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ON A TOTAL VERSION OF 1-2-3 CONJECTURE[1]

Olivier Baudon[1], Hervé Hocquard[1]

Antoni Marczyk[2], Monika Pilśniak[2]

Jakub Przybyło[2] and Mariusz Woźniak[2]

[1] *Université de Bordeaux and CNRS*
LaBRI, UMR5800
F-33400 Talence, France

[2] *Faculty Applied Mathematics*
AGH University of Science and Technology
Al. Mickiewicza 30, 30–059 Kraków, Poland

e-mail: baudon@labri.fr
hocquard@labri.fr
marczyk@agh.edu.pl
pilsniak@agh.edu.p
jakubprz@agh.edu.pl
mwozniak@agh.edu.pl

Abstract

A total k-coloring of a graph G is a coloring of vertices and edges of G using colors of the set $\{1, \ldots, k\}$. These colors can be used to distinguish adjacent vertices of G. There are many possibilities of such a distinction. In this paper, we focus on the one by the *full sum of colors* of a vertex, i.e., the sum of the color of the vertex, the colors on its incident edges and the colors on its adjacent vertices.

This way of distinguishing vertices has similar properties to the method when we only use incident edge colors and to the corresponding 1-2-3 Conjecture.

Keywords: neighbor sum distinguishing total coloring, general edge coloring, total coloring, neighbor-distinguishing index, neighbor full sum distinguishing total k-coloring.

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