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A CHARACTERIZATION OF 2-TREE PROBE INTERVAL GRAPHS

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Abstract

A graph is a *probe interval graph* if its vertices correspond to some set of intervals of the real line and can be partitioned into sets P and N so that vertices are adjacent if and only if their corresponding intervals intersect and at least one belongs to P. We characterize the 2-trees which are probe interval graphs and extend a list of forbidden induced subgraphs for such graphs created by Pržulj and Corneil in [2-*tree probe interval graphs have a large obstruction set*, Discrete Appl. Math. **150** (2005) 216–231].

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