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ON THE TOTAL k-DOMINATION IN GRAPHS

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Abstract

Let $G = (V, E)$ be a graph; a set $S \subseteq V$ is a total k-dominating set if every vertex $v \in V$ has at least k neighbors in S. The total k-domination number $\gamma_{kt}(G)$ is the minimum cardinality among all total k-dominating sets. In this paper we obtain several tight bounds for the total k-domination number of a graph. In particular, we investigate the relationship between the total k-domination number of a graph and the order, the size, the girth, the minimum and maximum degree, the diameter, and other domination parameters of the graph.

Keywords: k-domination, total k-domination, k-tuple domination, k-tuple total domination.

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