcal{G}(n+1) = \mathcal{G}(n) | \mathcal{G}(n)) \\ = \sum_{\mathcal{P}^i \in \tilde{\mathcal{Q}}_i, \mathcal{P}^i \neq \mathcal{P}_i(n)} \frac{e^{\mathcal{U}(\mathcal{G}(n))/\tau(n)}}{e^{\mathcal{U}(\mathcal{P}^i, \mathcal{G}_{-i}(n))/\tau(n)} + e^{\mathcal{U}(\mathcal{G}(n))/\tau(n)}} \cdot \frac{1}{\mathcal{L}_i}. \end{aligned} \quad (28)$$

Here  $\tau(n) := \frac{1}{n}$  is a positive smoothing factor and  $\mathcal{L}_i$  is a normalization factor for user  $i$ . This search algorithm will be referred to as **P\_RANDOM**. As before, the function  $\mathcal{U}()$  in the above equation is the objective in problem **(PL)** (resp. **(PU)**) if the algorithm is used to search for an optimal solution to problem **(PL)** (resp. **(PU)**).

**Theorem IV.9.** ***P\_RANDOM** converges to the optimal solution to the two approximate problems **(PL)** and **(PU)**.*

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Set  $\mathcal{P}_i(0) = \bar{\mathcal{P}}_i, \forall i \in \Omega$ .
temp =  $\mathcal{U}(\mathcal{P}(0))$ ,  $\epsilon = \text{temp}$ ,  $n = 0$ .
thrs =  $10^{-4}$ . (some small positive value)
while  $\epsilon > \text{thrs}$  do
   $n := n + 1$ ;
   $m := n \bmod N$ ;
  Set  $\mathcal{P}_m(n) = \mathcal{P}^m$  with probability
   $\frac{e^{\mathcal{U}(\mathcal{P}^m, \mathcal{G}_{-m}(n))/\tau(n)}}{e^{\mathcal{U}(\mathcal{P}^m, \mathcal{G}_{-m}(n))/\tau(n)} + e^{\mathcal{U}(\mathcal{G}(n))/\tau(n)}} \cdot \frac{1}{\mathcal{L}_m}, \forall \mathcal{P}^m \in \tilde{\mathcal{Q}}_m, \mathcal{P}^m \neq \mathcal{P}_m(n)$ .
  While  $\mathcal{P}_m(n)$  stays unchanged with probability
   $\frac{e^{\mathcal{U}(\mathcal{G}(n))/\tau(n)}}{e^{\mathcal{U}(\mathcal{P}^m, \mathcal{G}_{-m}(n))/\tau(n)} + e^{\mathcal{U}(\mathcal{G}(n))/\tau(n)}} \cdot \frac{1}{\mathcal{L}_m}$ .
  for  $j = 1 : N$  do
    if  $j \neq m$ 
       $\mathcal{P}_j(n) = \mathcal{P}_j(n-1)$ ;
  end for
   $\epsilon = |\mathcal{U}(\mathcal{P}(n)) - \text{temp}|$ ;
  temp =  $\mathcal{U}(\mathcal{P}(n))$ ;
end
    
```

---

### Algorithm 1: Pseudocode for Randomized Search

*Proof.* Due to the finiteness of the strategy spaces of all APs, we can form an  $N$ -dimensional positive recurrent Markov chain, with state at time  $n$  given by  $\mathcal{G}(n)$ ; there exists a stationary distribution of this Markov chain.

We next show that the following probability distribution  $\pi$  over the state space  $\tilde{\mathcal{Q}}$  is the stationary distribution of this Markov Chain.

$$\pi(\mathcal{G}) = T \cdot e^{\mathcal{U}(\mathcal{G})/\tau(n)}, \forall \mathcal{G} \in \tilde{\mathcal{Q}}, \quad (29)$$

where  $T$  is the normalization constant. As  $\sum_{\mathcal{G} \in \tilde{\mathcal{Q}}} \pi(\mathcal{G}) = 1$  the above is equivalent to  $T = \sum_{\mathcal{G} \in \tilde{\mathcal{Q}}} e^{\mathcal{U}(\mathcal{G})/\tau(n)}$ , and thus

$$\pi(\mathcal{G}) = \frac{e^{\mathcal{U}(\mathcal{G})/\tau(n)}}{\sum_{\mathcal{G} \in \tilde{\mathcal{Q}}} e^{\mathcal{U}(\mathcal{G})/\tau(n)}}, \forall \mathcal{G} \in \tilde{\mathcal{Q}}, \quad (30)$$

To prove this, consider the detailed balance equations. Specifically consider two states  $\mathcal{G}_1$  and  $\mathcal{G}_2$ . Note that, only when  $\mathcal{G}_1$  and  $\mathcal{G}_2$  differ in one element, the transition probability is positive; otherwise the transition probability is zero. We would like to show the following

$$\pi(\mathcal{G}_1) \cdot \mathbb{P}(\mathcal{G}_2 | \mathcal{G}_1) = \pi(\mathcal{G}_2) \cdot \mathbb{P}(\mathcal{G}_1 | \mathcal{G}_2), \forall \mathcal{G}_1, \mathcal{G}_2. \quad (31)$$

When there is only one element difference between the two states we know

$$\mathbb{P}(\mathcal{G}_2 | \mathcal{G}_1) = \mathcal{L} \cdot \frac{e^{\mathcal{U}(\mathcal{G}_2/\tau(n))}}{e^{\mathcal{U}(\mathcal{G}_2/\tau(n))} + e^{\mathcal{U}(\mathcal{G}_1/\tau(n))}} \quad (32)$$

$$\mathbb{P}(\mathcal{G}_1 | \mathcal{G}_2) = \mathcal{L} \cdot \frac{e^{\mathcal{U}(\mathcal{G}_1/\tau(n))}}{e^{\mathcal{U}(\mathcal{G}_1/\tau(n))} + e^{\mathcal{U}(\mathcal{G}_2/\tau(n))}} \quad (33)$$

with  $\mathcal{L}$  being some constant. Therefore (31) holds readily. It follows from standard result that  $\pi$  is indeed the stationary distribution.

Denote  $\mathcal{G}^* = \{\mathcal{P}^*\}$  as the set of global maximizers; i.e., the set of power profiles that maximizes our objectives in **(PL)** and

(PU) respectively. Suppose there is a state  $\mathcal{G}' \notin \mathcal{G}^*$ . We have

$$\pi(\mathcal{G}') = \frac{e^{\mathcal{U}(\mathcal{G}')/\tau(n)}}{\sum_{\mathcal{G} \in \tilde{\mathcal{Q}}} e^{\mathcal{U}(\mathcal{G})/\tau(n)}} = \frac{1}{\sum_{\mathcal{G} \in \tilde{\mathcal{Q}}} e^{(\mathcal{U}(\mathcal{G}) - \mathcal{U}(\mathcal{G}')/\tau(n))}} \quad (34)$$

For  $\mathcal{G} \in \mathcal{G}^*$  we have  $\mathcal{U}(\mathcal{G}) - \mathcal{U}(\mathcal{G}') > 0$  and

$$\lim_{n \rightarrow \infty} (\mathcal{U}(\mathcal{G}) - \mathcal{U}(\mathcal{G}')/\tau(n)) \rightarrow \infty \quad (35)$$

and hence we have  $\lim_{n \rightarrow \infty} \pi(\mathcal{G}') = 0$ ; furthermore by [1] we establish the following

$$\lim_{n \rightarrow \infty} \pi(\mathcal{G} \in \mathcal{G}^*) = 1 \quad (36)$$

i.e., the Markovian chain converges to the maximization states with probability 1.  $\square$

**Theorem IV.10.** *P\_RANDOM converges exponentially fast.*

*Proof.* Consider at time  $n$ , for any sub-optimal state  $\mathcal{G}$  we know there exists a positive constant  $\epsilon > 0$  such that

$$U^* \geq \mathcal{U}^* - \mathcal{U}(\mathcal{G}) \geq \epsilon \quad (37)$$

Therefore the probability of staying at state  $\mathcal{G}$  at time  $n$  is bounded by

$$\begin{aligned} \pi(\mathcal{G}) &= \frac{e^{\mathcal{U}(\mathcal{G}')/\tau(n)}}{\sum_{\mathcal{G} \in \tilde{\mathcal{Q}}} e^{\mathcal{U}(\mathcal{G})/\tau(n)}} \\ &\leq \frac{1}{\sum_{\mathcal{G} \in \tilde{\mathcal{Q}}} e^{(\mathcal{U}(\mathcal{G}) - \mathcal{U}(\mathcal{G}')/\tau(n))}} \leq e^{-\epsilon/\tau(n)} \end{aligned} \quad (38)$$

Therefore

$$U^* - E[\mathcal{U}(n)] \leq U^* \cdot e^{-\epsilon/\tau(n)} \quad (39)$$

Take  $\tau(n) = 1/n$  we have  $U^* - E[\mathcal{U}(n)] \leq U^* \cdot e^{-\epsilon n}$ .  $\square$

## V. NUMERICAL EXPERIMENTS

In this section, we provide simulation results to show system performance under the greedy and randomized search algorithms (denoted as ‘‘Greedy’’ and ‘‘P\_RANDOM’’ in the figures, respectively). We further compare them with the maximum transmission power strategy (‘‘Max’’), PPHY and PMAC respectively. The WLAN network’s topology used in the experiment is randomly generated, with 10 APs placed according to a uniform distribution in a square area; this topology is shown in Fig.2.

### A. Optimization with dual effects

We begin by comparing the computed power levels and the resulting system-wide throughput under the greedy and randomized search algorithms and the fixed, maximum power scheme.

For the first set of results, we fix  $p_c = 0.6$  and a maximum transmission power level of 15 for all APs. The resulting optimal power profile is depicted in Fig.3a. Here to get the ‘‘optimal solution’’ we utilize P\_RANDOM to solve (PL) and (PU) separately and then choose the one that gives us a better total

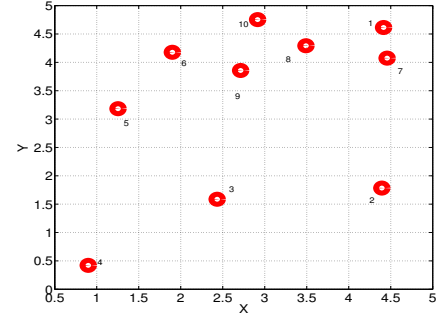


Fig. 2: Topology of the AP Network

throughput. We see that in this case APs (8, 9, 10)’s power levels are far short of the maximum level. This reflects the need to avoid excessive interference with each other as they are clustered in a relatively crowded neighborhood. APs (2, 4) are sitting relatively ‘‘alone’’ and thus they could transmit at a higher power. Similar observations can be made at each AP.

Next, the system performance is shown in Fig. 3b as a function of the attempt rate  $p_c$ . It is interesting to note the opposite trends exhibited by using optimal power tuning vs. always using maximum power levels as the attempt rate increases. As the network gets busier (more congested with higher attempt rate), the maximum power levels exacerbates the problem and the system throughput degrades even though the APs are trying harder. On the other hand, using optimal power-tune, as the network becomes more congested, the APs react by decreasing their transmission powers appropriately so that the system throughput actually improves. By either the greedy search or the randomized search algorithm, our optimal power-tune problem helps achieve a significant throughput performance improvement compared to the static maximum transmission power scheme.

We end this part with a look into the convergence performance of P\_RANDOM, shown in Fig.3c. It is seen that our randomized search algorithm converges quickly to the end solution; under the same simulation setting, the greedy policy converges to a solution of system throughput at around 1.6.

### B. Compare with PPHY and PMAC

We next compare our optimization model with dual effect to the model given by PPHY and PMAC, which tries to maximize the total rate at the physical layer and contention respectively. We present the total system throughput performance in Fig. 3d. We see PPHY and PMAC are clearly out-performed by our optimization model especially under higher attempt rate.

Moreover from our experiments we observe a clear difference in how power levels are tuned and the resulting throughput across different AP nodes. The reason is that under PPHY each AP treats all other APs as noise resources. However, due to CSMA/CA, no parallel transmission would be allowed for APs within the carrier sensing range and thus the first-order noises (those from the closest neighbors) could be removed.

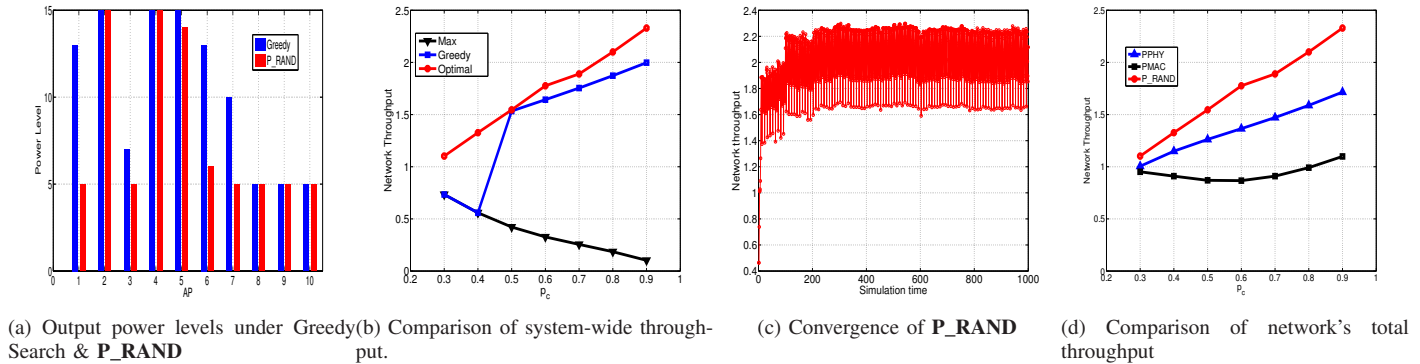


Fig. 3: Experiment results

Therefore APs could increase their power to some extent without contributing too much to their neighbors' noise level. This is why we observe a few APs with much higher power under **P\_RAND** than under **PPHY**. By contrast, with only PHY layer optimization APs cannot take full advantages of the noise-free property of CSMA, and therefore act conservatively.

On the other hand, we see that with **PMAC** in order to reduce the contention order the APs again act rather conservatively in reducing their power levels. This leads to a drop in noise resistance and the overall network throughput.

## VI. CONCLUSION

With the proliferation of densely-deployed WLANs, power tuning becomes a critical problem as it has major impacts on SNR as well as contention levels in these networks. Prior works mostly focused on one of these two issues in pursuit of either higher throughput or lower contention level, but not both. Our formulation and solution are given within a centralized framework. A natural next step is to examine distributed implementations of the solution, and similar optimization problems when users are strategic. These remain interesting directions of future research, but are out of the scope of the present paper.

In this paper, we have investigated the network throughput optimization problem by optimizing both spatial reuse (MAC) and SNR (PHY) performance at the same time. We have presented the complexity of solving the joint optimization problem and derived approximations to make it tractable. Then, by analyzing the problem structure, we have proposed efficient and near-optimal solutions. In order to demonstrate the effectiveness of our approach, we compared our results with several models optimizing on only either PHY or MAC layer. A clear advantage has been demonstrated for the dual efforts approach.

## ACKNOWLEDGMENT

This work was partly performed while the first author interned with Juniper Networks, Inc. We would like to extend our appreciations to David Aragon, Joe Williams, and many others for the inspiring discussions and helpful comments.

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