A Note on Martin Hellwig's "The Galí-Monacelli Model of a Small Open Economy Has No International Trade"

Jordi Galí[1]
Tommaso Monacelli[2]
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Hellwig (2015) points to a number of seeming inconsistencies/paradoxes associated with the equilibrium of our *RES* 2005 model (henceforth, the GM model). He claims that the source of those problems lies in the fact that the model assumes a double continuum of goods and countries and that an equilibrium price system may not have the "integral" representation assumed in the GM paper.

The main claim in Hellwig's comment can be summarized as follows: even though in GM it is assumed that the law of one price holds (at the level of each good variety), that assumption is inconsistent with the way we write households' budget constraints. In particular, Hellwig argues by means of an example that the relative price of two varieties (or, alternatively, of two bundles) produced in two different countries will differ across those two countries (i.e. their consumers will face different relative prices).[3]

Below we explain why Hellwig's claim is misleading and how the confusion may have arisen from the use of less-than-optimal notation in the original GM paper.

Let us start by writing total expenditures as shown in the budget constraint of the home country (h) consumer, as found in both GM and in Hellwig's note, though with more explicit notation here:[4]

$$\int_0^1 P_h(j) C_h^h(j) dj + \int_0^1 \int_0^1 P_i(j) C_i^h(j) dj di$$

where $i \in [0,1]$ indexes countries and $j \in [0,1]$ varieties. Since none of the issues raised in Hellwig's note pertain to the existence of a continuum of varieties produced in each country, we can use the well known result $\int_0^1 P_i(j) C_i^h(j) dj = P_i C_i^h$ for all $i \in [0,1]$ implied by the optimal allocation of expenditures across varieties produced in any given country (where $P_i \equiv \left(\int_0^1 P_i(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$ and $C_i^h \equiv \left(\int_0^1 C_i^h(j)^{1-\frac{1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$ are the usual price and consumption indexes) and

[1] CREI, UPF and Barcelona GSE

[2] Bocconi University and IGIER

[3] As Hellwig recognizes, the issues raised in his comment are orthogonal to the monetary and intertemporal nature of the model in GM, and thus can be discussed in the context of a static version of the model (with an arbitrary numéraire common to all countries, e.g. a common currency), as we do below.

[4] Hellwig writes expenditures on foreign goods as $\int_{[0,1]\setminus h} \int_0^1 P_i(j) C_i(j) dj di$ but this is equivalent to $\int_0^1 \int_0^1 P_i(j) C_i(j) dj di$ under the assumptions made in GM.

rewrite the above expression as

$$P_h C_h^h + \int_0^1 P_i C_i^h di \tag{1}$$

While not shown explicitly in the GM paper, by symmetry, expenditures of consumers in country k must be given by

$$P_k C_k^k + \int_0^1 P_i C_i^k di \tag{2}$$

Here is a simple way of stating Hellwig's paradox. Let us compare the cost of two bundles from the viewpoint of country h and country k consumers. The first bundle corresponds to one unit of the h-produced good. The second bundle corresponds to one unit of the k-produced good. From country h consumer's perspective, the cost of the two bundles appears to be P_h and $P_k di$, respectively. That is, the relative price of the h-produced bundle is infinite. From the viewpoint of country k's consumer, on the other hand, the cost of the two bundles is $P_h di$ and P_k, respectively, i.e. the relative price of the h-produced bundle is now zero!. The fact that the two consumers face different relative prices for the two bundles appears to violate the law of one price, which is awkward since the latter is *assumed*.

How can we make sense of this? The problem in Hellwig's reasoning is that he treats variable C_h^h in the first term of (1) as being measured in terms of the same units as C_h^k inside the integral in (2). *But this is not the case* (the same is true for C_k^k in (2) and C_k^h in (1)). The confusion is, most likely, the result of the suboptimal notation used in GM, even though that poor notation is of no consequence for the analysis and results derived therein. Next we use a *more explicit* notation to try to clarify the matter.

Consider a version of the global economy in GM with $N+1$ countries, indexed by $i \in \{1/(N+1), 2/(N+1), ..., 1\} \equiv I_N$. The expenditures of an *individual* consumer (i.e. the *per capita* expenditures) in country h can be written as

$$P_h C_h^h + \sum_{i \in I_N \setminus h} P_i C_i^h \tag{3}$$

The corresponding expenditures for an individual consumer in country k can be written as

$$P_k C_k^k + \sum_{i \in I_N \setminus k} P_i C_i^k \tag{4}$$

By assumption, variables C_h^h and C_h^k appearing in the two expressions above are meant to be measured in terms of the same "units" (e.g. number of cookies), and so are C_k^h and C_k^k. Going back to the exercise above: the relative price of any two bundles of h-produced and k-produced goods can be immediately seen to be identical for the consumers in both countries. Thus, the Hellwig paradox, if it exists, must have to do with the assumption of a continuum of countries and the use of integrals to represent expenditures found in the original GM model.

Next define $\mathcal{C}_i^h \equiv NC_i^h$ and $\mathcal{C}_i^k \equiv NC_i^k$. Using the previous transformation of variables we can now rewrite (3) and (4) as:

$$P_h C_h^h + \sum_{i \in I_N \setminus h} P_i \mathcal{C}_i^h \frac{1}{N}$$

$$P_k C_k^k + \sum_{i \in I_N \setminus k} P_i \mathcal{C}_i^k \frac{1}{N}$$

Taking the limit as N goes to infinity (and hence the size of each country becomes smaller and smaller):

$$P_h C_h^h + \int_0^1 P_i \mathcal{C}_i^h di \tag{5}$$

$$P_k C_k^k + \int_0^1 P_i \mathcal{C}_i^k di \tag{6}$$

Intuitively, as N increases *individual* consumption bundles C_i^h (for $i \neq h$) and C_i^k (for $i \neq k$) will become smaller and smaller at the rate N^{-1} (as each country i is becoming relatively less important in the global economy). But this will not generally be the case for the "transformed" variables $\mathcal{C}_i^h$ and $\mathcal{C}_i^k$, which will generally converge to a nonzero value, thus leading to well defined integrals in (5) and (4). Note also, and most importantly, that $\mathcal{C}_h^k$ and C_h^h refer to the same bundle of $h-$produced goods, *but their units of measurement are not the same now!* (so their cost cannot be directly compared).

Similarly, in the economy with a finite number of countries, the consumption index in country h consumer's utility function can be written as

$$C^h \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_h^h)^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_F^h)^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{7}$$

where

$$\begin{aligned} C_F^h &\equiv \left(\frac{1}{N^{\frac{1}{\gamma}}} \sum_{i \in I_N \setminus h} (C_i^h)^{1-\frac{1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \\ &= \left(\frac{1}{N} \sum_{i \in I_N \setminus h} (\mathcal{C}_i^h)^{1-\frac{1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \end{aligned} \tag{8}$$

Taking the limit as $N \to \infty$ we have

$$C_F^h \equiv \left(\int_0^1 (\mathcal{C}_i^h)^{1-\frac{1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}} \tag{9}$$

so that (7) (together with (9)) correspond to the consumption index that appears in the utility function of the home consumer in GM (though, admittedly, with a clearer notation here).

Note that the ultimate source of the asymmetry between domestic and imported goods in GM has to do with the assumption of a strong form of *home bias, as* captured by parameter α in the consumption index above. That assumption implies that, even though the size of each economy becomes negligible as $N \to \infty$, the share of its goods in *its own* consumers' expenditure does not vanish (in fact that share is equal to α in the symmetric steady state, as shown in GM). In a model without nontraded goods like GM's, an assumption of that sort is needed in order to match the fact that only a fraction of consumption in any country is accounted for by imports. It also plays an important role in the welfare and policy analysis in GM (since under our assumption the relative price of domestic vs imported goods–which can be influenced by a small country's central bank in the presence of price rigidities–will have an effect on domestic consumer's welfare).

Finally, it is important to note that the above digression has no bearing on any of the results in GM, which focus on the links between a small economy and the rest of the world (as opposed to bilateral trade and relative prices between any two countries). To see this note that the optimal allocation of expenditures across imported goods by country h consumer implies $\sum_{i \in I_N \setminus h} P_i C_i^h = P_F^h C_F^h$, where $P_F \equiv \left(\frac{1}{N} \sum_{i \in I_N \setminus h} (P_i)^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$ is country h's relevant price index for imported goods. Once we recognize this, the remaining problem consists of allocating total expenditures optimally between domestic and foreign goods:

$$\max \left[(1-\alpha)^{\frac{1}{\eta}} (C_h^h)^{1-\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_F^h)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

subject to a budget constraint that includes expenditures $P_h C_h^h + P_F^h C_F^h$. Thus the consumer's problem collapses to one involving two consumption bundles (domestic and imported), with no reference made to the existence of a continuum of countries. Given the focus in GM, one could have started from here and obtain the same aggregate equilibrium conditions that GM use in their subsequent analysis (thus avoiding all the confusing notation).

The discussion above has focused on the first part of Hellwig's note, which points to a potential flaw in the GM analysis having to do with the existence of a continuum of countries. The second part of Hellwig's note bypasses those issues by assuming a static version of the GM economy with a finite number of countries, while focusing on other questions. In our opinion the analysis and findings in this second part, which are largely unrelated to those in GM, have limited interest. More specifically, section 3 contains a digression on the indeterminacy of prices (as well as relative prices and quantities for the case in which a fraction of firms have predetermined prices). As far as we can tell this is a well known result, not at all specific to the GM model, and which would go away if the prices were expressed in terms of money and money supply and money demand equations were specified.[5] Section 4, on the other hand, analyzes

[5] In GM's intertemporal monetary model (like in most of the recent literature) the problem

the levels of trade and the terms of trade in Hellwig's version of the GM model with a finite (but possibly large) number of countries. The main findings (the possibility of vanishing trade under certain assumptions) differ from those in GM (as well as those implied by the finite country model discussed above) due to a different specification of preferences. If one uses instead the utility function given by (7) and (8) (which is the one whose limit corresponds to that in GM) it should be clear that the vanishing of trade result will never occur: a symmetric steady state exists, with unit relative prices and where the share of imports in GDP is given by α, independently of η and γ. Of course, there is nothing wrong with analyzing the consequences of alternative specifications of preferences for the level of trade in the steady state, but that analysis is unrelated to the GM paper, which focuses on very different issues.

References

Galí, Jordi and Tommaso Monacelli (2015): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies* (2005) 72, 707-734.

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of indeterminacy is "overcome" by specifying a suitable interest rate rule satisfying the so called Taylor principle.