Discussiones Mathematicae
Graph Theory 36 (2016) 643–659
doi:10.7151/dmgt.1882

TRIANGLE DECOMPOSITIONS OF PLANAR GRAPHS

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Abstract

A multigraph G is triangle decomposable if its edge set can be partitioned into subsets, each of which induces a triangle of G, and rationally triangle decomposable if its triangles can be assigned rational weights such that for each edge e of G, the sum of the weights of the triangles that contain e equals 1.

We present a necessary and sufficient condition for a planar multigraph to be triangle decomposable. We also show that if a simple planar graph is rationally triangle decomposable, then it has such a decomposition using only weights $0, 1$ and $\frac{1}{2}$. This result provides a characterization of rationally triangle decomposable simple planar graphs. Finally, if G is a multigraph with K_4 as underlying graph, we give necessary and sufficient conditions on the multiplicities of its edges for G to be triangle and rationally triangle decomposable.

Keywords: planar graphs, triangle decompositions, rational triangle decompositions.

2010 Mathematics Subject Classification: 05C10, 05C70.

[1]Supported by a Discovery Grant from the Natural Sciences and Engineering Research Council of Canada.

[2]Supported by a Julie Payette Research Scholarship and an André Hamer Postgraduate Prize from the Natural Sciences and Engineering Research Council of Canada.

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Received 15 April 2015
Revised 6 October 2015
Accepted 6 October 2015