

## GRAPHS WHOSE $A_\alpha$ -SPECTRAL RADIUS DOES NOT EXCEED 2

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*This paper is dedicated to the memory of our excellent colleague*  
*Slobodan K. Simić who recently passed away.*

### Abstract

Let  $A(G)$  and  $D(G)$  be the adjacency matrix and the degree matrix of a graph  $G$ , respectively. For any real  $\alpha \in [0, 1]$ , we consider  $A_\alpha(G) = \alpha D(G) + (1 - \alpha)A(G)$  as a graph matrix, whose largest eigenvalue is called the  $A_\alpha$ -spectral radius of  $G$ . We first show that the smallest limit point for the  $A_\alpha$ -spectral radius of graphs is 2, and then we characterize the connected graphs whose  $A_\alpha$ -spectral radius is at most 2. Finally, we show that all such graphs, with four exceptions, are determined by their  $A_\alpha$ -spectra.

**Keywords:**  $A_\alpha$ -matrix, Smith graphs, limit point, spectral radius, index.

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## REFERENCES

- [1] F. Belardo, E.M. Li Marzi, S.K. Simić and J.F. Wang, *Graphs whose signless Laplacian spectral radius does not exceed the Hoffman limit value*, Linear Algebra Appl. **435** (2011) 2913–2920.  
doi:10.1016/j.laa.2011.05.006
- [2] A.E. Brouwer and A. Neumaier, *The graphs with spectral radius between 2 and  $\sqrt{2 + \sqrt{5}}$* , Linear Algebra Appl. **114/115** (1989) 273–276.  
doi:10.1016/0024-3795(89)90466-7
- [3] S.M. Cioabă, E.R. van Dam, J.H. Koolen and J.-H. Lee, *Asymptotic results on the spectral radius and the diameter of graphs*, Linear Algebra Appl. **432** (2010) 722–737.  
doi:10.1016/j.laa.2009.09.016
- [4] D.M. Cvetković, M. Doob and I. Gutman, *On graphs whose spectral radius does not exceed  $(2 + \sqrt{5})^{1/2}$* , Ars Combin. **14** (1982) 225–239.
- [5] E.R. van Dam and W.H. Haemers, *Which graphs are determined by their spectrum?*, Linear Algebra Appl. **373** (2003) 241–272.  
doi:10.1016/S0024-3795(03)00483-X
- [6] E.R. van Dam and W.H. Haemers, *Developments on spectral characterizations of graphs*, Discrete Math. **309** (2009) 576–586.  
doi:10.1016/j.disc.2008.08.019
- [7] N. Ghareghani, G.R. Omid and B. Tayfeh-Rezaie, *Spectral characterization of graphs with index at most  $\sqrt{2 + \sqrt{5}}$* , Linear Algebra Appl. **420** (2007) 483–489.  
doi:10.1016/j.laa.2006.08.009
- [8] K. Guo and B. Mohar, *Digraphs with Hermitian spectral radius below 2 and their cospectrality with paths*, Discrete Appl. **340** (2017) 2616–2632.  
doi:10.1016/j.disc.2017.01.018
- [9] A.J. Hoffman, *On limit points on spectral radii of non-negative symmetric integral matrices*, in: Graph Theory and Applications, Y. Alavi, D.R. Lick, A.T. White (Ed(s)), Lecture Notes in Math. **303** (Springer-Verlag, Berlin, 1972) 165–172.
- [10] A.J. Hoffman and J.H. Smith, *On the spectral radii of topologically equivalent graphs*, in: Recent Advances in Graph Theory, M. Fiedler (Ed(s)), (Academia Praha, Prague, 1975) 273–281.
- [11] H.Q. Lin, X. Huang and J. Xue, *A note on the  $A_\alpha$ -spectral radius of graphs*, Linear Algebra Appl. **557** (2018) 430–437.  
doi:10.1016/j.laa.2018.08.008
- [12] H.Q. Lin, X.G. Liu and J. Liu, *Graphs determined by their  $A_\alpha$ -spectra*, Discrete Math. **342** (2019) 441–450.  
doi:10.1016/j.disc.2018.10.006
- [13] H.Q. Lin, J. Xue and J.L. Shu, *On the  $A_\alpha$ -spectra of graphs*, Linear Algebra Appl. **556** (2018) 210–219.  
doi:10.1016/j.laa.2018.07.003

- [14] X.G. Liu and S.Y. Liu, *On the  $A_\alpha$ -characteristic polynomial of a graph*, Linear Algebra Appl. **546** (2018) 274–288.  
doi:10.1016/j.laa.2018.02.014
- [15] L. Lu and S. Man, *Connected hypergraphs with small spectral radius*, Linear Algebra Appl. **509** (2016) 206–227.  
doi:10.1016/j.laa.2016.07.013
- [16] V. Nikiforov, *Merging the  $A$ - and  $Q$ -spectral theories*, Appl. Anal. Discrete Math. **11** (2017) 81–107.  
doi:10.2298/AADM1701081N
- [17] V. Nikiforov, G. Pastén, O. Rojo and R.L. Soto, *On the  $A_\alpha$ -spectra of trees*, Linear Algebra Appl. **520** (2017) 286–305.  
doi:10.1016/j.laa.2017.01.029
- [18] V. Nikiforov and O. Rojo, *A note on the positive semidefiniteness of  $A_\alpha(G)$* , Linear Algebra Appl. **519** (2017) 156–163.  
doi:10.1016/j.laa.2016.12.042
- [19] J.H. Smith, *Some properties of the spectrum of a graph*, in: Combinatorial Structures and their Applications, R. Guy, H. Hanani, N. Sauer, J. Schonheim (Ed(s)), (Gordon and Breach, New York, 1970) 403–406.
- [20] J.F. Wang and F. Belardo, *Spectral characterizations of graphs with small spectral radius*, Linear Algebra Appl. **437** (2012) 2408–2416.  
doi:10.1016/j.laa.2012.06.028
- [21] J.F. Wang, F. Belardo, Q.X. Huang and E.M. Li Marzi, *On graphs whose Laplacian index does not exceed 4.5*, Linear Algebra Appl. **438** (2013) 1541–1550.  
doi:10.1016/j.laa.2011.02.043
- [22] J.F. Wang, Q.X. Huang, F. Belardo and E.M. Li Marzi, *On graphs whose signless Laplacian index does not exceed 4.5*, Linear Algebra Appl. **431** (2009) 162–178.  
doi:10.1016/j.laa.2009.02.017
- [23] J.F. Wang, Q.X. Huang, X. An and F. Belardo, *Some notes on graphs whose spectral radius is close to  $\frac{3}{2}\sqrt{2}$* , Linear Algebra Appl. **429** (2008) 1606–1618.  
doi:10.1016/j.laa.2008.04.034
- [24] R. Woo and A. Neumaier, *On graphs whose spectral radius is bounded by  $\frac{3}{2}\sqrt{2}$* , Graphs Combin. **23** (2007) 713–726.  
doi:10.1007/s00373-007-0745-9

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