

RAINBOW VERTEX-CONNECTION AND FORBIDDEN SUBGRAPHS

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Abstract

A path in a vertex-colored graph is called *vertex-rainbow* if its internal vertices have pairwise distinct colors. A vertex-colored graph G is *rainbow vertex-connected* if for any two distinct vertices of G , there is a vertex-rainbow path connecting them. For a connected graph G , the *rainbow vertex-connection number* of G , denoted by $rvc(G)$, is defined as the minimum number of colors that are required to make G rainbow vertex-connected. In this paper, we find all the families $\mathcal{F}$ of connected graphs with $|\mathcal{F}| \in \{1, 2\}$, for which there is a constant $k_{\mathcal{F}}$ such that, for every connected $\mathcal{F}$ -free graph G , $rvc(G) \leq \text{diam}(G) + k_{\mathcal{F}}$, where $\text{diam}(G)$ is the diameter of G .

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