

SPECTRA OF ORDERS FOR k -REGULAR GRAPHS OF GIRTH g

ROBERT JAJCAY

*Department of Algebra and Geometry,
Comenius University, Bratislava
Mlynská dolina, 842 48 Bratislava, Slovakia*
e-mail: robert.jajcay@fmph.uniba.sk

AND

TOM RAIMAN

*Department of Applied Mathematics
VŠB – Technical University of Ostrava
17. listopadu 15/2172, 708 33 Ostrava-Poruba, Czech Republic*
e-mail: tom.raiman.st@vsb.cz

Abstract

A (k, g) -graph is a k -regular graph of girth g . Given $k \geq 2$ and $g \geq 3$, infinitely many (k, g) -graphs of infinitely many orders are known to exist. Our goal, for given k and g , is the classification of all orders n for which a (k, g) -graph of order n exists; we choose to call the set of all such orders the spectrum of orders of (k, g) -graphs. The smallest of these orders (the first element in the spectrum) is the order of a (k, g) -cage; the (k, g) -graph of the smallest possible order. The exact value of this order is unknown for the majority of parameters (k, g) . We determine the spectra of orders for $(2, g)$, $g \geq 3$, $(k, 3)$, $k \geq 2$, and $(3, 5)$ -graphs, as well as the spectra of orders of some families of $(k, 4)$ -graphs. In addition, we present methods for obtaining (k, g) -graphs that are larger than the smallest known (k, g) -graphs, but are smaller than (k, g) -graphs obtained by Sauer. Our constructions start from (k, g) -graphs that satisfy specific conditions derived in this paper and result in graphs of orders larger than the original graphs by one or two vertices. We present theorems describing ways to obtain ‘starter graphs’ whose orders fall in the gap between the well-known Moore bound and the constructive bound derived by Sauer and are the first members of an infinite sequence of graphs whose orders cover all admissible orders larger than those of the ‘starter graphs’.

Keywords: cage, k -regular graph, girth, Sauer bound.

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