Lattice Based Forward-Secure Identity Based Encryption Scheme with Shorter Ciphertext*

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Abstract

In MIST 2012 conference Singh et al [21] presented lattice based forward-secure identity based encryption schemes based on LWE assumption in the random oracle model as well as in the standard model. In this paper we propose lattice based forward-secure identity based encryption scheme with shorter ciphertext in the random oracle model. We have reduced size of the ciphertext C from $(m(i+2)+1)\times\|Z_q\|$ to $(m+1)\times\|Z_q\|$ where $\|Z_q\|$ is number of bits required to represent an element of Z_q.

Keywords: Lattice, Identity Based Encryption, Forward Security, Random Oracle Model, Learning With Error (LWE).

1 Introduction

In Crypto 84 conference Adi Shamir introduced the concept of identity-based cryptosystem [20]. Identity-based cryptosystem is a public key cryptosystem in which users' public key can be any arbitrary string. For example email or phone number can be public key. Private key coresponding to this arbitrary string are issued by Trusted Third Party (TTP) who knows some master secret key of the system. As a result, it significantly reduces system complexity and cost of establishing public key infrastructure. Although Shamir constructed an identity-based signature scheme using RSA function but he could not construct an identity based encryption and this became a long-lasting open problem. Only in 2001, Shamir's open problem was independently solved by Boneh and Franklin [8] and Cocks [12].
In 1994, Peter Shor showed that prime factorization and discrete logarithms problem can be solved in polynomial time on a quantum computer. In other words, once quantum computer comes into reality all of the public-key algorithms used to protect the Internet will be broken. It facilitated research on new cryptosystems that are secure from quantum computers, collectively called post-quantum cryptography. Lattice based problems are conjectured to remain secure in the advent of quantum computers. So the lattice based cryptography is one of the candidate for post quantum cryptography. Lattice based cryptography are also attractive due to their worst case hardness assumption. Recently Regev [19] defined

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the learning with errors (LWE) problem and proved that it enjoys similar worst-case hardness properties under a quantum reduction. A number of constructions of lattice identity based encryption is known [15, 10, 18, 1, 3].

Protecting secret keys is crucial for cryptography. There are some relatively insecure devices (smart cards, mobile phones etc.) which have threat of key exposure. The goal of the forward security is to protect security of past uses of key even if the current secret key is exposed. The notion of forward secrecy was first proposed by G*ü*nther [11] in 1989 and later by Diffie et al [13] in 1992 in the contexts of key exchange protocol. A key exchange protocol is said to provide forward secrecy if compromise of long term secret keys does not compromise the secrecy of the previously generated exchange keys, which can be converted as forward secure interactive public key encryption scheme. The notion of non-interactive forward security was proposed by Anderson [5] in 1997 and later formalized by Bellare and Miner [6]. In non-interactive forward security the lifetime of the system is divided into N time interval labled $0,1,...,N-1$. The device initially stores the secret key SK_0. After that device at beginning of interval i computes secret key SK_i at interval i using update algorithm $(SK_{i-1},..,)$ and then delete secret key SK_{i-1} at interval $i-1$. A forward secure encryption scheme guarantees that exposer of secret key at interval i will not compromise on the security of system for any prior time interval. But system can not prevent the adversary from breaking the security of system for any time interval greater than i. Forward secure encryption scheme in symmetric setting was proposed by Bellare and Yee [7]. The construction of forward secure encryption scheme in public key setting was a open problem since the question was first raised by Anderson [5]. Only in 2003, Anderson's open problem was solved by Canetti et al [9]. In this paper Canetti et al [9] constructed Binary Tree Encryption (BTE) based on billinear Diffi-Hellman assumption. They have also proposed a method to convert forward secure PKE scheme from any BTE scheme. Lu and Li [17] proposed efficient forward secure PKE scheme in standard model. Two forward secure encryption scheme in identity based setting have been proposed so far [16, 22].

Chris Peikert [18] proposed lattice based BTE scheme based on Learning With Error (LWE) assumption. Using Canetti et al [9], it can be converted into lattice based forward PKE scheme.

Our Contribution: In 2010 Eurocrypt conference Cash et al [10] and Peikert [18] presented basis delegation technique. This technique allows short basis (trapdoor) of a given lattice to derive short basis (trapdoor) of related lattice in secure way. Based on this technique Singh et al [21] constructed first lattice based forward secure encryption scheme in the random oracle model as well as in the standard model. In the above basis delegation technique the dimension of the short basis of related lattice is larger than the dimension of the parent lattice. Because of this, size of the ciphertext increases. In 2010 Crypto conference, Agrawal et al [2] proposed new basis delegation technique in which the size of short basis is constant. Based on this new technique we propose lattice based forward secure identity based encryption scheme with shorter ciphertext. We have reduced size of the ciphertext C from $(m(i+2)+1)\times\|Z_q\|$ to $(m+1)\times\|Z_q\|$ where $\|Z_q\|$ is the number of bits required to represent an element of Z_q.

Paper Outline: Our paper is organized as follows. In section 2, we describe basic definitions, security models, results and hard problems required to understand rest of the paper. Since our scheme is very similar to Singh et al [21] scheme so in section 3, we describe lattice based forward-secure identity based encryption scheme proposed by Singh et al [21]. In section 4, we describe our scheme. In section 5 we give conclusion and related open problems.

2 Preliminaries

2.1 Notation

We denote $[j] = \{0, 1, ..., j\}$ and $A_{id,[j]} = [A, A_{id,0}, A_{id,1}, ..., A_{id,j}] \in Z_q^{n\times(j+2)m}$ where $A_{id,i} \in Z_q^{n\times m}$. $\|S\|$ denotes the L_2 length of the longest vector in S, i.e. $\|S\| := max_i|s_i|$ for $1 \leq i \leq k$.
We say that *negl(n)* is a negligible function in n if it is smaller than the inverse of any polynomial function in n for sufficiently large n.
Gram Schmidt Orthogonalization: $\widetilde{S} := \{\widetilde{s_1}, ..., \widetilde{s_k}\} \subset R^m$ denotes the Gram-Schmidt orthogonalization of the set of linearly independently vectors $S = \{s_1, ..., s_k\} \subset R^m$. It is defined as follows: $\widetilde{s_1} = s_1$ and $\widetilde{s_i}$ is the component of s_i orthogonal to span$(s_1, ..., s_i)$ where $2 \leq i \leq k$. Since $\widetilde{s_i}$ is the component of s_i so $\|\widetilde{s_i}\| \leq \|s_i\|$ for all i.

2.2 Forward Secure IBE

Here definition of IBE is similar to [9, 16]. Forward secure IBE consists of five algorithms.

Setup(n, N): On input a security parameter n, outputs the master public key mpk and master secret key msk.

Extract(mpk,msk,id): On input master public key mpk, a master secret key msk, and an identity id $\in \{0,1\}^*$ outputs private key corresponding to an identity id.

Update(mpk,SK$_{id||i}$,$id||i$): On input master public key mpk, secret key SK$_{id||i}$ at i^{th} time period outputs secret key SK$_{id||(i+1)}$ at $(i+1)^{th}$ time period.

Encrypt($mpk, id||i$,M): On input master public key mpk, $id||i$ and a message M outputs ciphertext C.

Decrypt(C,mpk,SK$_{(id||i)}$): On input master public key mpk, a private key $SK_{id||i}$, and a ciphertext C outputs message M.

2.3 Selective-ID Security Model for Forward-Secure IBE

Security model is adapted from [9]. We define selective-ID security model using a game that the challenge ciphertext is indistinguishable from a random element in the ciphertext space. This property implies both semantic security and recipient anonymity. The game proceeds as follows.

Init: The adversary submits a target identity id^*.

Setup: The challenger runs Setup($1^n, N$) and gives the master public parameters (mpk) to adversary and keeps master secret key (msk) to itself.

Query Phase:

- Hash query: The adversary can issue hash query for any identity id. Adversary can repeat this polynomial number of times for different identities adaptively.

- Extraction query: The adversary can issue a query for a private key $SK_{id||0}$ corresponding to identity $id||0$. Adversary can repeat this polynomial number of times for different identities ($id \neq id^*$) adaptively.

Attack: The adversary issues one breakin(j) query and challenge(i,b) query, in either order, subject to $0 \leq i < j \leq N$. These queries are answered as follows:

- On query breakin(j), secret key $SK_{id||j}$ is computed and return to the adversary.
- On query challenge(i,b), the challenger picks a random bit $r \in \{0,1\}$ and a random ciphertext C. If $r = 0$ it sets the challenge ciphertext to $C^* = \text{Encrypt}(mpk, id^*||i, b)$. If $r = 1$ it sets the challenge ciphertext to $C^* = C$. It returns C^* as challenge to the adversary.

Guess: The adversary outputs a guess $r' \in \{0,1\}$, it succeeds if $r' = r$.
We refer an adversary A as an IND-sID-CPA adversary. We define the advantage of the adversary A in attacking fs-IBE scheme ξ as $Adv_{\xi,A}(n) = |Pr[r = r'] - 1/2|$.

Definition 1. We say that forward-secure IBE scheme ξ is selective-ID, indistinguishable from random if for all IND-sID-CPA PPT adversaries A we have $Adv_{\xi,A}(n)$ is a negligible function.

2.4 Integer Lattices

A lattice is defined as the set of all integer combinations

$$L(b_1, ..., b_n) = \left\{ \sum_{i=1}^{n} x_i b_i : x_i \in Z \text{ for } 1 \leq i \leq n \right\}$$

of n linearly independent vectors $b_1, ..., b_n \in R^n$. The set of vectors $\{b_1, ..., b_n\}$ is called a basis for the lattice. A basis can be represented by the matrix $B = [b_1, ..., b_n] \in R^{n \times n}$ having the basis vectors as columns. Using matrix notation, the lattice generated by a matrix $B \in R^{n \times n}$ can be defined as $L(B) = \{Bx : x \in Z^n\}$, where Bx is the usual matrix-vector multiplication. The determinant of a lattice is the absolute value of the determinant of the basis matrix $det(L(B)) = |det(B)|$.

Definition 2. For q prime, $A \in Z_q^{n \times m}$ and $u \in Z_q^n$, define:

$$\Lambda_q(A) := \{e \in Z^m \ \ s.t. \ \exists s \in Z_q^n \ where \ A^T s = e \ (mod \ q)\}$$
$$\Lambda_q^{\perp}(A) := \{e \in Z^m \ \ s.t. \ \ Ae = 0 \ (mod \ q)\}$$
$$\Lambda_q^{u}(A) := \{e \in Z^m \ \ s.t. \ \ Ae = u \ (mod \ q)\}$$

2.5 The Gram-Schmidt Norm of a Basis

Let S be a set of vectors S = $\{s_1, ..., s_k\}$ in R^m. We use the following notation:

- $|S|$ denotes the L_2 length of the longest vector in S, i.e. $\|S\| := max_i |s_i|$ for $1 \leq i \leq k$.
- $\widetilde{S} := \{\widetilde{s_1}, ..., \widetilde{s_k}\} \subset R^m$ denotes the Gram-Schmidt orthogonalization of the vector $s_1, ..., s_k$ taken in that order.

We refer to $\widetilde{\|S\|}$ as the Gram-Schmidt norm of S.

2.6 Discrete Gaussians

Let L be a subset of Z^m. For any vector $c \in R^m$ and any positive parameter $\sigma \in R > 0$, define:
$\rho_{\sigma,c}(x) = exp(-\pi \frac{\|x-c\|}{\sigma^2})$: a Gaussian-shaped function on R^m with center c and parameter σ,
$\rho_{\sigma,c}(L) = \sum_{x \in L} \rho_{\sigma,c}(x)$: the (always converging) $\rho_{\sigma,c}$ over L,
$D_{L,\sigma,c}$: the discrete Gaussian distribution over L with parameters σ and c,

$$\forall y \in L \ , \ D_{L,\sigma,c} = \frac{\rho_{\sigma,c}(y)}{\rho_{\sigma,c}(L)}$$

The distribution $D_{L,\sigma,c}$ will most often be defined over the Lattice $L = \Lambda_q^{\perp}$ for a matrix $A \in Z_q^{n \times m}$ or over a coset $L = t + \Lambda_q^{\perp}(A)$ where $t \in Z^m$.

Lemma 1 ([[14],Lemma 7.1]). Let Λ be an m-dimensional lattice. There is a deterministic polynomial-time algorithm ToBasis(S,s) that, given an arbitrary basis of Λ and a full-rank set $S = \{s_1, ..., s_m\}$ in Λ, returns a basis T of Λ satisfying

$$\|\widetilde{T}\| \leq \|\widetilde{S}\| \ \ and \ \ \|T\| \leq \|S\|\sqrt{m}/2$$

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RandBasis(S, s) ([[18]]) Randomized algorithm RandBasis(S, s) takes a basis S of some m-dimensional lattice $\wedge$ and a parameter $s \geq \|\widetilde{S}\|\sqrt{(\log n)}$, and outputs a new basis S of lattice $\wedge$, generated as follows.

1. For $i = 1, ..., m$:
   (a) Choose $v \leftarrow$ SampleBasis(S, s). If v is linearly independent of $\{v_1, ..., v_{i1}\}$, then let $v_i = v$ and go to the next value of i; otherwise, repeat this step.
2. Output $S' =$ ToBasis(V, S)

Lemma 2 ([[18],Lemma 3.3]). With overwhelming probability, $S' \leftarrow$ RandBasis(S, s) repeats Step 1a at most $O(m^2)$ times, and $||\widetilde{S'}|| \leq s\sqrt{m}$. Moreover,for any two bases S_0 , S_1 of the same lattice and any $s \geq ||\widetilde{S_i}||\sqrt{(log\ n)}$ for $i = \{0, 1\}$ RandBasis(S_0, s) and RandBasis(S_1, s) are within negl(n) statistical distance.

Theorem 1 ([[4], Theorem 3.2]). Let $q \geq 3$ be odd and $m := \lceil 6n \log q \rceil$.
There is probabilistic polynomial-time algorithm TrapGen(q, n) that outputs a pair $(A \in Z_q^{n \times m}, S \in Z^{n \times m})$ such that A is statistically close to a uniform matrix in $Z_q^{n \times m}$ and S is a basis for $\Lambda_q^{\perp}(A)$ satisfying

$$\|\widetilde{S}\| \leq O(\sqrt{n \log q}) \ \ and \ \ \|S\| \leq O(n \log q)$$

with all but negligible probability in n.

Theorem 2 ([[10], Lemma 3.3]). Let $A = [A_1, ..., A_k]$, where each $A_i \in Z_q^{n \times m}$. For $S \subseteq [k]$, $S = \{i_1, ..., i_j\}$, we write $A_S = [A_{i_1}, ..., A_{i_j}]$. Let n, q, m, k be positive integers with $q \geq 2$ and $m \geq 2n\ log\ q$. There exists a PPT algorithm ExtendBasis, that on input of $A \in Z_q^{n \times km}$, a set $S \subseteq [k]$, a basis B_S for $\Lambda_q^{\perp}(A_S)$, and an integer $L \geq \|\widetilde{B_S}\|.\sqrt{km}.w(\sqrt{km})$ outputs $B \leftarrow$ ExtendBasis(A, B_S, S, L). With overwhelming probability B is a basis of $\Lambda_q^{\perp}(A)$ with $\|\widetilde{B}\| \leq L$.

2.7 The LWE Hardness Assumption

The LWE (learning with error) hardness assumption is defined by Regev [19].

Definition 3. Consider a prime q, a positive integer n, and a distribution χ over Z_q, typically taken to be normal distribution. The input is a pair (A,v) from an unspecified challenge oracle $\bigcirc$, where $A \in Z_q^{m\times n}$ is chosen uniformly. v is chosen uniformly from Z_q^m or chosen to be $As+e$ for a uniformly chosen $s \in Z_q^n$ and a vector $e \in Z_q^m$. When v is chosen to be $As+e$ for a uniformly chosen $s \in Z_q^n$ and a vector $e \in Z_q^m$ an unspecified challenge oracle $\bigcirc$ is a noisy pseudo-random sampler $\bigcirc_s$. When v is chosen uniformly an unspecified challenge oracle $\bigcirc$ is a truly random sampler $\bigcirc_\$$.
Goal of the adversary is to distinguish with some non-negligible probability between these two cases.
Or we say that an algorithm A decides the (Z_q,n,χ)-LWE problem if $|Pr[A^{\bigcirc_s}=1]-Pr[A^{\bigcirc_\$}=1]|$ is non-negligible for a random $s \in Z_q^n$.

Definition 4. Consider a real parameter $\alpha=\alpha(n) \in \{0,1\}$ and a prime q. Denote by $T=R/Z$ the group of reals [0,1) with addition modulo 1. Denote by ψ_α the distribution over T of a normal variable with mean 0 and standard deviation $\alpha/\sqrt{2\pi}$ then reduced modulo 1. Denote by $\lfloor x \rceil = \lfloor x+\frac{1}{2} \rfloor$ the nearest integer to the real $x \in R$. We denote by $\overline{\psi}_\alpha$ the discrete distribution over Z_q of the random variable $\lfloor qX \rceil \bmod q$ where the random variable $X \in T$ has distribution ψ_α.

Theorem 3 ([19]). If there exists an efficient, possibly quantum algorithm for deciding the $(Z_q,n,\overline{\psi_\alpha})$-LWE problem for $q > 2\sqrt{n}/\alpha$ then there exists an efficient quantum algorithm for approximating the SIVP and GapSVP problems, to within $O(n/\alpha)$ factors in the l_2 norm, in the worst case.

2.8 Basis Delegation with Constant Dimension [2]

We briefly explain Agrawal et al [2]' basis delegation technique. Let $T_L=\{l_1,...,l_m\}$ be a short basis of lattice L in Z^m. Let R be "low norm" non singular matrix in $Z^{m\times m}$. Then $T_{L'}=\{Rl'_1,...,Rl'_m\}$ will be short basis of lattice $L'=RL$.
Let A be a matrix in $Z_q^{n\times m}$ and let T_A be a short basis of $\Lambda_q^\perp(A)$. Now we define $B=AR^{-1}$ in $Z_q^{n\times m}$. Since $AR^{-1}Rx=Ax=0$, so $T_B=RT_A$ will be a short basis of $\Lambda_q^\perp(B)$. Dimension of B is same as dimension of A. It is also required that it should be hard to recover short basis of $\Lambda_q^\perp(A)$ from the short basis of $\Lambda_q^\perp(B)$.
Now we describe new basis delegation algorithm as follows.

NewBasisDel (A,R,T_A,σ)**:** It works as follows.

1. Let $T_A=\{a_1,...,a_m\} \subseteq Z^m$. Calculate $T'_B=\{Ra_1,...,Ra_m\} \subseteq Z^m$.
2. Short basis T'_B is converted into T''_B of $\Lambda_q^\perp(B)$ using lemma 1.
3. Short basis $T_B=RandBasis(T''_B,\sigma)$. Output T_B.

Theorem 4. Suppose R is sampled from $D_{m\times m}$ and σ satisfies

$$\sigma > \|\widetilde{T_A}\|.\sigma_R\sqrt{m}\, w(log^{3/2}\, m).$$

Let T_B be the basis of $\Lambda_q^\perp(AR^{-1})$ output by *BasisDel*. Then T_B is distributed statistically close to the distribution $RandBasis(T,\sigma)$ where T is an arbitrary basis of $\Lambda_q^\perp(AR^{-1})$ satisfying $\|\widetilde{T_A}\| < \sigma/w(\sqrt{log\, m})$.

If R is a product of l matrices sampled from $D_{m\times m}$ then the bound on σ degrades to $\sigma > \|\widetilde{T}_A\|(\sigma_R\sqrt{m}w(log^{1/2}\ m))^l.w(log\ m)$.

3 Lattice Based Forward Secure IBE Schemes

Singh et al [21] recently proposed an lattice based forward secure IBE scheme in random oracle model. We first describe their construction.

3.1 Lattice Based Forward Secure IBE Scheme in Random Oracle Model

Setup(n,N): On input a security parameter n, we set the parameters q,m accordingly. Let $H:\{0,1\}^* \rightarrow Z_q^{n\times m}$ be hash function. Next we do the following.

1. Use algorithm TrapGen(q,n) to generate a matrix $A \in Z_q^{n\times m}$ and a short basis T_A for $\Lambda_q^{\perp}(A)$ such that $\|\widetilde{T_A}\| \leq O(\sqrt{n\log q})$.

2. For each $j \in [N] = \{0,1,2,...,N\}$ and an arbitary identity id we define the associated parity check matrix $A_{id,[j]}$
   $A_{id,[j]} = [A,A_{id,0},A_{id,1},...,A_{id,j}] \in Z_q^{n\times(j+2)m}$, where $A_{id,i} = H(id||i) \in Z_q^{n\times m}$

3. We choose $y \leftarrow Z_q^n$ uniformly.

4. Output the master public key and master secret key,
   mpk = (A,y), msk = T_A.

Extract(mpk,T_A,id): PKG generates the secret key for a user identity $id \in \{0,1\}^*$ by calling the function RandBasis(SampleBasis(A,T,$A_{id,0}$)) (theorem 2 and lemma 2). Output of function $SK_{id||0}$ is the secret key of this user.

Update(mpk,$\mathbf{SK}_{id||i}$,$id||i+1$): Given secret key at i^{th} time period $\mathrm{SK}_{id||i}$ user can find secret key at $i+1^{th}$ time period as follows.
$SK_{id||i+1}$ = RandBasis(ExtBasis($\mathrm{SK}_{id||i}$,$A_{id,[i+1]}$)) by theorem 2 and lemma 2.
Output $\mathrm{SK}_{id||i+1}$.

Encrypt(mpk,$id||i$,b): To encrypt a bit $b \in \{0,1\}$, we do the following.

1. We compute $A_{id,[i]} = [A,A_{id,0},A_{id,1},...,A_{id,i}] \in Z_q^{n\times(i+2)m}$, where $A_{id,i} = H(id||i) \in Z_q^{n\times m}$.

2. we choose $s \leftarrow Z_q^n$ uniformly.

3. Compute $p = A_{id,[i]}^T s + e \in Z_q^{m(i+2)}$, where $e \leftarrow \chi^{m(i+2)}$. Here $\chi^{m(i+2)}$ is error (gaussian) distribution.

4. Compute $c = y^T s + b\lfloor\frac{q}{2}\rfloor + \overline{e}$, where $\overline{e} \leftarrow \chi$. Here χ is error (gaussian) distribution.

5. Output the ciphertext $C = (p,c)$. Size of ciphertext $C = (m(i+2)+1) \times \|Z_q\|$

Decrypt(C,mpk,SK$_{(id||i)}$): To decrypt $C = (p,c)$, we do the following.

1. $s \leftarrow \text{invert}(mpk, \text{SK}_{(id||i)}, p)$.
2. Now we compute $b' = c - y^T s$.
3. If b' is closer to 0 than $\lfloor \frac{q}{2} \rfloor$ mod q output 0 otherwise output 1.

It is required that above forward secure IBE scheme has the correctness property, i.e, for any index $i \in [0,N]$ and secret key $SK_{id||i}$ and any message bit b we have $b = \text{Decryption}(mpk, \text{SK}_{id||i}, \text{Encryption}(mpk, id||i, \text{b}))$.

Theorem 5. If hash function H is modeled as random oracle, then Section 3.1 lattice based forward-secure IBE scheme is IND-sID-CPA (semantic) secure assuming the $LWE_{q,\chi}$ is hard or $Adv_{B,LWE_{q,\chi}}(n) = \frac{1}{N} Adv_{\chi,A}(n)$.

Proof: Here proof is similar to proof of theorem 4.1 of [10].
We now show semantic security of fs-IBE in the random oracle model. We will show that if there exist a PPT adversary A that breaks fs-IBE scheme with non-negligible probability then there must exist a PPT adversary B that solves LWE hard problem by simulating views of A. We assume that

- For each $i \in [N]$, A always makes polynomial number of Q_H different H-queries of interval i.
- Whenever A makes an H-query of interval i, we assume that it has queried H-query of interval $j < i$.
- Whenever A submits a user secret key query, we assume that it has made all relevant H queries beforehand.

Adversary A declares an identity ID^* that it intends to attack. Adversary B (works as challenger for adversary A) first pick $i^* \in [N]$. Here i^* is a guess for the i of challenge (i,b) query. Now B obtains $(i^*+2)(m+1)$ LWE samples i.e. $(u_i, v_i) \in Z_q^n \times Z_q$ $(0 \le i \le (i^*+2)(m+1)-1)$, which get parsed as $A_i = (u_{i\times n+1}, ..., u_{i\times n+n})$.

Setup: Adversary B sets master public key to be $mpk = A = A_0^* = \{u_1, ..., u_m\})$. Next Adversary B simulates the view of A as follows:

- Hash H Queries:
  1. A's hash query on $id^*||0$, adversary B returns $A_1 = \{u_{m+1}, ..., u_{2m}\}$. Similarly on A's hash query on $id^*||1, ..., id^*||i^*$, adversary B returns $A_2, A_3, ..., A_i$ respectively.
  2. For A's hash query on identity $id^*||j$, $(i+1 \le j \le N)$, adversary B runs the trapdoor algorithm TrapGen to generate $A \in Z_q^{n\times m}$ with corresponding trapdoor $T \in Z^{m\times m}$ and stores the tuple $(id^*||j, A_j, T_j)$ in list H. Adversary B returns the matrix A_j as a answer to hash query.
  3. For A's hash query on identity $id \ne id^*$, adversary B runs the trapdoor algorithm TrapGen to generate $A \in Z_q^{n\times m}$ with corresponding trapdoor $T \in Z^{m\times m}$. Adversary B returns matrix A and stores the tuple (id, A, T) in list H.
- Extraction Queries: When adversary A asks for the secret key for the identity $id \ne id^*$. As we have assumed that before extraction query adversary A would have made hash query for it, so adversary B will check the list H and returns the corresponding T to adversary A.

Attack:

- Challenge (i,b): When adversary A queries challenge (i,b), the adversary B picks a random bit $r \in \{0,1\}$ and a random ciphertext C. If $r = 0$ it returns challenge ciphertext to be (p^*, C^*) else it returns random ciphertext C.
- Breakin(j): When adversary A queries breakin(j), if $i < j \leq i^*$ then adversary B outputs a random bit and game abort (since B can not answer extraction queries for $j \leq i^*$). Otherwise adversary B will check the list H and returns the corresponding T_j to adversary A

Now adversary B proceeds as follows:

1. Set
$$c_1^* = \begin{bmatrix} v_1 \\ \vdots \\ v_{m(i+2)} \end{bmatrix} \in Z_q^{m(i+2)}$$
2. Blind the message bit by $c_0^* = v_0 + b^* \lfloor \frac{q}{2} \rceil$.
3. Set $CT^* = (c_0^*, c_1^*)$ and send it to adversary A.

When Oracle $\bigcirc$ is a pseudo-random LWE oracle then $c_0^* = v_0 + b^* \lfloor \frac{q}{2} \rceil = u_0^T + s + x + b^* \lfloor \frac{q}{2} \rceil$ for some $s \in Z_q^n$ and noise x. Similarly
$$c_1^* = \begin{bmatrix} v_1 \\ \vdots \\ v_{m(i+2)} \end{bmatrix} = A_{id^*||[i^*]}\ s + y$$
for some $s \in Z_q^n$ and noise y. So $CT^* = (c_0^*, c_1^*)$ is a valid encryption of b for $id^*||i^*$. When Oracle $\bigcirc$ is a random oracle then v_0, v^* are uniform and therefore $CT^* = (c_0^*, c_1^*)$ is a uniform in $(Z_q \times Z_q^m)$.

Finally adversary A terminates with some output, adversary B terminates with same output and ends the simulation. So if adversary A breaks the scheme then there exist adversary B which solves LWE hard problem.
Since probability that $i = i^*$ is $\frac{1}{N}$, so the probability that B does not abort during simulation is $\frac{1}{N}$. $Adv_{B,LWE_{q,\chi}}(n) = \frac{1}{N} Adv_{\chi,A}(n)$. Hence our scheme is semantic secure.

3.2 Lattice Based Forward Secure IBE Scheme in the Standard Model

Setup(n, N)**:** On input a security parameter n, we set the parameters q, m accordingly. Let $H : \{0,1\}^* \rightarrow \{0,1\}^\lambda$ be real. $H(id||i) = (t_1||t_2||...||t_\lambda)$ where $t_i \in \{0,1\}$. Next we do following.

1. Use algorithm TrapGen(q,n) to generate a matrix $A \in Z_q^{n \times m}$ and a short basis T_A for $\Lambda_q^\perp(A)$ such that $\|\widetilde{T_A}\| \leq O(\sqrt{n \log q})$.
2. For $0 \leq i \leq N,\ 1 \leq u \leq \lambda$ and $b \in \{0,1\}$ sample the matrices $C_{i,u,b} \in Z_q^{n \times m}$ uniformly and independently and also sample $y \in Z_q^n$ uniformly. For each $j \in [N] = \{0,1,2,...,N\}$ and an arbitary identity id we define the associated parity check matrix $A_{id,[j]}$ as
$A_{id,[j]} = [A, A_{id,0}, A_{id,1}, ..., A_{id,j}] \in Z_q^{n \times ((j+1)\lambda+1)m}$,
where $A_{id,j} = [C_{j,1,t_1}, ..., C_{j,\lambda,t_\lambda}] \in Z_q^{n \times \lambda m}$
for $H(id||j) = (t_1||t_2||...t_\lambda) \in \{0,1\}^\lambda$

3. Output the master public key and master secret key,

$$mpk = (A, y, (C_{i,u,b})_{\substack{0\le i\le N \\ 1\le u\le \lambda \\ b\in\{0,1\}}}),\ msk = T_A$$

Extract(mpk,T,id), Update(mpk,$SK_{id||i}, id||i$), Encrypt(mpk,$id||i$,b) and Decrypt(C,mpk,$SK_{(id||i)}$) function is same as Extract, Update, Encrypt and Decrypt function of section 3 lattice based forward-secure IBE scheme.

Theorem 6: Section 3.2 lattice based forward-secure IBE is IND-sID-CPA (semantic) secure assuming the $LWE_{q,\chi}$ is hard or $Adv_{B,LWE_{q,\chi}}(n) = \frac{1}{N} Adv_{\chi,A}(n)$.

Proof: Here proof is same as proof of theorem 4 except how adversary B sets the matrix $A_{id||j}$. Adversary A declares an identity ID^* that it intends to attack. Adversary B first pick $i^* \in [N]$. Here i^* is a guess for the i of challenge(i,b) query. Now B obtains $2^\lambda(i^*+1)(m\lambda+1)$ LWE samples, which get parsed as
$C_{j,u,t_i} \in Z_q^{n\times m}$ is mu_i vectors from the sample for $(0 \le j' \le i^*, 1 \le u \le \lambda, t_i \in \{0,1\})$ and . Adversary B sets master public key to be $mpk = A = A_0^*$ (m u_i vectors from LWE oracle). Based on hash value $H(id||j) = (t_1, t_2, ..., t_\lambda) \in \{0,1\}^\lambda$, adversary B sets the matrix $A_{id||j}$.
This proof differs from proof of theorem 4 by number of LWE samples required for adversary B. Number of LWE samples required in this proof is approximately equal to 2^λ times LWE samples required in proof of the previous theorem 4.

4 Lattice Based Forward Secure IBE Scheme with Shorter Ciphertext

Our scheme is similar to scheme of [21] [2]. In this scheme size of ciphertext $C = (m+1) \times \|Z_q\|$ whereas size of ciphertext in previous schemes is equal to $(m(i+2)+1) \times \|Z_q\|$. Now we describe our new forward secure-IBE scheme with shorter ciphertext as follows.

Setup(n, N): On input a security parameter n, we set the parameters q, m accordingly. Let $H : \{0,1\}^* \to Z_q^{m\times m}$ be hash function. Next we do the following.

1. Use algorithm TrapGen(q, n) to generate a matrix $A \in Z_q^{n\times m}$ and a short basis T_A for $\Lambda_q^\perp(A)$ such that $\|\widetilde{T_A}\| \le O(\sqrt{n \log q})$.

2. We choose $y \leftarrow Z_q^n$ uniformly.

3. Output the master public key and master secret key,
mpk = (A, y), msk = T_A.

Extract(mpk,T,id): PKG generates the secret key for a user identity $id \in \{0,1\}^*$ as follows.
Let $R_{id||0} = H(id||0) \in Z^{m\times m}$ and $A_{id||0} = AR_{id||0}^{-1}$. PKG calls the function $NewBasisDel(A_{id||0}, R_{id||0}, T_A, \sigma)$. Output of the function $SK_{id||0}$ is the secret key of this user.

Update(mpk,$\mathbf{SK}_{id||i}$,$id||i$): Given secret key at i^{th} time period $\mathrm{SK}_{id||i}$ user can find secret key at $i+1^{th}$ time period as follows.

1. Let $R_{id||i+1} = H(id||i+1)...H(id||0) \in Z^{m\times m}$ and $A_{id||i+1} = A(R_{id||i+1})^{-1}$.
2. Evaluate $SK_{id||i+1} \leftarrow NewBasisDel(A_{id||i+1}, R_{id||i+1}, SK_{id||i+1}, \sigma)$.
3. Output $\mathrm{SK}_{id||i+1}$.

When $i = 0$ and $SK_{id||0} = T_A$ then algorithm Update() works as Extract().

Encrypt(mpk,$id||i$,b): To encrypt a bit $b \in \{0,1\}$, we do the following.

1. We compute $R_{id||i} = H_{id,i}...H_{id,0} \in Z_q^{m\times m}$ and $A_{id||i} = A(R(id||i))^{-1}$.
2. we choose $s \leftarrow Z_q^n$ uniformly.
3. Compute $p = A_{id||i}^T s + e \in Z_q^m$, where $e \leftarrow \chi^m$. Here χ^m is error (gaussian) distribution.
4. Compute $c = y^T s + b\lfloor \frac{q}{2} \rfloor + \overline{e}$, where $\overline{e} \leftarrow \chi$. Here χ is error (gaussian) distribution.
5. Output the ciphertext $C = (p,c)$. Size of ciphertext $C = (m+1) \times \|Z_q\|$.

Decrypt(C,mpk,$\mathbf{SK}_{(id||i)}$): To decrypt $C = (p,c)$, we do the following.

1. $s \leftarrow \mathrm{invert}(mpk, \mathrm{SK}_{(id||i)}, p)$.
2. Now we compute $b' = c - y^T s$.
3. If b' is closer to 0 than $\lfloor \frac{q}{2} \rfloor$ mod q output 0 otherwise output 1.

It is required that above forward secure IBE scheme has the correctness property, i.e, for any index $i \in [0,N]$ and secret key $\mathrm{SK}_{id||i}$ and any message bit b we have $b = \mathrm{Decryption}(mpk, \mathrm{SK}_{id||i}, \mathrm{Encryption}(mpk, id||i, \mathrm{b}))$.

Theorem 7. If hash function H is modeled as random oracle, then our lattice forward-secure IBE with constant ciphertext is IND-sID-CPA (semantic) secure assuming the $LWE_{q,\chi}$ is hard or $Adv_{B,LWE_{q,\chi}}(n) = \frac{1}{N} Adv_{\chi,A}(n)$.

Proof: Here proof is similar to proof of theorem 4 of [21] and theorem 5 of [2].
We now show semantic security of fs-IBE in the random oracle model. We will show that if there exist a PPT adversary A that breaks fs-IBE scheme with non-negligible probability then there must exist a PPT adversary B that solves LWE hard problem by simulating views of A. We assume that

- For each $i \in [N]$, A always makes polynomial number of Q_H different H-queries of interval i.
- Whenever A makes an H-query of interval i, we assume that it has queried H-query of interval $j < i$.
- Whenever A submits a user secret key query, we assume that it has made all relevant H queries beforehand.

Adversary A declares an identity ID^* that it intends to attack. Adversary B (works as challenger for adversary A) first pick $i^* \in [N]$. Here i^* is a guess for the i of challenge (i,b) query. Now B obtains $(m+1)$ LWE samples, which get parsed as
$(u_i, v_i) \in Z_q^n \times Z_q$ $(0 \le i \le n)$.

Setup: Adversary B runs the trapdoor algorithm TrapGen to generate $A \in Z_q^{n\times m}$ with corresponding trapdoor $T \in Z_q^{n\times m}$. Now adversary sets master public key to be $mpk = A$ and master secret key to be trapdoor T. Next Adversary B simulates the view of A as follows:

- Hash H Queries:
  1. A's hash query on $id^*||i^*$, adversary B returns A_{i^*} where A_{i^*} is m-samples (n-vector u_i where $(1 \leq i \leq n)$) from LWE oracle.
  2. A's hash query on any $id||0$, adversary B will choose low norm matrix $R_{id||0} \in Z_q^{m\times m}$ and runs algorithm $NewBasisDel(A, R_{id^*||0}, T_A, \sigma)$ to generate matrix $A_{id||0} = AR_{id||0}^{-1}$ and basis $T_{id||0}$ of $\Lambda_q^{\perp}(A_{id||0})$. Adversary B returns the matrix $A_{id||0}$ as a answer to hash query and stores the tuple $(id||0, A_{id||0}, T_{id||0})$ in list H.
  3. A's hash query on $id(\neq id^*)||i$ where $i > 0$. Since we have assumed that A would have made hash query on $id||i-1$, adversary B will choose low norm matrix $R_{id||i} \in Z_q^{m\times m}$ and runs algorithm $NewBasisDel(A_{id||i-1}, R_{id||i}, T_{id||i-1}, \sigma)$ to return matrix $A_{id||i}$ and basis $T_{id||i}$ of $\Lambda_q^{\perp}(A_{id||i})$. Adversary B returns the matrix $A_{id||i}$ as a answer to hash query and stores the tuple $(id||i, A_{id||i}, T_{id||i})$ in list H.
  4. A's hash query on $id^*||i$ where $0 < i < i^*$. Adversary will choose a matrix $R_{id^*||i} \in Z_q^{m\times m}$ uniformly and returns to adversary A.
  5. A's hash query on $id^*||i^*+1$, Adversary B runs the trapdoor algorithm TrapGen to generate $A_{id^*||i^*+1} \in Z_q^{n\times m}$ with corresponding trapdoor $T_{id^*||i^*+1} \in Z_q^{m\times m}$ and returns matrix $A_{id^*||i^*+1}$ and stores the tupple $id^*||i^*+1, A_{id^*||i^*+1}, T_{id^*||i^*+1}$ in list H.
  6. A's hash query on $id^*||i$, where $i > i^*+1$. Since we have assumed that A would have made hash query on $id^*||i-1$, adversary B will choose low norm matrix $R_{id^*||i} \in Z_q^{m\times m}$ and runs algorithm $NewBasisDel(A_{id^*||i-1}, R_{id^*||i}, T_{id^*||i-1}, \sigma)$ to generate matrix $A_{id^*||i}$ and basis $T_{id^*||i}$ of $\Lambda_q^{\perp}(A_{id^*||i})$. Adversary B returns the matrix $A_{id^*||i}$ as a answer to hash query and stores the tuple $(id^*||i, A_{id^*||i}, T_{id^*||i})$ in list H.
- Extraction Queries: When adversary A asks for the secret key for the identity $id \neq id^*$. As we have assumed that before extraction query adversary A would have made hash query for it, so adversary B will check the list H and returns the corresponding T to adversary A.

Attack:

- Challenge (i,b): When adversary A queries challenge (i,b), the adversary B picks a random bit $r \in \{0,1\}$ and a random ciphertext C. If $r = 0$ it returns challenge ciphertext to be (p^*, C^*) else it returns random ciphertext C.
- Breakin(j): When adversary A queries breakin(j), if $i < j \leq i^*$ then adversary B outputs a random bit and game abort (since B can not answer extraction queries for $j \leq i^*$). Otherwise adversary B will check the list H and returns the corresponding T to adversary A.

Now adversary B proceeds as follows:

1. Set
$$c_1^* = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} \in Z_q^m$$

2. Blind the message bit by $c_0^* = v_0 + b^* \lfloor \frac{q}{2} \rceil$.

3. Set $CT^* = (c_0^*, c_1^*)$ and send it to adversary A.

When Oracle $\bigcirc$ is a pseudo-random LWE oracle then $c_0^* = v_0 + b^* \lfloor \frac{q}{2} \rceil = u_0^T + s + x + b^* \lfloor \frac{q}{2} \rceil$ for some $s \in Z_q^n$ and noise x. Similarly

$$c_1^* = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} = A_{id^*||i^*}\ s + y$$

for some $s \in Z_q^n$ and noise y. So $CT^* = (c_0^*, c_1^*)$ is a valid encryption of b for $id^*||i^*$. When Oracle $\bigcirc$ is a random oracle then v_0, v^* are uniform and therefore $CT^* = (c_0^*, c_1^*)$ is a uniform in $(Z_q \times Z_q^m)$.

Finally adversary A terminates with some output, adversary B terminates with same output and ends the simulation. So if adversary A breaks the scheme then there exist adversary B which solves LWE hard problem.
Since probability that $i = i^*$ is $\frac{1}{N}$, so the probability that B does not abort during simulation is $\frac{1}{N}$. $Adv_{B,LWE_{q,\chi}}(n) = \frac{1}{N} Adv_{\chi,A}(n)$. Hence our scheme is semantic secure.

5 Conclusion

We have proved our efficient scheme to be semantically secure in selective-ID. In some cases adversary already may have private keys of users ID's of his choice. So security must be strengthened a bit. Security must allow the adversary to obtain the private key associated with any identity ID of his choice then adversary can declare the identity to be challenged [8]. The scheme which is secure against these kind of attack is called adaptive-ID (IND-ID-CPA) secure scheme. Construction of adaptively secure lattice-based forward-secure IBE scheme is open problem. Construction of CCA (IND-ID-CCA) secure lattice-based forward-secure IBE scheme is another open problem.

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