Understanding the Gains from Wage Flexibility: the Exchange Rate Connection.*

Jordi Galí , CREI, Universitat Pompeu Fabra and Barcelona GSE.
Tommaso Monacelli, Università Bocconi and IGIER.

Online Appendix

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A Medium Scale DSGE Model of a Small Open Economy

We describe a medium-sized DSGE model of a small open economy inhabited by a representative household and by a continuum of firms each producing a differentiated variety. The household invests in financial assets and accumulates physical capital. In this version of the model all goods are traded. See Appendix C for an extension to a two-sector structure with both traded and non-traded goods.

A.1 Households

The household features a continuum of members, indexed by $j \in [0,1]$. Each household member is specialized in a differentiated labor service, which she supplies in an amount $\mathcal{N}_t(j)$. Household preferences are given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(\widetilde{C}_t, \{\mathcal{N}_t(j)\}; Z_t) \right\} \tag{1}$$

where $\widetilde{C}_t(j) \equiv C_t(j) - hC_{t-1}$ measures (external) habit-adjusted consumption, $h \in [0,1]$, and Z_t is an exogenous preference shifter.

We specialize utility to take the following expression:

$$U(\widetilde{C}_t, \{\mathcal{N}_t(j)\}; Z_t) = \left(\log \widetilde{C}_t - \frac{1}{1+\varphi} \int_0^1 \mathcal{N}_t(j)^{1+\varphi} dj \right) Z_t,$$

The consumption index is defined by

$$C_t \equiv \left((1-\upsilon)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + \upsilon^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ is an index of domestic goods consumption given by the CES function $C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$, with $i \in [0,1]$ denoting the good variety, and $C_{F,t}$ is the quantity consumed of a composite foreign good. Parameter $\epsilon_p > 1$ denotes the elasticity of substitution between varieties produced domestically. Parameter $\upsilon \in [0,1]$ can be interpreted as a measure of openness. The investment index can be defined in a completely analogous way.

The optimal allocation of consumption between domestic and imported goods requires:

$$C_{H,t} = (1-\upsilon)(P_{H,t}/P_t)^{-\eta} C_t \qquad ; \qquad C_{F,t} = \upsilon (P_{F,t}/P_t)^{-\eta} C_t, \tag{3}$$

where $P_t \equiv \left((1-\upsilon) P_{H,t}^{1-\eta} + \upsilon P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}$ is the consumer price index (CPI, for short). Analogous expressions hold for the investment good.

The sequence of budget constraints assumes the following form (expressed in units of the aggregate consumption basket, and abstracting from the specification of state contingent assets):

$$C_t + \frac{B_{H,t}/P_t}{R_t} + I_t \leq \frac{W_t}{P_t} N_t + \frac{\mathcal{R}_{k,t}}{P_t} K_{t-1} + \tau_t + \frac{B_{H,t-1}}{P_t} + \frac{\int_0^1 \Gamma_t(i)}{P_t} \tag{4}$$

where $B_{H,t}$ denote Home holdings of a riskless bond denominated in domestic currency, $R_t^{-1}$is the price of that bond, τ_t are government net transfers of domestic currency, $\mathcal{R}_{k,t}$ is the nominal rental rate of capital, and $\Gamma_t(i)$ are the profits of monopolistic firm i, whose shares are owned by the domestic residents.

The accumulation of capital obeys:

$$K_{t+1} = (1-\delta)K_t + I_t \left[1 - \Omega\left(\frac{I_t}{I_{t-1}} - 1\right)\right] \tag{5}$$

where $\Omega(\cdot)$ is increasing and convex, and such that $\Omega(0) = \Omega'(0) = 0$ and $\Omega''(0) = 0$.

Equilibrium conditions Let λ_t and $\lambda_t \psi_t$ be the Lagrange multipliers on constraints (4) and (5) respectively. Hence λ_t denotes the shadow value of one unit or real income. First order conditions with respect to N_t, C_t, $B_{H,t}$, I_t, K_t read:

$$\left(C_t - h\overline{C}_{t-1}\right)^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{6}$$

$$Z_t \left(C_t - h\overline{C}_{t-1}\right)^{-\sigma} = \lambda_t$$

$$\lambda_t = \beta R_t E_t \left\{\lambda_{t+1} \frac{P_t}{P_{t+1}}\right\}$$

$$\psi_t \left[1 - \Omega(\cdot) - \Omega'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] = 1 - \beta E_t \left\{\psi_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{I_{t+1}}{I_t}\right)^2 \Omega'\left(\frac{I_{t+1}}{I_t}\right)\right\}$$

$$\psi_t = \beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\left[r_{k,t+1} + (1-\delta)\psi_{t+1}\right]\right\}$$

where $r_{k,t+1} \equiv \mathcal{R}_{k,t+1}/P_{t+1}$.

Foreign Households The portfolio choice by households in Foreign implies the following Euler condition:

$$U_{c,t}^* = \beta R_t^* E_t \left\{U_{c,t+1}^* \frac{P_t^*}{P_{t+1}^*}\right\}$$

A.2 Production and Price Setting

Each monopolistic firm i in Home produces a homogenous good according to the *CRS* production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} K_t^{\alpha}(i) \tag{7}$$

where A_t is a labor productivity shifter (common across firms). The cost minimizing choice of labor and capital input implies:

$$\frac{W_t}{P_{H,t}(i)} = \frac{MC_t}{P_{H,t}(i)}\ A_t(1-\alpha)\left(\frac{K_t(i)}{N_t(i)}\right)^{\alpha} \tag{8}$$

$$\frac{\mathcal{R}_{k,t}}{P_{H,t}(i)} = \frac{MC_t}{P_{H,t}(i)}\ A_t\alpha\left(\frac{N_t(i)}{K_t(i)}\right)^{1-\alpha} \tag{9}$$

where MC denotes the nominal marginal cost. Notice that the above conditions imply

$$MC_t = \frac{W_t^{1-\alpha}\mathcal{R}_{k,t}^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_t} \tag{10}$$

Hence, and due to the CRS assumption, the nominal marginal cost is the same across firms. In equilibrium, this implies that also the capital-labor ratio is common across firms.

Optimal Pricing Each domestic firm can revise its price at random intervals. Let $(1-\theta_p)$ be the probability that a firm can reoptimize its price at any given time t. The first order condition with respect to $\overline{P}_{H,t}$ for profit maximization reads:

$$\underbrace{E_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{t,t+k}Y_{t+k|t}\overline{P}_{H,t}\right\}}_{LHS} \tag{11}$$

$$= \underbrace{\mathcal{M}_p E_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{t,t+k}Y_{t+k|t}MC_{t+k|t}\right\}}_{RHS}$$

where (from equilibrium)

$$\nu_{t,t+k} = \beta^k\frac{U_{c,t+k}P_t}{U_{c,t}P_{t+k}},$$

and $MC_{t+k|t}$ is the nominal marginal cost at $t+k$ of a firm that last reset its price at time t. Notice that, using (10), it holds

$$MC_{t+k|t} = MC_{t+k}$$

The above equivalence is an implication of the assumption of constant return to scale in production.

Dividing through by $P_{H,t}$ we can write the LHS of the above equation as follows:

$$LHS \equiv \left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{1-\varepsilon_p} E_t \left\{ \sum_{k=0}^{\infty} \theta_p^k \nu_{t,t+k} Y_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{\varepsilon_p} \right\}$$

Consider next the RHS of (11):

$$\begin{aligned} RHS &\equiv \mathcal{M}_p \frac{1}{P_{H,t}} E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon_p} Y_{t+k} MC_{t+k} \right\} \\ &\equiv \mathcal{M}_p \frac{1}{P_{H,t}} E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon_p} Y_{t+k} mc_{t+k} P_{H,t} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right) \right\} \\ &\equiv \mathcal{M}_p \left(\frac{\bar{P}_{H,t}}{P_{H,t}} \right)^{-\varepsilon_p} E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} Y_{t+k} mc_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{1+\varepsilon_p} \right\}. \end{aligned}$$

Equating LHS and RHS and rearranging we finally obtain:

$$\begin{aligned} \left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right) \ E_t \sum_{k=0}^{\infty} \theta_p^k \nu_{t,t+k} Y_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{\varepsilon_p} &= \\ \mathcal{M}_p E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} Y_{t+k} mc_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{1+\varepsilon_p} \right\} \end{aligned}$$

Recursive representation of the pricing block Define

$$\begin{aligned} \mathcal{K}_{p,t} &\equiv E_t \sum_{k=0}^{\infty} \theta_p^k \nu_{t,t+k} Y_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{\varepsilon_p} \\ \mathcal{Z}_{p,t} &\equiv E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} Y_{t+k} mc_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{1+\varepsilon_p} \right\} \end{aligned}$$

Express recursively as:

$$\mathcal{K}_{p,t} = Y_t + \theta_p \underbrace{\left(\beta \frac{U_{c,t+1}}{U_{c,t} \Pi_{t+1}} \right)}_{\nu_{t,t+1}} \Pi_{H,t+1}^{\varepsilon_p} \mathcal{K}_{p,t+1}$$

Similarly

$$\mathcal{Z}_{p,t} = Y_t mc_t + \theta_p \left(\beta \frac{U_{c,t+1}}{U_{c,t}\Pi_{t+1}} \right) \Pi_{H,t+1}^{1+\varepsilon_p} \mathcal{Z}_{p,t+1}$$

We also have:

$$1 = \theta_p (\Pi_{H,t})^{\varepsilon_p - 1} + (1-\theta_p) \left(\frac{\overline{P}_{H,t}}{P_{H,t}} \right)^{1-\varepsilon_p} . \tag{12}$$

A.3 Terms of Trade and Exchange Rate Pass-Through

The *terms of trade* is the relative price of imported goods:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{13}$$

Under the assumption $P_t^* = P_{F,t}^*$, the real exchange rate is defined as

$$\mathcal{Q}_t \equiv \frac{\mathcal{E}_t}{P_t} = \frac{\mathcal{E}_t}{P_t}$$

where $\mathcal{E}_t$ is the nominal exchange rate (the price of unit of foreign currency expressed in units of domestic currency), and where the second equality holds under the assumption that the rest of the world is an approximately closed economy.

The terms of trade can be related to the CPI-PPI ratio as follows:

$$\frac{P_t}{P_{H,t}} = [(1-\upsilon) + \upsilon {S_t}^{1-\eta}]^{\frac{1}{1-\eta}} \equiv q(S_t), \tag{14}$$

with $q_{s,t} \equiv \partial q(S_t)/\partial S_t > 0$.

Law of-one-price gap Nominal stickiness in import prices and the presence of local distribution costs (modeled below) motivate deviations from the law of one price. Let the *law-of-one-price gap* be denoted by:

$$\Phi_{F,t} \equiv \frac{\mathcal{E}_t}{P_{F,t}}.$$

The expression for the real exchange rate becomes:

$$\begin{aligned} \mathcal{Q}_t &= \frac{\mathcal{E}_t}{P_t} \\ &= \Phi_{F,t} \frac{\mathcal{S}_t}{q(\mathcal{S}_t)} \end{aligned} \tag{15}$$

In the case of *complete* pass-through, $\Phi_{F,t} = 1$ for all t.

A.4 Optimal import pricing

Each variety produced in the rest of the world is distributed to the final consumer by a local importer. Distributing C_F units of imported variety f to the local consumer requires combining $M_{F,t}$ units of a homogeneous imported input with labor, according to the following constant return to scale production function:

$$C_{F,t}(f) = N_t(f)^{1-\alpha_F} M_{F,t}(f)^{\alpha_F} \tag{16}$$

where $M_{F,t}(f)$ and $N_t(f)$ denote the quantity of imported input and of labor respectively employed by the intermediate local importer f.

Let $P^*_{F,t}(f)$ be the "dock price" of the imported input (expressed in units of foreign currency), and let $P_{F,t}(f)$ be the local currency price of the distributed variety. The local currency price of the distributed imported variety, $P_{F,t}(f)$, can be changed only at random intervals with probability $(1 - \theta_{F,p})$.

The cost minimizing choice of imported inputs and labor requires:

$$\frac{W_t}{P_{F,t}(f)} = \frac{MC_{F,t}}{P_{F,t}(f)} (1 - \alpha_F) \left(\frac{M_{F,t}(f)}{N_t(f)} \right)^{\alpha_F} \tag{17}$$

$$\Phi_{F,t}(f) = \frac{MC_{F,t}}{P_{F,t}(f)} \alpha_F \left(\frac{N_t(f)}{M_{F,t}(f)} \right)^{1-\alpha_F} \tag{18}$$

where MC_F denotes the nominal marginal cost of local importer f.

The above conditions imply:

$$MC_{F,t} = \frac{W_t^{1-\alpha_F} \left(\mathcal{E}_t P^*_{F,t} \right)^{\alpha_F}}{\alpha_F^{\alpha_F} (1 - \alpha_F)^{1-\alpha_F}} \tag{19}$$

Hence, and due to the CRS assumption, the nominal marginal cost is the same across local importers.

The intermediate local importer solves:

$$\max \; E_t \sum_{k=0}^{\infty} \theta^k_{F,p} \nu_{t,t+k} \left\{ \left[\overline{P}_{F,t}(f) - MC_{F,t+k} \right] C_{F,t+k}(f) \right\}$$

$$\text{s.t. } (51),$$

and to the optimal demand function for variety f :

$$C_{F,t+k}(f) = \left(\frac{\overline{P}_{F,t}(f)}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k} \ . \tag{20}$$

The first order condition with respect to $\overline{P}_{F,t}(f)$ reads:

$$\begin{aligned} & E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} C_{F,t+k} \overline{P}_{F,t}(f) \right\} \qquad (21) \\ = & \left(\frac{\varepsilon_p}{\varepsilon_p - 1} \right) E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} C_{F,t+k} MC_{F,t+k} \right\} \end{aligned}$$

The real marginal cost for the local intermediate importer reads:

$$\begin{aligned} mc_{F,t} & \equiv \frac{MC_{F,t}}{P_{F,t}} \\ & = \left(\frac{w_t \, q(S_t)}{S_t} \right)^{1-\alpha_F} \Phi_{F,t}^{\alpha_F} \end{aligned}$$

where $w_t = W_t / P_t$ is the real CPI wage.

Dividing through by $P_{F,t}$ in (21), and using (20) the above pricing condition can be written:

$$\begin{aligned} & \left(\frac{\overline{P}_{F,t}}{P_{F,t}} \right)^{1-\varepsilon_p} E_t \left\{ \sum_{k=0}^{\infty} \theta_{F,p}^k \, \nu_{t,t+k} C_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{\varepsilon_p} \right\} \\ = & \left(\frac{\varepsilon_p}{\varepsilon_p - 1} \right) \frac{1}{P_{F,t}} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\frac{\bar{P}_{F,t}}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k} MC_{F,t+k} \right\} \\ = & \left(\frac{\varepsilon_p}{\varepsilon_p - 1} \right) \frac{1}{P_{F,t}} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\frac{\bar{P}_{F,t}}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k} mc_{F,t+k} P_{F,t} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right) \right\} \\ = & \left(\frac{\varepsilon_p}{\varepsilon_p - 1} \right) \left(\frac{\bar{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_p} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} C_{F,t+k} mc_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{1+\varepsilon_p} \right\}. \end{aligned}$$

Simplifying:

$$\begin{aligned} & \left(\frac{\overline{P}_{F,t}}{P_{F,t}} \right) E_t \left\{ \sum_{k=0}^{\infty} \theta_{F,p}^k \, \nu_{t,t+k} C_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{\varepsilon_p} \right\} \\ = & \left(\frac{\varepsilon_p}{\varepsilon_p - 1} \right) E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} C_{F,t+k} mc_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{1+\varepsilon_p} \right\}. \end{aligned}$$

Expressed in recursive form the above condition reads:

$$\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)\mathcal{K}_{F,t} = \mathcal{M}_p \mathcal{Z}_{F,t}$$

where

$$\mathcal{K}_{F,t} \equiv C_{F,t} + \theta_{F,p}\left(\beta \frac{U_{c,t+1}}{U_{c,t}\Pi_{t+1}}\right)\Pi_{F,t+1}^{\varepsilon_p}\mathcal{K}_{F,t+1}$$

$$\mathcal{Z}_{F,t} \equiv C_{F,t}\left[\left(w_t \frac{g(S_t)}{S_t}\right)^{1-\alpha_F}\Phi_{F,t}^{\alpha_F}\right] + \theta_{F,p}\left(\frac{\beta U_{c,t+1}}{U_{c,t}\Pi_{t+1}}\right)\Pi_{F,t+1}^{1+\varepsilon_p}\mathcal{Z}_{F,t+1}$$

Furthermore:

$$1 = \theta_{F,p}(\Pi_{F,t})^{\varepsilon_p - 1} + (1-\theta_{F,p})\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{1-\varepsilon_p}.$$

A.5 Export Demand

We assume that aggregate export demand, X_t, takes the following form (assuming $P_t^* = P_{F,t}^* = 1$ for all t):

$$\begin{aligned} X_t &= \upsilon\left(\frac{P_{H,t}}{\mathcal{E}_t}\right)^{-\eta} Y_t^* \\ &= \upsilon\left(\mathcal{S}_t \Phi_{F,t}\right)^{\eta} Y_t^* \end{aligned}$$

where the second equality has used (15).

A.6 Wage Setting

Let $N_t(i)$ be the labor *demand* by firm i. Each firm i employs *all* differentiated labor types. Hence *total* labor demand by firm i can be written:

$$N_t(i) = \left(\int_0^1 N_t(i,j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

where $N_t(i,j)$ is demand by firm i of labor type j.

Optimal demand for labor type j by firm i reads:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(i) \tag{22}$$

Integrating across domestic good producing firms, we can derive the equilibrium total demand for each labor type j (using (22) above):

$$\begin{aligned}
\underbrace{N_t(j)}_{\substack{\text{total demand}\\ \text{for labor type j}}} &= \underbrace{\int_0^1 N_t(i,j)di}_{\substack{\text{integrating}\\ \text{across firms}}} \\
&= \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(i)di \\
&= \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} \int_0^1 N_t(i)di \\
&= \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t
\end{aligned} \tag{23}$$

The above expression would hold in the absence of labor distribution costs for local importers. The optimal demand for labor type j by the intermediate importer f reads:

$$N_t(f,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(f) \tag{24}$$

Hence the *total* demand for each labor type j reads:

$$\begin{aligned}
N_t(j) &= \underbrace{\int_0^1 N_t(i,j)di}_{\substack{\text{intermediate good}\\ \text{producers in Home}}} + \underbrace{\int_0^1 N_t(f,j)df}_{\substack{\text{intermediate local}\\ \text{importers}}} \\
&= \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(i)di + \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(f)df \\
&= \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} \left[\int_0^1 N_t(i)di + \int_0^1 N_t(f)df\right] \\
&= \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t
\end{aligned} \tag{25}$$

where now $N_t \equiv \int_0^1 N_t(i)di + \int_0^1 N_t(f)df$.

Optimal wage setting problem Next, consider the optimal wage setting problem for household j:

$$\max E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U(\widetilde{C}_{t+k|t}(j), \mathcal{N}_{t+k|t}(j))$$

where $\mathcal{N}_{t+k|t}(j)$ is time $t+k$ labor supply by household type j who last reset her wage in time t.

At the chosen wage $\overline{W}_t(j)$, household type j is assumed to supply enough labor to satisfy demand. The constraint reads, using (23):

$$\begin{aligned}
\underbrace{\mathcal{N}_{t+k|t}(j)}_{\substack{\text{total supply}\\ \text{of labor type j}}} &= \underbrace{N^d_{t+k|t}(j)}_{\substack{\text{total demand for}\\ \text{for labor type j}}} \\
&= \left(\frac{\overline{W}_t(j)}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}
\end{aligned}$$

Notice that N_{t+k} bears the index $t+k$ (and *not* $t+k|t$) because it corresponds to aggregate (or average) labor demand.

The additional household's constraint is the budget constraint:

$$P_{t+k}C_{t+k|t}(j) + E_t\left\{\nu_{t+k,t+k+1}\mathcal{B}_{t+k+1|t}\right\} \leq \mathcal{B}_{t+k|t} + \overline{W}_t(j)N_{t+k|t}(j) - T_{t+k}$$

where we now make explicit that the households can pool labor income risk through state contingent assets $\mathcal{B}_t$. Each household j reoptimizing the wage at a given time t will choose the same optimal wage. It is therefore convenient to *abstract from index j.*

Household problem The (relevant portion of the) Lagrangian of the household's problem is

$$\mathcal{L}^w = E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\left\{U\left(C_{t+k|t},\ \mathcal{N}_{t+k|t}\right) - \widetilde{\lambda}_{t+k|t}\left[P_{t+k}C_{t+k|t} - \overline{W}_t\ \mathcal{N}_{t+k|t}\right]\right\}. \tag{26}$$

where $\widetilde{\lambda}_{t+k}$ is the shadow value of one unit of *nominal* income at $t+k$.

The FOC of the problem with respect to $\overline{W}_t$ is:

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{U_{\mathcal{N},t+k|t}\frac{\partial\mathcal{N}_{t+k|t}}{\partial\overline{W}_t} + \widetilde{\lambda}_{t+k|t}\left(N_{t+k|t} + \overline{W}_t\frac{\partial\mathcal{N}_{t+k|t}}{\partial\overline{W}_t}\right)\right\} = 0$$

Notice:

$$\begin{aligned}
\frac{\partial\mathcal{N}_{t+k|t}}{\partial\overline{W}_t} &= -\varepsilon_w\left(\frac{\overline{W}_t}{W_{t+k}}\right)^{-\varepsilon_w-1}\frac{N_{t+k}}{W_{t+k}} \\
&= -\varepsilon_w\mathcal{N}_{t+k|t}\frac{1}{\overline{W}_t}
\end{aligned}$$

Hence we can write:

$$-\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{U_{\mathcal{N},t+k|t}\ \varepsilon_w \mathcal{N}_{t+k|t}\frac{1}{\overline{W}_t}+\lambda_{t+k|t}\mathcal{N}_{t+k|t}\left(\varepsilon_w-1\right)\right\}=0$$

Under complete markets and separable utility we have $U_{c,t+k}(C_{t+k|t},\mathcal{N}_{t+k|t})=U_{c,t+k}(C_{t+k})$. In addition, equilibrium implies $U_{c,t+k}=\widetilde{\lambda}_{t+k}P_{t+k}$ (since $\widetilde{\lambda}_{t+k}$ is the shadow value of one unit of *nominal* income at $t+k$). Recall that under our calibration $U_{c,t}\equiv Z_t\left(C_t^{-\sigma}-h\overline{C}_{t-1}\right)=\lambda_t=\widetilde{\lambda}_t P_t$.

Hence we have:

$$-\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{U_{\mathcal{N},t+k|t}\mathcal{N}_{t+k|t}\mathcal{M}_w+U_{c,t+k}\mathcal{N}_{t+k|t}\frac{\overline{W}_t}{P_{t+k}}\right\}=0$$

where $\mathcal{M}_w\equiv\varepsilon_w/(\varepsilon_w-1)$.

The above expression can be rewritten:

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\left\{U_{c,t+k}\mathcal{N}_{t+k|t}\left[\frac{\overline{W}_t}{P_{t+k}}+\frac{U_{\mathcal{N},t+k|t}}{U_{c,t+k}}\mathcal{M}_w\right]\right\}=0 \tag{27}$$

Recursive representation Condition (27) reads:

$$\underbrace{E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\mathcal{N}_{t+k|t}U_{c,t+k}\frac{\overline{W}_t}{P_{t+k}}}_{LHS}=\underbrace{E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\mathcal{N}_{t+k|t}\mathcal{M}_w(-U_{\mathcal{N},t+k|t})}_{RHS}$$

Using the optimal labor demand condition

$$\mathcal{N}_{t+k|t}=\left(\frac{\overline{W}_t}{W_{t+k}}\right)^{-\varepsilon_w}N_{t+k}, \tag{28}$$

we can write the LHS as follows:

$$\begin{aligned} LHS &\equiv \left(\frac{\overline{W}_t}{P_t}\right)^{1-\varepsilon_w}\left\{\begin{array}{l}\left(\frac{W_t}{P_t}\right)^{\varepsilon_w}N_tU_{c,t}+\beta\theta_w\left(\frac{W_{t+1}}{P_{t+1}}\right)^{\varepsilon_w}\Pi_{t+1}^{\varepsilon_w-1}N_{t+1}U_{c,t+1}+\\ +(\beta\theta_w)^2\left(\frac{W_{t+2}}{P_{t+2}}\right)^{\varepsilon_w}\left(\Pi_{t+1}\Pi_{t+2}\right)^{\varepsilon_w-1}N_{t+2}U_{c,t+2}+...\end{array}\right\}\\ &= \overline{w}_t^{1-\varepsilon_w}E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k w_{t+k}^{\varepsilon_w}\left(\prod_{s=1}^{k}\Pi_{t+s}\right)^{\varepsilon_w-1}N_{t+k}U_{c,t+k},\end{aligned}$$

where $\overline{w}_t\equiv\overline{W}_t/P_t$.

Next consider RHS:

$$RHS \equiv -\left(\frac{\overline{W}_t}{P_t}\right)^{-\varepsilon_w} \left\{ \begin{array}{c} \left(\frac{W_t}{P_t}\right)^{\varepsilon_w} N_t \mathcal{M}_w U_{\mathcal{N},t|t} \\ +\beta\theta_w \left(\frac{W_{t+1}}{P_{t+1}}\right)^{\varepsilon_w} N_{t+1}\Pi_{t+1}^{\varepsilon_w}\mathcal{M}_w U_{\mathcal{N},t+1|t} + ... \end{array} \right\}$$

This can be written

$$RHS \equiv \overline{w}_t^{-\varepsilon_w} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k w_{t+k}^{\varepsilon_w} \left(\prod_{s=1}^{k} \Pi_{t+s}\right)^{\varepsilon_w} N_{t+k}\mathcal{M}_w(-U_{N,t+k|t})$$

Under the assumption that $U_{\mathcal{N}}(\bullet)$ is homogenous of degree φ in $\mathcal{N}$ we have (using (28)):

$$\begin{aligned} -U_{\mathcal{N},t+k|t} &= \left(\frac{\overline{W}_t}{W_{t+k}}\right)^{-\varepsilon_w\varphi} (-U_{\mathcal{N},t+k}(N_{t+k})) \\ &= \left(\frac{\overline{W}_t/P_t}{W_{t+k}/P_{t+k}}\right)^{-\varepsilon_w\varphi} \left(\prod_{s=1}^{k}\Pi_{t+s}\right)^{\varepsilon_w\varphi} (-U_{\mathcal{N},t+k}(N_{t+k})) \end{aligned}$$

Substituting:

$$RHS \equiv \overline{w}_t^{-\varepsilon_w(1+\varphi)} E_t \sum_{k=0}^{\infty}(\beta\theta_w)^k w_{t+k}^{\varepsilon_w(1+\varphi)} N_{t+k}\mathcal{M}_w \left(\prod_{s=1}^{k}\Pi_{t+s}\right)^{\varepsilon_w(1+\varphi)} (-U_{\mathcal{N},t+k}(N_{t+k}))$$

Combining LHS and RHS we obtain:

$$\begin{aligned} &\overline{w}_t^{1+\varepsilon_w\varphi} \underbrace{E_t \sum_{k=0}^{\infty}(\beta\theta_w)^k w_{t+k}^{\varepsilon_w}\left(\prod_{s=1}^{k}\Pi_{t+s}\right)^{\varepsilon_w-1} N_{t+k}U_{c,t+k}}_{\mathcal{K}_t^w} \\ &= \mathcal{M}_w \underbrace{E_t\sum_{k=0}^{\infty}(\beta\theta_w)^k w_{t+k}^{\varepsilon_w(1+\varphi)} N_{t+k}\left(\prod_{s=1}^{k}\Pi_{t+s}\right)^{\varepsilon_w(1+\varphi)} (-U_{\mathcal{N},t+k}(N_{t+k}))}_{\mathcal{Z}_t^w} \end{aligned}$$

We can rewrite recursively:

$$\mathcal{K}_{w,t} = w_t^{\varepsilon_w} N_t U_{c,t} + \beta\theta_w \Pi_{t+1}^{\varepsilon_w-1}\mathcal{K}_{w,t+1}$$

$$\mathcal{Z}_{w,t} = w_t^{\varepsilon_w(1+\varphi)} N_t(-U_{\mathcal{N},t}(N_t)) + \beta\theta_w\Pi_{t+1}^{\varepsilon_w(1+\varphi)}\mathcal{Z}_{w,t+1}$$

Hence the first order condition can be written in compact form:

$$\overline{w}_t^{1+\varepsilon_w\varphi}\mathcal{K}_{w,t} = \mathcal{M}_w\mathcal{Z}_{w,t}$$

A.7 Price Dispersion, Wage Dispersion, and Equilibrium

Market clearing for each individual domestic variety implies:

$$\underbrace{A_t K_t^{\alpha}(i) N_t(i)^{1-\alpha}}_{\substack{\text{supply of} \\ \text{variety i}}} = \underbrace{\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p} Y_t}_{\substack{\text{demand of} \\ \text{variety i}}} \tag{29}$$

where $N_t(i)$ denotes the total amount of labor employed by firm i. Rearranging:

$$N_t(i) = \left[\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p} \frac{Y_t}{A_t(K_t/N_t)^{\alpha}}\right]$$

where we used the fact that, in equilibrium, all firms choose the same capital labor ratio. Integrating across all producers:

$$\begin{aligned} \int_0^1 N_t(i)di &= \int_0^1 \left[\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p} \frac{Y_t}{A_t(K_t/N_t)^{\alpha}}\right] di & (30) \\ &= \frac{Y_t}{A_t(K_t/N_t)^{\alpha}} \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p} di = \frac{Y_t}{A_t(K_t/N_t)^{\alpha}} \Delta_{p,t}, & (31) \end{aligned}$$

where $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{PH_{,t}}\right)^{-\varepsilon_p} di$ measures the dispersion of relative prices across domestic producers. In a more compact form:

$$N_t = \frac{Y_t}{A_t(K_t/N_t)^{\alpha}} \Delta_{p,t},$$

where $N_t \equiv \int_0^1 N_t(i)di$.

Hence we can finally write:

$$A_t K_t^{\alpha} N_t^{1-\alpha} = Y_t \Delta_{p,t} \tag{32}$$

Expressing $\Delta_{p,t}$ in recursive form:

$$\begin{aligned} \Delta_{p,t} &= \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon_p} di \\ &= \int_{1-\theta_p} \left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon_p} di + \left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\varepsilon_p} \int_{\theta_p} \left(\frac{P_{H,t-1}(i)}{P_{H,t-1}}\right)^{-\varepsilon_p} di \\ &= (1-\theta_p)\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon_p} + \theta_p \Pi_{H,t}^{\varepsilon_p} \Delta_{p,t-1} \end{aligned}$$

Market clearing Total demand for domestically produced goods reads:

$$\begin{aligned} Y_t &\equiv C_{H,t} + I_{H,t} + X_t \\ &= (1-\upsilon)q(\mathcal{S}_t)^{\eta}\,(C_t + I_t) \;\;+\;\; \upsilon\,(\mathcal{S}_t \Phi_{F,t})^{\eta}\, Y_t^{*} \end{aligned} \tag{33}$$

Hence condition (60) becomes:

$$A_t K_t^{\alpha} N_t^{1-\alpha} = [(1-\upsilon)q(\mathcal{S}_t)^{\eta}\,(C_t + I_t) \;\;+\;\; \upsilon\,(\mathcal{S}_t \Phi_{F,t})^{\eta}\, Y_t^{*}]\,\Delta_{p,t} \tag{34}$$

Let $\mathcal{N}_t(j)$ denote labor supply by each differentiated household. Since each household is assumed to satisfy labor demand at the given posted wage, equilibrium in the labor market requires:

$$\mathcal{N}_t(j) = N_t(j)$$

Aggregating across each household j one obtains, using (23):

$$\begin{aligned} \mathcal{N}_t &\equiv \int_0^1 \mathcal{N}_t(j)dj = \int_0^1 N_t(j) \\ &= \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} dj\;N_t \end{aligned}$$

where $\mathcal{N}_t$ is an index of *aggregate* labor supply. By defining $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w}$ as an index of wage dispersion, the above equation becomes.

$$\mathcal{N}_t = \Delta_{w,t} N_t. \tag{35}$$

Notice that by substituting (62) into (60) one obtains:

$$\mathcal{N}_t = \Delta_{w,t}\left(\frac{Y_t \Delta_{p,t}}{A_t {K_t}^{\alpha}}\right)^{\frac{1}{1-\alpha}} \tag{36}$$

which shows that the relationship between aggregate employment $\mathcal{N}_t$ and aggregate output Y_t depends on both price and wage dispersion.

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$$\frac{\Phi_{F,t}}{\Phi_{F,t-1}} = \frac{(\mathcal{E}_t/\mathcal{E}_{t-1})\,\Pi^{*}_{F,t}}{\Pi_{F,t}}$$

$$\frac{S_t}{S_{t-1}} = \frac{(\mathcal{E}_t/\mathcal{E}_{t-1})}{(\Phi_{F,t}/\Phi_{F,t-1})\,\Pi_{H,t}}$$

A.8 Wage dispersion and welfare

Each household is a monopolistic supplier of a specialized labor type. While households can pool consumption uncertainty (so that the marginal utility of nominal income is the same across households), they cannot pool employment uncertainty. Hence labor supply is *heterogenous* in equilibrium. Under the assumption of separable preferences, and in particular of isoelastic disutility from labor, the intertemporal utility for household $j \in [0,1]$ reads:

$$\mathcal{V}_t(j) \equiv E_t \sum_{k=0}^{\infty} \beta^k \left\{ U\left(\widetilde{C}_{t+k}(j)\right) - \frac{\mathcal{N}_{t+k}(j)^{1+\varphi}}{1+\varphi} \right\}$$

where, under consumption pooling,

$$U\left(\widetilde{C}_t(j)\right) = U\left(\widetilde{C}_t\right) \text{ for all } j \tag{37}$$

We wish to evaluate an *aggregate* measure of household's welfare:

$$\mathcal{V}_t \equiv \int_0^1 \mathcal{V}_t(j)dj = E_t \sum_{k=0}^{\infty} \beta^k \left\{ U\left(\widetilde{C}_{t+k}\right) - \int_0^1 \frac{\mathcal{N}_{t+k}(j)^{1+\varphi}}{1+\varphi} dj \right\},$$

where we have made use of (37).

In equilibrium, using (23):

$$\begin{aligned} \mathcal{N}_t(j) &= N_t(j) \\ &= \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t \text{ for all } t \end{aligned}$$

Thus we can write:

$$\mathcal{V}_t = E_t \sum_{k=0}^{\infty} \beta^k \left\{ U\left(\widetilde{C}_{t+k}\right) - \frac{N_{t+k}^{1+\varphi}}{1+\varphi} \int_0^1 \left(\frac{W_{t+k}(j)}{W_{t+k}}\right)^{-\varepsilon_w(1+\varphi)} dj \right\},$$

Let the *welfare relevant* measure of wage dispersion be:

$$\widetilde{\Delta}_{w,t}^{1+\varphi} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w(1+\varphi)} dj$$

Thus we can write:

$$\mathcal{V}_t = E_t \sum_{k=0}^{\infty} \beta^k \left\{ U\left(\widetilde{C}_{t+k}\right) - \frac{\widetilde{N}_{t+k}^{1+\varphi}}{1+\varphi} \right\},$$

where

$$\widetilde{\mathcal{N}}_t = N_t \widetilde{\Delta}_{w,t}^{1+\varphi} \text{ for all t}$$

is the aggregate employment index that is relevant for aggregate (average) welfare. Notice that in the case $\varphi = 0$, i.e., of linear disutility of labor, $\widetilde{\Delta}_{w,t} = \Delta_{w,t}$.

A recursive expression for $\widetilde{\Delta}_{w,t}^{1+\varphi}$ reads:

$$\begin{aligned}
\widetilde{\Delta}_{w,t}^{1+\varphi} &= \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w(1+\varphi)} dj \\
&= \int_{1-\theta_w} \left(\frac{\overline{W}_t}{W_t} \right)^{-\varepsilon_w(1+\varphi)} dj + \left(\frac{W_{t-1}}{W_t} \right)^{-\varepsilon_w(1+\varphi)} \int_{\theta_w} \left(\frac{W_{t-1}(j)}{W_{t-1}} \right)^{-\varepsilon_w(1+\varphi)} dj \\
&= (1-\theta_w) \left(\frac{\overline{w}_t}{w_t} \right)^{-\varepsilon_w(1+\varphi)} + \theta_w \left(\frac{w_t \Pi_t}{w_{t-1}} \right)^{\varepsilon_w(1+\varphi)} \widetilde{\Delta}_{w,t-1}^{1+\varphi}
\end{aligned}$$

where $w_t \equiv W_t/P_t$. Notice that, for the measure $(1-\theta_w)$ of reoptimizing households, $\overline{W}_t(j) = \overline{W}_t$.

A.9 Price setting problem with indexation

In this section we lay out the problem of domestic good producers in the case of partial indexation. A similar structure applies to the price setting problem of domestic importers in the case of distribution costs and incomplete exchange rate pass-through.

We assume that those domestic producers which cannot reoptimize their price choose to raise prices by a fraction $\chi_p \in [0,1]$ of the domestic price level. Formally:

$$P_{H,t+1}(j) = \Pi_{H,t}^{\chi_p} P_{H,t}(j)$$

If the firm is unable to reoptimize for k periods, this will become:

$$P_{H,t+k}(j) = P_{H,t}(j) \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p} \right)$$

We use the convention of setting:

$$\prod_{s=1}^{0} \Pi_{H,t+s-1}^{\chi_p} = 1$$

The problem of the reoptimizing firm becomes:

$$\max_{\overline{P}_{H,t}} E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left[\overline{P}_{H,t} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p} \right) Y_{t+k|t} - MC_{t+k|t} Y_{t+k|t} \right] \right\},$$

subject to the demand constraints:

$$Y_{t+k|t}=\left(\frac{\overline{P}_{H,t}\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_p}\right)}{P_{H,t+k}}\right)^{-\varepsilon_p}Y_{t+k}.$$

The associated first order condition is given by:

$$E_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{t,t+k}Y_{t+k|t}\overline{P}_{H,t}\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_p}\right)\right\}=\mathcal{M}_pE_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{t,t+k}Y_{t+k|t}MC_{t+k|t}\right\}$$

Dividing both sides by $P_{H,t}$ and substituting out the demand for firm i conditional on not readjusting the price for k periods yields:

$$\frac{1}{P_{H,t}}E_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{t,t+k}\left(\frac{\overline{P}_{H,t}\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_p}\right)}{P_{H,t+k}}\right)^{-\varepsilon_p}Y_{t+k}\overline{P}_{H,t}\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_p}\right)\right\}$$

$$=\frac{1}{P_{H,t}}\mathcal{M}_pE_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{t,t+k}\left(\frac{\overline{P}_{H,t}\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_p}\right)}{P_{H,t+k}}\right)^{-\varepsilon_p}Y_{t+k}MC_{t+k|t}\right\}$$

Notice that we can write:

$$P_{H,t+k}=\frac{P_{H,t+k}}{P_{H,t+k-1}}\cdots\frac{P_{H,t+1}}{P_{H,t}}P_{H,t}=P_{H,t}\prod_{s=1}^{k}\frac{P_{H,t+s}}{P_{H,t+s-1}}=P_{H,t}\prod_{s=1}^{k}\Pi_{H,t+s}$$

We follow the convention of setting:

$$\prod_{s=1}^{0}\Pi_{H,t+s}=1$$

We are left with:

$$\frac{\left(\overline{P}_{H,t}\right)^{1-\varepsilon_p}}{P_{H,t}}E_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{H,t,t+k}\left(P_{H,t+k}\right)^{\varepsilon_p}Y_{t+k}\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_p}\right)^{1-\varepsilon_p}\right\}=$$

$$=\mathcal{M}_pE_t\sum_{k=0}^{\infty}\theta_p^k\left\{\nu_{t,t+k}\left(\frac{\overline{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon_p}Y_{t+k}mc_{t+k}\left(\prod_{s=1}^{k}\Pi_{H,t+s-1}^{\chi_p}\right)^{-\varepsilon_p}\left(\prod_{s=1}^{k}\Pi_{H,t+s}\right)\right\}$$

Next, multiply the LHS by $1 = P_{H,t}^{\varepsilon_p - \varepsilon_p}$ so that we can write:

$$\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{1-\varepsilon_p} E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left(\frac{P_{H,t+k}}{P_{H,t}}\right)^{\varepsilon_p} Y_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p}\right)^{1-\varepsilon_p} \right\} =$$

$$= \mathcal{M}_p E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon_p} Y_{t+k} mc_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p}\right)^{-\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{H,t+s}\right) \right\}$$

Following similar steps for the RHS we obtain:

$$\frac{\overline{P}_{H,t}}{P_{H,t}} E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s}\right)^{\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p}\right)^{1-\varepsilon_p} Y_{t+k} \right\} =$$

$$= \mathcal{M}_p E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} Y_{t+k} mc_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p}\right)^{-\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{H,t+s}\right)^{1+\varepsilon_p} \right\}$$

Now let:

$$\mathcal{F}_{p,t} \equiv E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s}\right)^{\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p}\right)^{1-\varepsilon_p} Y_{t+k} \right\}$$

$$\mathcal{K}_{p,t} \equiv E_t \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} Y_{t+k} mc_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p}\right)^{-\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{H,t+s}\right)^{1+\varepsilon_p} \right\}$$

This implies that we can rewrite the first order condition as:

$$\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right) \mathcal{F}_{p,t} = \mathcal{M}_p \mathcal{K}_{p,t}$$

Consider $\mathcal{F}_{p,t}$ and the following facts:

$$\nu_{t,t} = \beta^0 \frac{U_{c,t}}{U_{c,t}} \frac{P_t}{P_t} = 1$$

$$\theta_p^0 = 1$$

$$\prod_{s=1}^{0} \Pi_{H,t+s} = 1$$

Moreover,

$$\frac{U_{c,t+k}}{U_{c,t}} = \prod_{s=1}^{k} \frac{U_{c,t+s}}{U_{c,t+s-1}}$$

$$\nu_{t,t+k} = \beta^k \left(\prod_{s=1}^{k} \frac{U_{c,t+s}}{U_{c,t+s-1}} \right) \frac{1}{\prod_{s=1}^{k} \Pi_{t+s}} = \nu_{t,t+1} \cdots \nu_{t+k-1,t+k}$$

The above results together imply:

$$\begin{aligned}
\mathcal{F}_{p,t} &= Y_t + E_t \sum_{k=1}^{\infty} \theta_p^k \left\{ \nu_{t,t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_p} \right)^{1-\varepsilon_p} Y_{t+k} \right\} \\
&= Y_t + E_t \nu_{t,t+1} \theta_p \Pi_{H,t+1}^{\varepsilon_p} \sum_{k=1}^{\infty} \theta_p^{k-1} \left\{ \nu_{t+1,t+k} \left(\prod_{s=2}^{k} \Pi_{H,t+s} \right)^{\varepsilon_p} \left(\prod_{s=2}^{k} \Pi_{H,t+s}^{\chi_p} \right)^{1-\varepsilon_p} Y_{t+k} \right\} \\
&= Y_t + E_t \beta \theta_p \frac{U_{c,t+1}}{U_c} \frac{1}{\Pi_{t+1}} \Pi_{H,t+1}^{\varepsilon_p} \left(\Pi_{H,t}^{\chi_p} \right)^{1-\varepsilon_p} \times \\
&\times \sum_{k=0}^{\infty} \theta_p^k \left\{ \nu_{t+1,t+1+k} \underbrace{\left(\prod_{s=2}^{k+1} \Pi_{H,t+s} \right)^{\varepsilon_p}}_{\left(\prod_{s=1}^{k} \Pi_{H,t+1+s}\right)^{\varepsilon_p}} \underbrace{\left(\prod_{s=2}^{k} \Pi_{H,t+s}^{\chi_p} \right)^{1-\varepsilon_p}}_{\left(\prod_{s=1}^{k} \Pi_{H,t+s}^{\chi_p}\right)^{1-\varepsilon_p}} Y_{t+1+k} \right\}
\end{aligned}$$

Finally, we can use the law of iterated expectations and write:

$$\mathcal{F}_{p,t} = Y_t + \beta \theta_p \frac{U_{c,t+1}}{U_c} \frac{1}{\Pi_{t+1}} \Pi_{H,t+1}^{\varepsilon_p} \left(\Pi_{H,t}^{\chi_p} \right)^{1-\varepsilon_p} \mathcal{F}_{p,t+1}$$

A similar argument shows:

$$\mathcal{K}_{p,t} = Y_t mc_t + \beta \theta_p \frac{U_{c,t+1}}{U_c} \frac{1}{\Pi_{t+1}} \Pi_{H,t+1}^{1+\varepsilon_p} \left(\Pi_{H,t}^{\chi_p} \right)^{-\varepsilon_p} K_{p,t+1}$$

Finally, we have assumed that those firms that are not allowed to reoptimize will set their prices according to

$$P_{H,t}(j) = \Pi_{H,t-1}^{\chi_p} P_{H,t-1}(j)$$

Hence:

$$P_{H,t}^{1-\varepsilon_p} = \theta_p \Pi_{H,t-1}^{\chi_p(1-\varepsilon_p)} P_{H,t-1}^{1-\varepsilon_p} + (1-\theta_p) \overline{P}_{H,t}^{1-\varepsilon_p}$$

This can be rewritten as:

$$1 = \theta_p \Pi_{H,t-1}^{\chi_p(1-\varepsilon_p)} \Pi_{H,t}^{\varepsilon_p - 1} + (1-\theta_p) \left(\frac{\overline{P}_{H,t}}{P_{H,t}} \right)^{1-\varepsilon_p}$$

The index of price dispersion of domestic good prices follows:

$$
\begin{aligned}
\Delta_{p,t} &= \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di \\
&= (1-\theta_p)\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon_p} + \theta_p(1-\theta_p)\left(\frac{\overline{P}_{H,t-1}\Pi_{H,t-1}^{\chi_p}}{P_{H,t}}\right)^{-\varepsilon_p} + \theta_p^2(1-\theta_p)\left(\frac{\overline{P}_{H,t-2}\Pi_{H,t-1}^{\chi_p}\Pi_{H,t-2}^{\chi_p}}{P_{H,t}}\right)^{-\varepsilon_p} + \\
&= (1-\theta_p)\sum_{k=0}^{\infty}\theta_p^k\left(\frac{\overline{P}_{H,t-k}\left(\prod_{s=1}^k \Pi_{H,t-k+s-1}^{\chi_p}\right)}{P_{H,t}}\right)^{-\varepsilon_p}
\end{aligned}
$$

Therefore we have:

$$
\begin{aligned}
\Delta_{p,t} &= (1-\theta_p)\sum_{k=0}^{\infty}\theta_p^k\left(\frac{\overline{P}_{H,t-k}\left(\prod_{s=1}^k \Pi_{H,t-k+s-1}^{\chi_p}\right)}{P_{H,t}}\right)^{-\varepsilon_p} \\
&= (1-\theta_p)\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon_p} + (1-\theta_p)\sum_{k=1}^{\infty}\theta_p^k\left(\frac{\overline{P}_{H,t-k}\left(\prod_{s=1}^k \Pi_{H,t-k+s-1}^{\chi_p}\right)}{P_{H,t}}\right)^{-\varepsilon_p} \\
&= (1-\theta_p)\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon_p} + (1-\theta_p)\sum_{k=0}^{\infty}\theta_p^{k+1}\left(\frac{P_{H,t-1}}{P_{H,t-1}}\frac{\overline{P}_{H,t-k-1}}{P_{H,t}}\left(\prod_{s=2}^{k}\Pi_{H,t-k+s}^{\chi_p}\right)\right)^{-\varepsilon_p} \\
&= (1-\theta_p)\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon_p} + \theta_p\Pi_{H,t}^{\varepsilon_p}\left(\Pi_{H,t-1}^{\chi_p}\right)^{-\varepsilon_p}\underbrace{(1-\theta_p)\sum_{k=0}^{\infty}\theta_p^k\left(\frac{\overline{P}_{H,t-k-1}}{P_{H,t-1}}\left(\prod_{s=1}^{k}\Pi_{H,t-k+s-1}^{\chi_p}\right)\right)^{-\varepsilon_p}}_{=\Delta_{p,t-1}}
\end{aligned}
$$

Hence:

$$
\Delta_{p,t} = (1-\theta_p)\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)^{-\varepsilon_p} + \theta_p\Pi_{H,t}^{\varepsilon_p}\left(\Pi_{H,t-1}^{\chi_p}\right)^{-\varepsilon_p}\Delta_{p,t-1}
$$

A.10 Import pricing problem with indexation

We that the domestic importers who cannot reoptimize raise their price by a fraction $\chi_{pf} \in [0,1]$ of the PPI of the importer. Formally, the price of importer j at $t+1$, if not reoptimized, will be:

$$
P_{F,t+1}(j) = \Pi_{F,t}^{\chi_{pf}} P_{F,t}(j)
$$

In general:

$$
P_{F,t+k}(j) = P_{F,t}(j)\left(\prod_{s=1}^{k}\Pi_{F,t+s-1}^{\chi_{pf}}\right).
$$

We use the convention of setting:

$$\prod_{s=1}^{0} \Pi_{F,t+s-1}^{\chi_{pf}} = 1$$

The problem of the reoptimizing importer becomes:

$$\max_{\overline{P}_{F,t}} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left[\overline{P}_{F,t} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right) C_{F,t+k|t} - MC_{F,t+k} C_{F,t+k} \right] \right\}$$

subject to:

$$C_{F,t}(f) = N_t(f)^{1-\alpha_F} M_{F,t}(f)^{\alpha_F}$$

and the demand constraints:

$$C_{F,t+k}(f) = \left(\frac{\overline{P}_{F,t} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k}$$

The associated first order condition is given by:

$$E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} C_{F,t+k} \overline{P}_{F,t} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right) \right\} = \mathcal{M}_p E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} C_{F,t+k} MC_{t+k} \right\}$$

Dividing both sides by $P_{F,t}$ and substituting out the demand for firm i conditional on not readjusting the price for k periods, we obtain . We are left with:

$$\frac{1}{P_{F,t}} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\frac{\overline{P}_{F,t} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k} \overline{P}_{F,t} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right) \right\}$$

$$= \frac{1}{P_{F,t}} \mathcal{M}_p E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\frac{\overline{P}_{F,t} \left(\prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_{pf}} \right)}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k} MC_{F,t+k} \right\}$$

The real marginal cost for the importer reads:

$$mc_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$$

Also, we can write:

$$P_{F,t+k} = \frac{P_{F,t+k}}{P_{F,t+k-1}} \cdots \frac{P_{F,t+1}}{P_{F,t}} P_{F,t} = P_{F,t} \prod_{s=1}^{k} \frac{P_{F,t+s}}{P_{F,t+s-1}} = P_{F,t} \prod_{s=1}^{k} \Pi_{F,t+s}$$

We follow the convention of setting:

$$\prod_{s=1}^{0} \Pi_{F,t+s} = 1$$

Then, we are left with:

$$\frac{\left(\overline{P}_{F,t}\right)^{1-\varepsilon_p}}{P_{F,t}} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(P_{F,t+k}\right)^{\varepsilon_p} C_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{1-\varepsilon_p} \right\} =$$

$$= \mathcal{M}_p E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\frac{\overline{P}_{F,t}}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k} mc_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{-\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right) \right\}$$

Next, multiply the LHS by $1 = P_{F,t}^{\varepsilon_p - \varepsilon_p}$ so that we can write:

$$\left(\frac{\overline{P}_{F,t}}{P_{F,t}} \right)^{1-\varepsilon_p} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{\varepsilon_p} C_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{1-\varepsilon_p} \right\} =$$

$$= \mathcal{M}_p \left(\frac{\overline{P}_{F,t}}{P_{F,t}} \right)^{-\varepsilon_p} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} C_{F,t+k} mc_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{-\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{1+\varepsilon_p} \right\}$$

Hence, simplifying:

$$\frac{\overline{P}_{F,t}}{P_{F,t}} E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{\varepsilon_p} C_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{1-\varepsilon_p} \right\} =$$

$$= \mathcal{M}_p E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} C_{F,t+k} mc_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{-\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{1+\varepsilon_p} \right\}$$

Now let:

$$\mathcal{K}_{F,t} \equiv E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{\varepsilon_p} C_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{1-\varepsilon_p} \right\}$$

$$\mathcal{Z}_{F,t} \equiv E_t \sum_{k=0}^{\infty} \theta_{F,p}^k \left\{ \nu_{t,t+k} C_{F,t+k} mc_{F,t+k} \left(\prod_{s=1}^{k} \Pi_{F,t+s-1}^{\chi_{pf}} \right)^{-\varepsilon_p} \left(\prod_{s=1}^{k} \Pi_{F,t+s} \right)^{1+\varepsilon_p} \right\}$$

This implies that we can rewrite the first order condition as:

$$\left(\frac{\overline{P}_{H,t}}{P_{H,t}}\right)\mathcal{K}_{F,t} = \mathcal{M}_p \mathcal{Z}_{F,t}$$

We can write $\mathcal{K}_{F,t}$ and $\mathcal{Z}_{F,t}$ recursively as:

$$\mathcal{K}_{F,t} = C_{F,t} + \theta_{F,p}\beta\frac{U_{c,t+1}}{U_{c,t}\Pi_{t+1}}\Pi_{F,t+1}^{\varepsilon_p}\Pi_{F,t}^{\chi_{pf}(1-\varepsilon_p)}\mathcal{K}_{F,t+1}$$

$$\mathcal{Z}_{F,t} = C_{F,t}\left[\left(w_t\frac{g(S_t)}{S_t}\right)^{1-\xi}\Phi_{F,t}^{\xi}\right] + \theta_{F,p}\beta\frac{U_{c,t+1}}{U_{c,t}\Pi_{t+1}}\Pi_{F,t+1}^{1+\varepsilon_p}\Pi_{F,t}^{-\chi_{pf}\varepsilon_p}\mathcal{Z}_{F,t+1}$$

We have assumed that those firms that are not allowed to reoptimize will set their prices according to

$$P_{F,t}(j) = \Pi_{F,t-1}^{\chi_p}P_{F,t-1}(j)$$

Hence:

$$P_{F,t}^{1-\varepsilon_p} = \theta_{F,p}\Pi_{F,t-1}^{\chi_p(1-\varepsilon_p)}P_{F,t-1}^{1-\varepsilon_p} + (1-\theta_{F,p})\,\overline{P}_{F,t}^{1-\varepsilon_p}$$

This can be rewritten as:

$$1 = \theta_{F,p}\Pi_{F,t-1}^{\chi_p(1-\varepsilon_p)}\Pi_{F,t}^{\varepsilon_p-1} + (1-\theta_{F,p})\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{1-\varepsilon_p}$$

A.11 Wage setting problem with indexation

Households who do not reoptimize are not allowed to change their wage and update it according to the following rule:

$$W_{t+1}(j) = \Pi_t^{\chi_w}W_t(j)$$

Notice that Π_t is CPI inflation. Mor generally, this reads:

$$W_{t+k}(j) = W_t(j)\left(\prod_{s=1}^{k}\Pi_{t+s-1}^{\chi_w}\right)$$

We use the convention of setting:

$$\prod_{s=1}^{0}\Pi_{t+s-1}^{\chi_p} = 1$$

The problem reads (abstracting from consumption habits):

$$\max_{\overline{W}_t} E_t\sum_{k=0}^{\infty}(\beta\theta_p)^k\left\{U\left(C_{t+k|t}, N_{t+k|t}\right) - \tilde{\lambda}_{t+k|t}\left[P_{t+k}C_{t+k|t} - \overline{W}_t\left(\prod_{s=1}^{k}\Pi_{t+s-1}^{\chi_w}\right)N_{t+k|t}\right]\right\}$$

The demand for labor of type j conditioning on the household not reoptimizing for k periods reads:

$$N_{t+k|t} = \left(\frac{\overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k}$$

Hence the first order condition reads:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ U_{n,t+k|t} \frac{\partial N_{t+k|t}}{\partial \overline{W}_t} + \tilde{\lambda}_{t+k|t} \left(N_{t+k|t} + \overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right) \frac{\partial N_{t+k|t}}{\partial \overline{W}_t} \right) \right\} = 0$$

Notice that:

$$\frac{\partial N_{t+k|t}}{\partial \overline{W}_t} = -\varepsilon_w \left(\frac{\overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}{W_{t+k}} \right)^{-\varepsilon_w - 1} \frac{N_{t+k}}{W_{t+k}} = -\varepsilon_w N_{t+k|t} \frac{1}{\overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}$$

Using the last expression and rearranging:

$$-\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ U_{n,t+k|t} \varepsilon_w N_{t+k|t} \frac{1}{\overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)} + \tilde{\lambda}_{t+k|t} N_{t+k|t} (\varepsilon_w - 1) \right\} = 0$$

Dividing through by $\varepsilon_w - 1$, multiplying by $\overline{W}_t$, and using $U_{c,t+k} = \tilde{\lambda}_{t+k} P_{t+k}$, it yields:

$$-\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ U_{n,t+k|t} N_{t+k|t} \mathcal{M}_w \frac{1}{\left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)} + U_{c,t+k} N_{t+k|t} \frac{\overline{W}_t}{P_{t+k}} \right\} = 0$$

Finally, rearranging:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ U_{c,t+k} N_{t+k|t} \left[\frac{U_{n,t+k|t}}{U_{c,t+k}} \mathcal{M}_w \frac{1}{\left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)} + \frac{\overline{W}_t}{P_{t+k}} \right] \right\} = 0$$

Rewrite the above condition as:

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U_{c,t+k} N_{t+k|t} \frac{\overline{W}_t}{P_{t+k}} = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k N_{t+k|t} \mathcal{M}_w \left(-U_{n,t+k|t} \right) \frac{1}{\left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}$$

Recall the optimal labor demand condition

$$N_{t+k|t} = \left(\frac{\overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k}$$

So, the LHS can be written as

$$
\begin{aligned}
LHS &\equiv E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U_{c,t+k} \left(\frac{\overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k} \frac{\overline{W}_t}{P_{t+k}} \\
&= \left(\frac{\overline{W}_t}{P_t} \right)^{1-\varepsilon_w} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U_{c,t+k} N_{t+k} \left(\frac{W_{t+k}}{P_{t+k}} \right)^{\varepsilon_w} \left(\prod_{s=1}^{k} \Pi_{t+s} \right)^{\varepsilon_w - 1} \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)^{-\varepsilon_w}
\end{aligned}
$$

Next, the RHS can be expressed as:

$$
RHS = \left(\frac{\overline{W}_t}{P_t} \right)^{-\varepsilon_w} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{W_{t+k}}{P_{t+k}} \right)^{\varepsilon_w} \left(\prod_{s=1}^{k} \Pi_{t+s} \right)^{\varepsilon_w} \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)^{-\varepsilon_w} N_{t+k} \mathcal{M}_w \left(-U_{n,t+k|t} \right)
$$

Under the assumption that $U_n(\cdot)$ is homogeneous of degree φ in N we have that:

$$
\begin{aligned}
-U_{n,t+k|t} &= \left(\frac{\overline{W}_t \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}{W_{t+k}} \right)^{-\varepsilon_w \varphi} \left(-U_{n,t+k} \left(N_{t+k} \right) \right) \\
&= \left(\frac{\frac{\overline{W}_t}{P_t} \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)}{\frac{W_{t+k}}{P_{t+k}}} \right)^{-\varepsilon_w \varphi} \left(\prod_{s=1}^{k} \Pi_{t+s} \right)^{\varepsilon_w \varphi} \left(-U_{n,t+k} \left(N_{t+k} \right) \right)
\end{aligned}
$$

Substituting, we obtain:

$$
RHS = \left(\frac{\overline{W}_t}{P_t} \right)^{-\varepsilon_w (1+\varphi)} \mathcal{M}_w E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{W_{t+k}}{P_{t+k}} \right)^{(1+\varphi)\varepsilon_w} N_{t+k} \left(\prod_{s=1}^{k} \Pi_{t+s} \right)^{\varepsilon_w (1+\varphi)} \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)^{-\varepsilon_w (1+\varphi)}
$$

Hence the whole optimality condition reads:

$$
\left(\frac{\overline{W}_t}{P_t} \right)^{1+\varphi \varepsilon_w} \mathcal{F}_{w,t} = \mathcal{M}_w \mathcal{K}_{w,t},
$$

where

$$
\mathcal{F}_{w,t} \equiv E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U_{c,t+k} N_{t+k} \left(\frac{W_{t+k}}{P_{t+k}} \right)^{\varepsilon_w} \left(\prod_{s=1}^{k} \Pi_{t+s} \right)^{\varepsilon_w - 1} \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)^{-\varepsilon_w}
$$

$$
\mathcal{K}_{w,t} \equiv E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{W_{t+k}}{P_{t+k}} \right)^{(1+\varphi)\varepsilon_w} N_{t+k} \left(\prod_{s=1}^{k} \Pi_{t+s} \right)^{\varepsilon_w (1+\varphi)} \left(-U_{n,t+k} \left(N_{t+k} \right) \right) \left(\prod_{s=1}^{k} \Pi_{t+s-1}^{\chi_w} \right)^{-\varepsilon_w (1+\varphi)}
$$

The above conditions can be written recursively as:

$$\mathcal{F}_{w,t} = \left(\frac{W_t}{P_t}\right)^{\varepsilon_w} N_t U_{c,t} + \beta\theta_w \Pi_{t+1}^{\varepsilon_w - 1} \left(\Pi_t^{\chi_w}\right)^{-\varepsilon_w} \mathcal{F}_{w,t+1}$$

$$\mathcal{K}_{w,t} = \left(\frac{W_t}{P_t}\right)^{\varepsilon_w(1+\varphi)} N_t \left(-U_{n,t}\left(N_t\right)\right) + \beta\theta_w \Pi_{t+1}^{(1+\varphi)\varepsilon_w} \left(\Pi_t^{\chi_w}\right)^{-\varepsilon_w(1+\varphi)} \mathcal{K}_{w,t+1}$$

A household who does not reoptimize will set the real wage equal to:

$$w_{t+1}(j) \equiv \frac{W_{t+1}(j)}{P_{t+1}} = \frac{\Pi_t^{\chi_w} W_t(j)}{P_{t+1}} \frac{P_t}{P_t} = \frac{W_t(j)}{P_t} \frac{\Pi_t^{\chi_w}}{\Pi_t} = w_t(j) \frac{\Pi_t^{\chi_w}}{\Pi_t}$$

The average real wage in any given period t depends on the previous period average real wage and on the reoptimized one:

$$\left(\frac{W_t}{P_t}\right)^{1-\varepsilon_w} = \theta_w \left(\frac{W_{t-1}}{P_{t-1}} \frac{\Pi_t^{\chi_w}}{\Pi_t}\right)^{1-\varepsilon_w} + (1-\theta_w)\left(\frac{\overline{W}_t}{P_t}\right)^{1-\varepsilon_w}$$

The dispersion in wages effectively drives a (time-varying) wedge between the supply and demand of labor:[1]

$$N_t(i) = \int_0^1 N_t^d(i,j)\,dj = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t^d(i)\,dj = N_t^d(i) \underbrace{\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} dj}_{\Delta_{w,t}}$$

Clearly, we can integrate across all firms, to obtain:

$$N_t = \int_0^1 N_t(i)\,di = \Delta_{w,t} \int_0^1 N_t^d(i)\,di = \Delta_{w,t} N_t^d$$

The wage dispersion index $\Delta_{w,t}$ can then be written:

$$\begin{aligned}
\Delta_{w,t} &= \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} dj \\
&= (1-\theta_w)\left(\frac{\overline{W}_t}{W_t}\right)^{-\varepsilon_w} + \theta_w(1-\theta_w)\left(\frac{\overline{W}_{t-1}\Pi_{t-1}^{\chi_w}}{W_t}\right)^{-\varepsilon_w} + \theta_w^2(1-\theta_w)\left(\frac{\overline{W}_{t-2}\Pi_{t-1}^{\chi_w}\Pi_{t-2}^{\chi_w}}{W_t}\right)^{-\varepsilon_w} + \ldots \\
&= (1-\theta_w)\sum_{k=0}^{\infty}\theta_w^k \left(\frac{\overline{W}_{t-k}\left(\prod_{s=1}^{k}\Pi_{t+s-1}^{\chi_w}\right)}{W_t}\right)^{-\varepsilon_w}
\end{aligned}$$

[1] This is true for all labor types that are hired by each firm i. Indeed, all firms hire all types of labor.

B A small open economy with traded and non-traded goods

In this section we extend our baseline DSGE model to allow for traded and non-traded goods. Let consumption demand be given by:

$$C_t \equiv \left((1-\gamma)^{\frac{1}{\xi}} C_{N,t}^{1-\frac{1}{\xi}} + \gamma^{\frac{1}{\xi}} C_{T,t}^{1-\frac{1}{\xi}}\right)^{\frac{\xi}{\xi-1}} \quad \xi > 0 \tag{38}$$

where $C_{N,t}$ is a composite index of consumption of non-traded goods, and $C_{T,t}$ is a composite index of traded goods, in turn given by:

$$C_{T,t} \equiv \left((1-\upsilon)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + \upsilon^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \quad \eta > 0, \tag{39}$$

where $C_{\iota,t} \equiv \left(\int_0^1 C_{\iota,t}(i)^{\frac{\epsilon_p-1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$ is an index of domestic goods consumption in the domestic sector $\iota = H, N$,with $i \in [0,1]$ denoting the good variety. $C_{F,t} \equiv \left(\int_0^1 C_{F,t}(f)^{\frac{\epsilon_p-1}{\epsilon_p}} df\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$ is the quantity consumed of a composite foreign good. Parameter $\epsilon_p > 1$ denotes the elasticity of substitution between varieties produced domestically (whether traded or non-traded).

Optimal consumption demand for variety i in sector $\iota = H, N$ requires:

$$C_{\iota,t}(i) = (P_{\iota,t}(i)/P_{\iota,t})^{-\varepsilon_p} C_{\iota,t} \quad (\iota = H, N) \tag{40}$$

Optimal demand for imported variety f reads:

$$C_{F,t}(f) = (P_{F,t}(f)/P_{F,t})^{-\varepsilon_p} C_{F,t} \tag{41}$$

Also:

$$C_{H,t} = (1-\upsilon)(P_{H,t}/P_{T,t})^{-\eta} C_{T,t} \quad ; \quad C_{F,t} = \upsilon(P_{F,t}/P_{T,t})^{-\eta} C_{T,t} \tag{42}$$

$$C_{N,t} = (1-\gamma)(P_{N,t}/P_t)^{-\xi} C_t \quad ; \quad C_{T,t} = \gamma(P_{T,t}/P_t)^{-\xi} C_t \tag{43}$$

where

$$P_{\iota,t} \equiv \left(\int_0^1 P_{\iota,t}^{1-\varepsilon_p}(i)\right)^{\frac{1}{1-\varepsilon_p}} di$$

is the utility based price index in sector $\iota = H, N$,

$$P_{T,t} \equiv \left((1-\upsilon) P_{H,t}^{1-\eta} + \upsilon P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

is the utility-based price index of traded goods and

$$P_t \equiv \left((1-\gamma)P_{N,t}^{1-\xi} + \gamma P_{T,t}^{1-\xi}\right)^{\frac{1}{1-\xi}}$$

is the utility-based aggregate CPI index.

Using the above equilibrium conditions it holds:

$$P_{N,t}C_{N,t} + P_{T,t}C_{T,t} = P_t C_t$$

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_{T,t}C_{T,t}$$

$$\int_0^1 P_{N,t}(i)C_{N,t}(i)di = P_{N,t}C_{N,t}$$

$$\int_0^1 P_{H,t}(i)C_{H,t}(i) + P_{F,t}(i)C_{F,t}(i)di = P_{T,t}C_{T,t}$$

Terms of trade, relative price of tradables and real exchange rate We define the *terms of trade* as the relative price of the imported good

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}.$$

We also define:

$$\frac{P_{T,t}}{P_{H,t}} = [(1-\upsilon) + \upsilon \mathcal{S}_t{}^{1-\eta}]^{\frac{1}{1-\eta}} \equiv q(\mathcal{S}_t) \tag{44}$$

with $q^{'}(\mathcal{S}_t) > 0$.

The *relative price of tradables* is :

$$\mathcal{T}_t \equiv \frac{P_{T,t}}{P_{N,t}}$$

We also define

$$\frac{P_t}{P_{N,t}} = [(1-\gamma) + \gamma \mathcal{T}_t{}^{1-\xi}]^{\frac{1}{1-\xi}} \equiv h(\mathcal{T}_t) \tag{45}$$

with $h^{'}(\mathcal{T}_t) > 0$.

Since $P_t^* = P_{F,t}^*$, and under the assumption $P_{F,t}^* = 1$, the consumption real exchange rate reads:

$$\mathcal{Q}_t \equiv \frac{\mathcal{E}_t}{P_t}.$$

B.1 Optimal pricing

Each monopolistic firm i in sector ι (i.e., domestic *tradable* and *nontradable)* produces a differentiated good according to the *CRS* production function:

$$Y_{\iota,t}(i) = N_{\iota,t}(i), \quad \iota = N, H \tag{46}$$

The cost minimizing choice of labor implies:

$$\frac{W_t}{P_{\iota,t}(i)} = \frac{MC_{\iota,t}}{P_{\iota,t}(i)} \tag{47}$$

where MC_ι denotes the nominal marginal cost in sector ι.

Optimal pricing in sector j implies:

$$\overline{p}_{\iota,t} \ = \mathcal{M}_p \frac{E_t\left\{\sum_{k=0}^{\infty}\theta_{p,\iota}^k \nu_{t,t+k} Y_{\iota,t+k} mc_{\iota,t+k}\left(\prod_{s=1}^k \Pi_{\iota,t+s}\right)^{1+\varepsilon_p}\right\}}{E_t\left\{\sum_{k=0}^{\infty}\theta_{p,\iota}^k \nu_{t,t+k} Y_{\iota,t+k}\left(\prod_{s=1}^k \Pi_{\iota,t+s}\right)^{\varepsilon_p}\right\}}$$

where $\overline{p}_{\iota,t} \equiv \overline{P}_{\iota,t}/P_{\iota,t}$ is the optimally chosen individual price in sector j (expressed as a ratio to the average price in sector ι), and $mc_{\iota,t}$ is the real marginal cost expressed in units of goods produced in sector j.

Define:

$$\mathcal{K}_{p_\iota,t} \equiv E_t \sum_{k=0}^{\infty}\theta_{p,\iota}^k \nu_{t,t+k} Y_{\iota,t+k}\left(\prod_{s=1}^k \Pi_{\iota,t+s}\right)^{\varepsilon_p}$$

$$\mathcal{Z}_{p_\iota,t} \equiv E_t \sum_{k=0}^{\infty}\theta_{p,\iota}^k \left\{\nu_{t,t+k} Y_{\iota,t+k} mc_{\iota,t+k}\left(\prod_{s=1}^k \Pi_{\iota,t+s}\right)^{1+\varepsilon_p}\right\}$$

Therefore the optimal pricing condition reads:

$$\overline{p}_{\iota,t} \ = \mathcal{M}_p \frac{\mathcal{Z}_{p_\iota,t}}{\mathcal{K}_{p_\iota,t}}$$

Expressing recursively as:

$$\mathcal{K}_{p_\iota,t} = Y_{\iota,t} + \theta_{p,\iota} E_t\left\{\left(\beta\frac{U_{c,t+1}}{U_{c,t}\Pi_{t+1}}\right)\Pi_{\iota,t+1}^{\varepsilon_p}\mathcal{K}_{p_\iota,t+1}\right\}$$

Similarly

$$\mathcal{Z}_{p_\iota,t} = Y_{\iota,t} mc_{\iota,t} + \theta_{p,\iota} E_t\left\{\left(\beta\frac{U_{c,t+1}}{U_{c,t}\Pi_{t+1}}\right)\Pi_{\iota,t+1}^{1+\varepsilon_p}\mathcal{Z}_{p_\iota,t+1}\right\}$$

We also have:

$$1=\theta_{p,\iota}(\Pi_{\iota,t})^{\varepsilon_p-1}+(1-\theta_{p,\iota})\overline{p}_{\iota,t}^{1-\varepsilon_p}. \tag{48}$$

Finally, and letting $w_t \equiv W_t/P_t$ denote the CPI real wage, notice that:

$$mc_{\iota,t}=\begin{cases} w_t h(\mathcal{T}_t) & \text{if } \iota = N \\ w_t h(\mathcal{T}_t) q(\mathcal{S}_t)/\mathcal{T}_t & \text{if } \iota = H \end{cases}$$

B.2 Export Demand

We assume that total export demand (for domestically produced traded good i), X_t, takes the following form (recalling $P_t^* = P_{F,t}^* = 1$ for all t):

$$\begin{aligned} X_t &= \upsilon \cdot \left(\frac{P_{H,t}}{\mathcal{E}_t}\right)^{-\eta} Y_t^* \\ &= \upsilon \cdot \mathcal{S}_t^{\eta} Y_t^* \end{aligned}$$

B.3 Equilibrium with tradables and nontradables

Equilibrium in the market for each differentiated variety i in sector ι requires:

$$N_{\iota,t}(i)=\left(\frac{P_{\iota,t}(i)}{P_{\iota,t}}\right)^{-\varepsilon_p} Y_{\iota,t}^d$$

where $Y_{\iota,t}^d$ is total demand in sector $\iota = H, N$.

Integrating it yields:

$$\int_0^1 N_{\iota,t}(i)di \equiv N_{\iota,t} = \Delta_{p_\iota,t} Y_{\iota,t}^d$$

where $\Delta_{p_\iota,t} \equiv \int_0^1 \left(\frac{P_{\iota,t}(i)}{P_{\iota,t}}\right)^{-\varepsilon_p} di$ denotes price dispersion in the sector ι, and where

$$Y_{\iota,t}^d \equiv \begin{cases} (1-\gamma)h(\mathcal{T}_t)^{\xi} C_t & \text{if } \iota = N \\ (1-\upsilon)q(\mathcal{S}_t)^{\eta}\gamma\left(\frac{h(\mathcal{T}_t)}{\mathcal{T}_t}\right)^{\xi} C_t + \upsilon \mathcal{S}_t^{\eta} Y_t^* & \text{if } \iota = H \end{cases}$$

B.4 Equilibrium conditions

Let the pricing block in sector $\iota = H, N$ be defined by the set of processes:

$$\mathcal{P}_{\iota,t} \equiv \left\{\overline{p}_{\iota,t},\ \mathcal{K}_{p_\iota,t}, \mathcal{Z}_{p_\iota,t},\ \Delta_{p_\iota,t}\right\}$$

An equilibrium in the sticky-price model with tradables and nontradables is a set of processes

$$\{\mathcal{P}_{\iota,t},\ mc_{\iota,t},\ \Pi_{\iota,t}N_{\iota,t},\ N_t, w_t,\ \mathcal{S}_t, \mathcal{T}_t, \Pi_t,\Pi_{T,t}, C_t, R_t\}$$

which solve:

- Pricing block in sector $\iota = N, H$

$$\overline{p}_{\iota,t}\ \mathcal{K}_{p_\iota,t} = \mathcal{M}_p \mathcal{Z}_{p_\iota,t}$$

$$\mathcal{K}_{p_\iota,t} = N_{\iota,t} + \theta_{p,\iota} E_t \left\{ \left(\beta \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}\Pi_{t+1}} \right) \Pi_{\iota,t+1}^{\varepsilon_p} \mathcal{K}_{p_\iota,t+1} \right\}$$

$$\mathcal{Z}_{p_\iota,t} = N_{\iota,t}\ mc_{\iota,t} + \theta_{p,\iota} E_t \left\{ \left(\beta \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}\Pi_{t+1}} \right) \Pi_{\iota,t+1}^{1+\varepsilon_p} \mathcal{Z}_{p_\iota,t+1} \right\}$$

$$1 = \theta_{p,\iota}(\Pi_{\iota,t})^{\varepsilon_p - 1} + (1-\theta_{p,\iota})\overline{p}_{\iota,t}^{1-\varepsilon_p}.$$

$$\Delta_{p_\iota,t} = (1-\theta_{p,\iota}) \left(\overline{p}_{\iota,t}\right)^{-\varepsilon_p} + \theta_{p,\iota}\Pi_{\iota,t}^{\varepsilon_p}\Delta_{p_\iota,t-1}$$

$$mc_{\iota,t} = \begin{cases} w_t h(\mathcal{T}_t) & \text{if } \iota = N \\ w_t h(\mathcal{T}_t) q(\mathcal{S}_t)/\mathcal{T}_t & \text{if } \iota = H \end{cases}$$

- Goods market equilibrium: nontradables

$$N_{N,t} = \Delta_{p_N,t}(1-\gamma)h(\mathcal{S}_t)^{\xi} C_t$$

- Goods market equilibrium: tradables

$$N_{H,t} = \Delta_{p_H,t} \left[(1-\upsilon) q(\mathcal{S}_t)^{\eta} \gamma \left(\frac{h(\mathcal{T}_t)}{\mathcal{T}_t} \right)^{\xi} C_t \right] + \upsilon \mathcal{S}_t^{\eta} Y_t^*$$

- Labor market equilibrium

$$\underbrace{N_t}_{\text{labor supply}} = \sum_{\iota = N,H} N_{\iota,t}$$

- Risk-sharing

$$\left(\frac{C_t^*}{C_t} \right)^{-\sigma} = \left(\frac{\mathcal{S}_t}{q(\mathcal{S}_t)} \right) \left(\frac{\mathcal{T}_t}{h(\mathcal{T}_t)} \right)$$

- Consumption Euler

$$C_t^{-\sigma} = E_t \left\{ \frac{C_{t+1}^{-\sigma} R_t}{\Pi_{t+1}} \right\}$$

- Consumption-leisure

$$C_t^{\sigma} N_t^{\varphi} = w_t$$

- CPI inflation

$$\Pi_t = \frac{h(\mathcal{T}_t)}{h(\mathcal{T}_{t-1})} \Pi_{N,t}$$

- Traded good inflation

$$\Pi_{T,t} = \frac{q(\mathcal{S}_t)}{q(\mathcal{S}_{t-1})} \Pi_{H,t}$$

- Relative price of tradables

$$\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\Pi_{T,t}}{\Pi_{N,t}}$$

- Monetary policy rule

$$R_t = f(\Pi_t)$$

B.5 Deviations from the law of one price

In the presence of nominal stickiness in import prices and/or local distribution costs the law of one price in tradables ceases to hold. Let the *law-of-one-price gap* be denoted by:

$$\Phi_{F,t} \equiv \frac{\mathcal{E}_t}{P_{F,t}}$$

Next we show how to relate the consumption real exchange rate to the three key relative prices of our setup: the terms of trade $\mathcal{S}_t$, the relative price of tradables $\mathcal{S}_{T,t}$ and the law of one price gap $\Phi_{F,t}$. From the definition of consumption real exchange rate:

$$\mathcal{Q}_t = \Phi_{F,t} \frac{\mathcal{S}_t}{P_t / P_{H,t}} \tag{49}$$

Notice that can write:

$$\begin{aligned} \frac{P_t}{P_{H,t}} &= \frac{P_t}{P_{N,t}} \frac{P_{N,t}}{P_{T,t}} \frac{P_{T,t}}{P_{H,t}} \\ &= \frac{h(\mathcal{T}_t)}{\mathcal{T}_t} q(\mathcal{T}_t) \end{aligned}$$

Combining we can finally write:

$$\mathcal{Q}_t = \underbrace{\left(\frac{\mathcal{S}_t}{q(\mathcal{S}_t)}\right)}_{\substack{\propto \text{terms of} \\ \text{trade}}} \times \underbrace{\Phi_{F,t}}_{\text{lop gap}} \times \underbrace{\left(\frac{\mathcal{T}_t}{h(\mathcal{T}_t)}\right)}_{\substack{\propto \text{relative price} \\ \text{of tradables}}} \equiv \mathcal{Q}(\mathcal{S}_t, \Phi_{F,t}, \mathcal{T}_t) \tag{50}$$

The above expression shows how movements in the consumption real exchange rate can be decomposed, respectively, into movements of the terms of trade (captured by the term $\mathcal{S}_t/q(\mathcal{S}_t)$), deviations from the law of one price in tradables (captured by the term $\Phi_{F,t}$), and movements in the relative price of tradables (captured by the term $\mathcal{T}_t/h(\mathcal{T}_t)$). Notice that in the case of law of one price holding at all times, $\Phi_{F,t} = 1$ for all t. Also, in the case in which all goods are tradable, $\mathcal{T}_t = h(\mathcal{T}_t) = 1$.

B.6 Distribution costs in tradables

Each variety produced in the rest of the world is distributed to the final consumer by a differentiated local importer. Distributing C_F units of imported good to the local consumer requires combining $M_{F,t}$ units of a homogeneous imported input with labor, according to the following constant return to scale production function:

$$C_{F,t}(f) = N_{F,t}(f)^{1-\alpha_F} M_{F,t}^{\alpha_F}(f) \tag{51}$$

where $N_{F,t}(f)$ and $M_{F,t}(f)$ denote the quantity of imported input and of labor respectively employed by the local importer f.

Let $P^*_{F,t} = 1$ be the "dock price" of the imported input (expressed in units of foreign currency), and let $P_{F,t}(f)$ be the local currency price of the distributed imported variety. We assume that the import prices are flexible also at the consumer level.

Conditional on (51), the cost minimizing choice of imported inputs and labor requires:

$$\frac{W_t}{P_{F,t}(f)} = \frac{MC_{F,t}(f)}{P_{F,t}(f)} (1-\alpha_F) \left(\frac{M_{F,t}(f)}{N_{F,t}(f)}\right)^{\alpha_F} \tag{52}$$

$$\Phi_{F,t} = \frac{MC_{F,t}(f)}{P_{F,t}(f)} \alpha_F \left(\frac{N_{F,t}(f)}{M_{F,t}(f)}\right)^{1-\alpha_F}, \tag{53}$$

where MC_F denotes the nominal marginal cost of the local importer.

The above conditions imply:

$$MC_{F,t}(f) = \frac{W_t^{1-\alpha_F} \mathcal{E}_t^{\alpha_F}}{\alpha_F^{\alpha_F} (1-\alpha_F)^{1-\widetilde{\alpha}_{\iota F}}} \equiv MC_{F,t} \tag{54}$$

Hence the nominal marginal cost is common across local importers.

Conditional on the optimality condition (54), the local importer solves:

$$\max \; E_t \sum_{k=0}^{\infty} \nu_{t,t+k} \left\{ \left[P_{F,t+k}(f) - MC_{F,t+k} \right] C_{F,t+k}(f) \right\}$$

subject to the optimal demand function for the imported good:

$$C_{F,t+k}(f) = \left(\frac{P_{F,t+k}(f)}{P_{F,t+k}} \right)^{-\varepsilon_p} C_{F,t+k} \tag{55}$$

The first order condition of this problem requires:

$$P_{F,t}(f) = P_{F,t} = \frac{\varepsilon_p}{\varepsilon_p - 1} MC_{F,t}$$

Expressing in real terms:

$$\begin{aligned} mc_{F,t} &\equiv \frac{MC_{F,t}}{P_{F,t}} \\ &= \left(\frac{w_t \; q(\mathcal{S}_t)}{\mathcal{S}_t} \quad \underbrace{\frac{h(\mathcal{T}_t)}{\mathcal{T}_t}}_{\substack{=1 \text{ if all} \\ \text{goods tradables}}} \right)^{1-\alpha_F} \Phi_{F,t}^{\alpha_F} \end{aligned}$$

Hence, besides the real consumption wage, movements in both the terms of trade and the relative price of tradables affect the importer's marginal cost.

The presence of distribution costs implies that aggregate export demand should be written:

$$\begin{aligned} X_t &= \upsilon \cdot \left(\frac{P_{H,t}}{\mathcal{E}_t} \right)^{-\eta} Y_t^* \\ &= \upsilon \cdot \mathcal{S}_t^{\eta} \Phi_{F,t}^{\eta} Y_t^* \end{aligned}$$

Market clearing for variety i:

$$N_{\iota,t}(i) = \left(\frac{P_{\iota,t}(i)}{P_{\iota,t}} \right)^{-\varepsilon_p} Y_{\iota,t}^d$$

Integrating

$$N_{\iota,t} \equiv \int_0^1 N_{\iota,t}(i) di = \Delta_{p_\iota,t} Y_{\iota,t}^d$$

where

$$Y^d_{\iota,t} \equiv \begin{cases} (1-\gamma)h(\mathcal{T}_t)^{\xi}C_t & \text{if } \iota = N \\ (1-\upsilon)q(\mathcal{S}_t)^{\eta}\gamma\left(\frac{h(\mathcal{T}_t)}{\mathcal{T}_t}\right)^{\xi}C_t + \mathcal{S}_t^{\eta}\Phi^{\eta}_{F,t}Y^*_t & \text{if } \iota = H \end{cases}$$

B.7 Full model with capital and nontradables

We assume that physical capital is employed in the non-traded sector only. The production function in sector ι reads:

$$Y_{\iota,t}(i) = A_{\iota,t}K^{\widetilde{\alpha}_{\iota}}_{\iota,t}(i)N_{\iota,t}(i)^{1-\widetilde{\alpha}_{\iota}} \tag{56}$$

where $\widetilde{\alpha}_{\iota} \equiv \zeta\alpha_{\iota}$, and with $\zeta = 1$ if $\iota = N$, and $\zeta = 0$ if $\iota = H$.

Market clearing for each individual domestic variety i in sector $\iota = H, N$ implies:

$$\underbrace{Y_{\iota,t}(i)}_{\substack{\text{supply of}\\ \text{variety i}\\ \text{in sector } \iota}} = \underbrace{\left(\frac{P_{\iota,t}(i)}{P_{\iota,t}}\right)^{-\varepsilon_p} Y^d_{\iota,t}}_{\substack{\text{demand of}\\ \text{variety i in}\\ \text{sector } \iota}} \tag{57}$$

Rearranging:

$$N_{\iota,t}(i) = \left[\left(\frac{P_{\iota,t}(i)}{P_{\iota,t}}\right)^{-\varepsilon_p} \frac{Y^d_{\iota,t}}{A_t(K_{\iota,t}/N_{\iota,t})^{\widetilde{\alpha}_{\iota}}}\right]$$

where we used the fact that, in equilibrium, all firms in sector ι choose the same capital labor ratio.

Integrating across all producers in sector ι:

$$\int_0^1 N_{\iota,t}(i)di = \int_0^1 \left[\left(\frac{P_{\iota,,t}(i)}{P_{\iota,,t}}\right)^{-\varepsilon_p} \frac{Y^d_{\iota,t}}{A_{\iota,t}(K_{\iota,t}/N_{\iota,t})^{\widetilde{\alpha}_{\iota}}}\right] di \tag{58}$$

$$= \frac{Y_{\iota,t}}{A_{\iota,t}(K_{\iota,t}/N_{\iota,t})^{\widetilde{\alpha}_{\iota}}} \int_0^1 \left(\frac{P_{\iota,,t}(i)}{P_{\iota,,t}}\right)^{-\varepsilon_p} di = \frac{Y_t}{A_{\iota,t}(K_{\iota,t}/N_{\iota,t})^{\widetilde{\alpha}_{\iota}}}\Delta_{p_{\iota},,t}, \tag{59}$$

where $\Delta_{p_{\iota,t}} \equiv \int_0^1 \left(\frac{P_{\iota,,t}(i)}{P_{\iota,t}}\right)^{-\varepsilon_p} di$ measures the dispersion of relative prices across domestic producers. In a more compact form:

$$N_{\iota,t} = \frac{Y^d_{\iota,t}}{A_{\iota,t}(K_{\iota,t}/N_{\iota,t})^{\widetilde{\alpha}_{\iota}}}\Delta_{p_{\iota},,t},$$

where $N_{\iota,t} \equiv \int_0^1 N_{\iota,t}(i)di$.

Hence we can finally write, for each sector ι:

$$A_{\iota,t}K_{\iota,t}^{\widetilde{\alpha}_\iota}N_{\iota,t}^{1-\widetilde{\alpha}_\iota} = Y_{\iota,t}^d \Delta_{p_\iota,t} \tag{60}$$

Expressing $\Delta_{p,t}$ in recursive form:

$$\Delta_{p_\iota,t} = (1-\theta_{p,\iota})\left(\frac{\overline{P}_{\iota,t}}{P_{\iota,t}}\right)^{-\varepsilon_p} + \theta_{p,\iota}\Pi_{\iota,,t}^{\varepsilon_p}\Delta_{p_\iota,,t-1}$$

Market clearing Market clearing in sector $\iota \in H, N$

$$A_{\iota,t}K_{\iota,t}^{\widetilde{\alpha}_\iota}N_{\iota,t}^{1-\widetilde{\alpha}_\iota} = Y_{\iota,t}^d \Delta_{p_\iota,t} \tag{61}$$

where

$$Y_{\iota,t}^d \equiv \begin{cases} (1-\gamma)h(\mathcal{T}_t)^\xi (C_t + I_t) & \text{if } \iota = N \\ (1-\upsilon)q(\mathcal{S}_t)^\eta \gamma \left(\frac{h(\mathcal{T}_t)}{\mathcal{T}_t}\right)^\xi (C_t + I_t) + \mathcal{S}_t^\eta \Phi_{F,t}^\eta Y_t^* & \text{if } \iota = H \end{cases}$$

Labor market equilibrium *Total* demand for each labor type j reads:

$$\begin{aligned} N_t(j) &= \left(\sum_{\iota\in H,N}\int_0^1 N_{\iota,t}(i,j)di\right) + \int_0^1 N_t(f,j)df \\ &= \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w}\left[\left(\sum_{\iota\in H,N}\int_0^1 N_{\iota,t}(i)di\right) + \int_0^1 N_t(f)df\right] \\ &= \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t. \end{aligned}$$

where

$$N_t \equiv \left(\sum_{\iota\in H,N} N_{\iota,t}\right) + N_{F,t},$$

and $N_{\iota,t} \equiv \int_0^1 N_{\iota,t}(i)di$, $N_{F,t} \equiv \int_0^1 N_t(f)df$.

Let $\mathcal{N}_t(j)$ denote labor supply by each differentiated household. Since each household is assumed to satisfy labor demand at the given posted wage, equilibrium in the labor market requires:

$$\mathcal{N}_t(j) = N_t(j)$$

Aggregating across each household j one obtains, using (23):

$$\mathcal{N}_t \equiv \int_0^1 \mathcal{N}_t(j)dj = \int_0^1 N_t(j) = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} dj\ N_t$$

where $\mathcal{N}_t$ is an index of *aggregate* labor supply. By defining $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w}$ as an index of wage dispersion, the above equation becomes.

$$\mathcal{N}_t = \Delta_{w,t} N_t. \tag{62}$$

B.8 Full set of equilibrium conditions

- Pricing block in sector $\iota = N, H$

$$\overline{p}_{\iota,t}\ \mathcal{K}_{p_\iota,t} = \mathcal{M}_p \mathcal{Z}_{p_\iota,t}$$

$$\mathcal{K}_{p_\iota,t} = Y_{\iota,t} + \theta_{p,\iota} E_t \left\{ \left(\beta \frac{C_{t+1}^{\sigma}}{C_t^{\sigma} \Pi_{t+1}} \right) \Pi_{\iota,t+1}^{\varepsilon_p} \mathcal{K}_{p_\iota,t+1} \right\}$$

$$\mathcal{Z}_{p_\iota,t} = Y_{\iota,t}\ mc_{\iota,t} + \theta_{p,\iota} E_t \left\{ \left(\beta \frac{C_{t+1}^{\sigma}}{C_t^{\sigma} \Pi_{t+1}} \right) \Pi_{\iota,t+1}^{1+\varepsilon_p} \mathcal{Z}_{p_\iota,t+1} \right\}$$

$$1 = \theta_{p,\iota} (\Pi_{\iota,t})^{\varepsilon_p - 1} + (1 - \theta_{p,\iota}) \overline{p}_{\iota,t}^{1-\varepsilon_p}.$$

$$\Delta_{p_\iota,t} = (1 - \theta_{p,\iota}) \left(\overline{p}_{\iota,t}\right)^{-\varepsilon_p} + \theta_{p,\iota} \Pi_{\iota,t}^{\varepsilon_p} \Delta_{p_\iota,t-1}$$

- Firms' efficiency conditions in each sector

$$w_t \frac{h(\mathcal{T}_t) q(\mathcal{S}_t)}{\mathcal{T}_t} = mc_{H,t}\ A_{H,t} (1 - \widetilde{\alpha}_\iota) \left(N_{H,t}\right)^{\widetilde{\alpha}_\iota - 1} \tag{63}$$

$$w_t h(\mathcal{T}_t) = mc_{N,t}\ A_{N,t} (1 - \widetilde{\alpha}_\iota) \left(\frac{K_{N,t}}{N_{N,t}} \right)^{\widetilde{\alpha}_\iota} \tag{64}$$

$$r_{k,t} \frac{h(\mathcal{T}_t) q(\mathcal{S}_t)}{\mathcal{T}_t} = mc_{H,t}\ A_{H,t} \widetilde{\alpha}_\iota \left(\frac{N_{N,t}}{K_{N,t}(i)} \right)^{1-\widetilde{\alpha}_\iota} \tag{65}$$

- Production function

$$Y_{\iota,t} = A_{\iota,t} K_{\iota,t}^{\widetilde{\alpha}_\iota} N_{\iota,t}^{1-\widetilde{\alpha}_\iota} \quad \iota = H, N$$

- Investment efficiency conditions

$$\psi_t\left[1-\Omega\left(\cdot\right)-\Omega'\left(\frac{I_t}{I_{t-1}}-1\right)\frac{I_t}{I_{t-1}}\right]=1-\beta E_t\left\{\psi_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{I_{t+1}}{I_t}\right)^2\Omega'\left(\frac{I_{t+1}}{I_t}-1\right)\right\}$$

$$\psi_t=\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[r_{k,t+1}+(1-\delta)\psi_{t+1}\right]\right\}$$

- Goods market equilibrium: nontradables

$$Y_{N,t}=\Delta_{p_N,t}(1-\gamma)h(\mathcal{T}_t)^{\xi}(C_t+I_t)$$

- Goods market equilibrium: tradables

$$Y_{H,t}=\Delta_{p_H,t}\left[(1-\upsilon)q(\mathcal{S}_t)^{\eta}\gamma\left(\frac{h(\mathcal{T}_t)}{\mathcal{T}_t}\right)^{\xi}C_t+\mathcal{S}_t^{\eta}\Phi_{F,t}^{\eta}Y_t^*\right]$$

- Labor market equilibrium

$$\underbrace{N_t}_{\substack{\text{labor}\\ \text{supply}}}=\sum_{\iota=N,H}N_{\iota,t}+N_{F,t}$$

- Risk-sharing

$$\left(\frac{C_t^*}{C_t}\right)^{-\sigma}=\left(\frac{\mathcal{S}_t}{q(\mathcal{S}_t)}\right)\Phi_{F,t}\left(\frac{\mathcal{T}_t}{h(\mathcal{T}_t)}\right)$$

- Consumption Euler

$$C_t^{-\sigma}=E_t\left\{\frac{C_{t+1}^{-\sigma}R_t}{\Pi_{t+1}}\right\}$$

- Consumption-leisure

$$C_t^{\sigma}N_t^{\varphi}=w_t$$

- CPI inflation

$$\Pi_t=\frac{h(\mathcal{T}_t)}{h(\mathcal{T}_{t-1})}\Pi_{N,t}$$

- Traded good inflation

$$\Pi_{T,t}=\frac{q(\mathcal{S}_t)}{q(\mathcal{S}_{t-1})}\Pi_{H,t}$$

- Relative price of tradables

$$\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\Pi_{T,t}}{\Pi_{N,t}}$$

- Monetary policy rule

$$R_t = f(\Pi_t, \Delta\mathcal{E}_t)$$

- Deviations from law of one price

$$\frac{\Phi_{F,t}}{\Phi_{F,t-1}} = \frac{\Delta\mathcal{E}_t}{\Pi_{F,t}}$$

- Pricing in the import sector (flex prices)

$$\left(\frac{w_t\ q(\mathcal{S}_t)}{\mathcal{S}_t}\frac{h(\mathcal{T}_t)}{\mathcal{T}_t}\right)^{1-\tilde{\alpha}_F} \Phi_{F,t}^{\tilde{\alpha}_F} = \frac{\varepsilon_p - 1}{\varepsilon_p}$$

- Evolution of the terms of trade

$$\frac{S_t}{S_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}}$$

- Choice of labor in import sector

$$w_t \frac{h(\mathcal{T}_t)q(\mathcal{T}_t)}{\mathcal{T}_t S_t} = mc_{F,t}\ (1-\alpha_F)\left(\frac{M_{F,t}}{N_{F,t}}\right)^{\tilde{\alpha}_F}$$

- Choice of imported input

$$\Phi_{F,t} = mc_{F,t}\alpha_F\left(\frac{N_{F,t}}{M_{F,t}}\right)^{1-\tilde{\alpha}_F},$$

- Marginal cost in the import sector

$$mc_{F,t} = \left(\frac{w_t\ q(\mathcal{S}_t)}{\mathcal{S}_t}\frac{h(\mathcal{T}_t)}{\mathcal{T}_t}\right)^{1-\alpha_F} \Phi_{F,t}^{\alpha_F}$$

B.9 Calibration

The baseline calibration is identical to the one employed in the one-sector version of the model (see the main text and Appendix B). What remains to be specified is the value of a few additional parameters: the preference share of *traded* goods in the consumption aggregator (γ), the elasticity of substitution between traded and non-traded goods (ξ), and the sectoral degree of price stickiness $\theta_{p,\iota}$, $\iota \in (H, N)$.

We set $\gamma = 0.6$, which corresponds to the GIPS average share of goods (as opposed to services) in the HICP price index in 2009 (source Eurostat). Following Mendoza (1991) and Corsetti et al. (2008), we set the elasticity of substitution between traded and non-traded goods $\xi = 0.74$. We set the degree of price rigidity in the domestic traded sector equal to $\theta_{p,H} = 0.8$ (consistent with our baseline calibration, which strikes a balance between micro and macro-based empirical studies). Concerning the non-traded sector, the micro-based evidence of Alvarez et al. (2006) suggests that, on average in the Euro area, the frequency of price changes in the service sector is almost half the one in the industrial goods sector (our proxies for non-traded and traded goods respectively). For the GIPS, this difference is however less stark, implying that if, in the baseline setting, prices are on average sticky for five quarters, they should remain sticky in the non-traded sector for 5.75 quarters, which requires to set $\theta_{p,N} = 0.83$. Our results are, however, largely insensitive to the choice of the sectoral relative degree of price stickiness.

We calibrate the shock process for sectoral productivity as follows. We compute, for each sector $\iota \in (H, N)$ and each country $i \in GIPS$, the productivity measure $a^i_{\iota,t} = y^i_{\iota,t} - 0.75 n^i_{\iota,t}$, where $y^i_{\iota,t}$ is (the log of hp-filtered) real gross value added (after deflating by the national GDP deflator), and $n^i_{\iota,t}$ is (the log of hp-filtered) employment (thousands of hours worked).[2] This measure assumes that the labor share is equal in both sectors, which is consistent with the evidence in the GIPS from OECD data.

We then estimate the AR(1) process for each country i and sector ι:

$$a^i_{\iota,t} = \rho^i_{a,\iota}\ a^i_{\iota,t-1} + \varepsilon^i_{a,\iota,t},$$

and set $\rho_{a,\iota}$ and $\sigma_{a,\iota}$ (the standard deviation of the innovation) in the model equal to the estimated average value across $i \in GIPS$ within each sector. Those values are reported in Table A1. Our settings are summarized in Table C1 below.

[2] All data are from Eurostat (Quarterly National Accounts, basic breakdown of main GDP aggregates and employment by industry). The sectoral breakdown is based on the classification of economic activities NACE Rev.2. In our measure, the *tradable* sector is manufacturing (C). The *non-tradable* sector is an aggregate of: construction (F) + wholesale and retail trade+ transport, accomodation and food service activities (G-I) + information and communication (J) + financial and insurance activities (K) + real estate activities (L) + professional, scientific and technical activities + administrative and support service activities (M-N) + public administration, defence, education, human health and social work activities (O-Q)+ arts, entertainment and recreation + activities of household and extra-territorial organizations and bodies (R-U).

	Table C1. Calibration	
Parameter	*Description*	*Value*
	DSGE Model with Traded and Non-Traded Goods	
γ	Share of traded goods in consumption basket	0.6
ξ	Elasticity of substitution between traded and non traded goods	0.74
$\theta_{p,H}$	Calvo index of price rigidities: traded sector	0.8
$\theta_{p,N}$	Calvo index of price rigidities: non-traded sector	0.83
$\rho_{H,a}$	Persistence of technology process: traded sector	0.58
$\rho_{N,a}$	Persistence of technology process: non-traded sector	0.82
$\sigma_{H,a}$	St.dev. of innovation of technology process: traded sector	0.019
$\sigma_{N,a}$	St.dev. of innovation of technology process: non-traded sector	0.009

Note: all remaining parameter values as in the baseline calibration. See main text.

B.10 Results

Figures (1) and (3) describe, for the case of a currency union ($\phi_e = 1$) and in the extended DSGE model with traded and non-traded goods, the effect on welfare losses of variations in the degree of wage rigidity, θ_w, conditional on all shocks and on each individual shock respectively. In all cases, welfare losses are expressed as a ratio to its value under the baseline wage rigidity ($\theta_w = 0.8$). The price rigidity parameter is kept unchanged at its baseline setting of $\theta_p = 0.8$. As it is clear, the main message of our previous analysis is largely confirmed. Figure (3) displays the effects on welfare of varying, simultaneously, both the wage rigidity and the price rigidity parameter. Also in this case, the results of our baseline DSGE model are largely confirmed.

C Welfare and Wage Flexibility in Large Recessions

In this section we are interested in assessing the effect on welfare of varying the degree of wage rigidity *conditional* on the economy being subject to a negative shock of particular magnitude. This exercise speaks to the following question: how advantageous is it, for a small open economy belonging to a currency area facing a large adverse shock, to enjoy a higher degree of wage flexibility?

To address this point, we compute the consumption compensating variation that, conditional on the same negative realization of the exogenous state variables, would make the domestic household indifferent in the following two economies: one with a given degree of wage rigidity θ_w, and one where θ_w is set to its baseline value $\overline{\theta}_w = 0.8$. Throughout this exercise we employ our baseline DSGE model presented in the main text.

Formally, let $\boldsymbol{S}_t^{(-)}$ denote the state vector conditional on a (two-standard deviation)

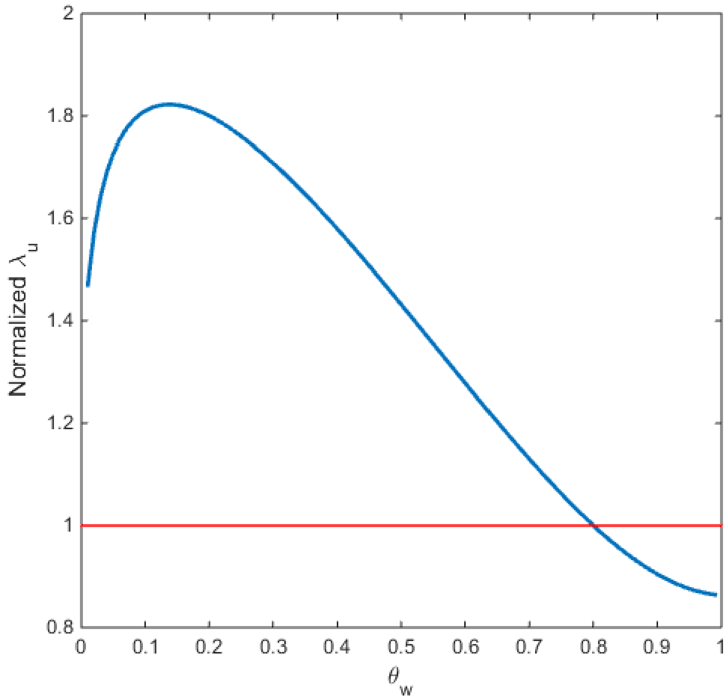

Figure 1: Wage Rigidities and Welfare in a Currency Union: DSGE Model with Traded and Non-Traded Goods (all shocks)

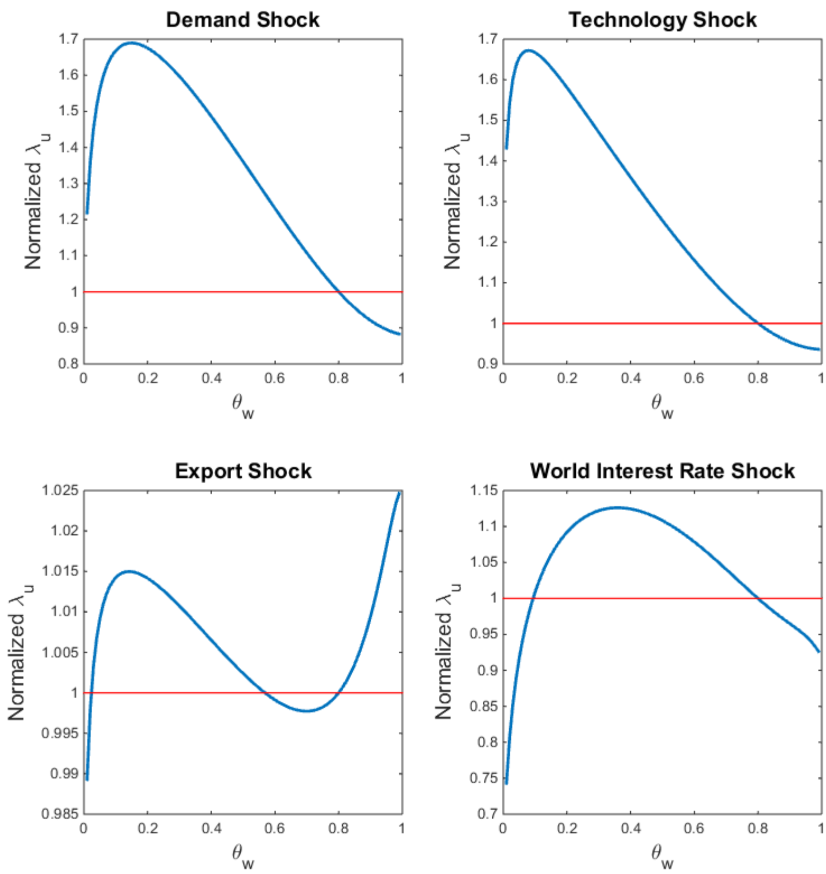

Figure 2: Wage Rigidities and Welfare in a Currency Union: DSGE Model with Traded and Non-Traded Goods (conidtional on shock at the time).

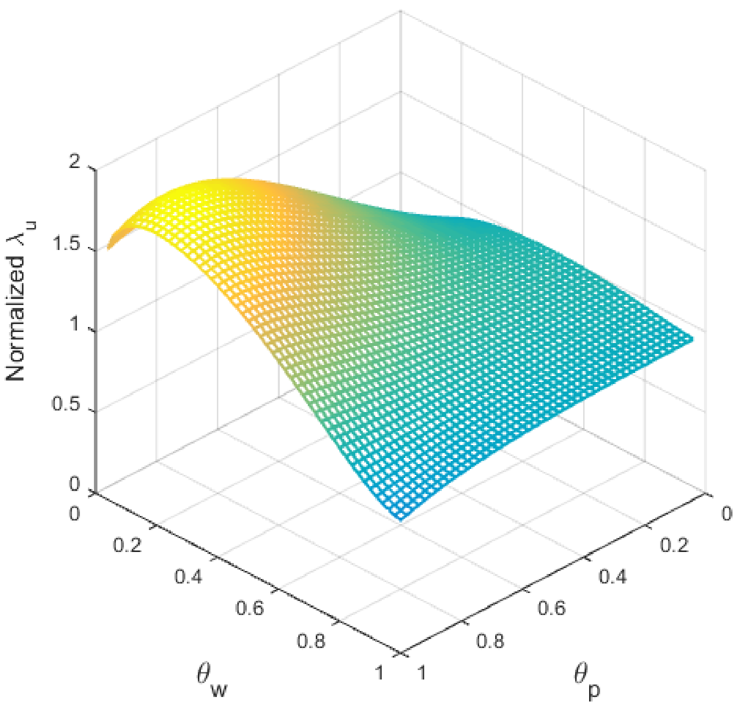

Figure 3: Nominal Rigidities and Welfare in a Currency Union: DSGE Model with Traded and Non-Traded Goods (all shocks).

negative realization of the shock(s), and let:

$$\mathbb{V}_t^{(\theta_w)}(\boldsymbol{S}_t^{(-)}, \lambda_{\theta_w}) \equiv \left\{ E_t \sum_{t=0}^{\infty} U\left(\widetilde{C}_t^{(\theta_w)}(1+\lambda_{\theta_w}), \{\mathcal{N}_t^{(\theta_w)}(j)\}\right) \mid \boldsymbol{S}_t^{(-)} \right\}$$

denote the conditional expected present discounted value of utility (our measure of welfare) associated to a given degree of wage rigidity θ_w. We compute the value of λ_{θ_w} that solves:

$$\mathbb{V}_t^{(\theta_w)}(\boldsymbol{S}_t^{(-)}, \lambda_{\theta_w}) = \mathbb{V}_t^{(\overline{\theta}_w)}(\boldsymbol{S}_t^{(-)}, 0)$$

In practice, we need to compute, under alternative values of θ_w, the impulse response of the variable $\mathbb{V}_t$ conditional on a two-standard deviation negative realization of (a generic component k of) the exogenous state vector.[3] Since, in a second-order approximation of the model, the impulse response of $\mathbb{V}_t$ is state dependent, we numerically proceed as follows. We let the system begin in the deterministic steady state, and draw a series of random shocks e_t for T periods. Based on a given draw, we derive two simulations for the vector $\mathbb{Y}_t$ of the endogenous variables. The first is a baseline simulation called $\mathbb{Y}_{1,t}(T \mid e_t)$; the second, $\mathbb{Y}_{2,t}(T \mid e_t + \varepsilon_k)$, is a simulation obtained by adding a deterministic negative impulse ε_k (of size two standard deviations) to the component k of vector e_t in period $T - p + 1$. The impulse response to shock k is computed as $\mathbb{Y}_{2,t} - \mathbb{Y}_{1,t}$. We repeat this exercise for $\mathbb{Z}$ times, compute the average, and drop the first $T - p$ observations. De facto, this procedure amounts to computing an average (i.e., generalized) impulse response to a two standard deviation innovation when the state vector is initialized at its ergodic mean (via a suitable choice of T).

Figure 4, 5 and 6 display the value of λ_{θ_w} (expressed in percentage) for alternative values of θ_w in the case of a domestic demand shock, export shock and world interest rate shock respectively. By construction this measure is equal to zero in the baseline case of $\overline{\theta}_w = 0.8$. The figure reports the computed λ_{θ_w} conditional on a two standard deviation negative realization of each type of innovation. A positive value on the vertical axis therefore corresponds to the household's welfare loss (expressed in units of consumption compensating variations) of having a degree of wage rigidity θ_w *relative* to the baseline $\overline{\theta}_w$ when the economy is hit by a negative shock of particular magnitude.

We clearly see that, starting from $\overline{\theta}_w = 0.8$, *reducing* the degree of wage rigidity generates a welfare loss. This is in line with our central results previously obtained for unconditional welfare measures. Absolute welfare losses from higher wage flexibility can be particularly large in the case of demand shocks - as high as 1.5 percent of consumption relative to the allocation under the baseline degree of wage rigidity), although they remain relatively small in the case of export and world interest rate shocks (respectively up to 0.2 and 0.1 percent of consumption relative to the baseline allocation).

[3] We choose to report the impact response, which means that welfare $\mathbb{V}_t$ is evaluated conditional on the value of the *endogenous* state vector being at its ergodic mean.

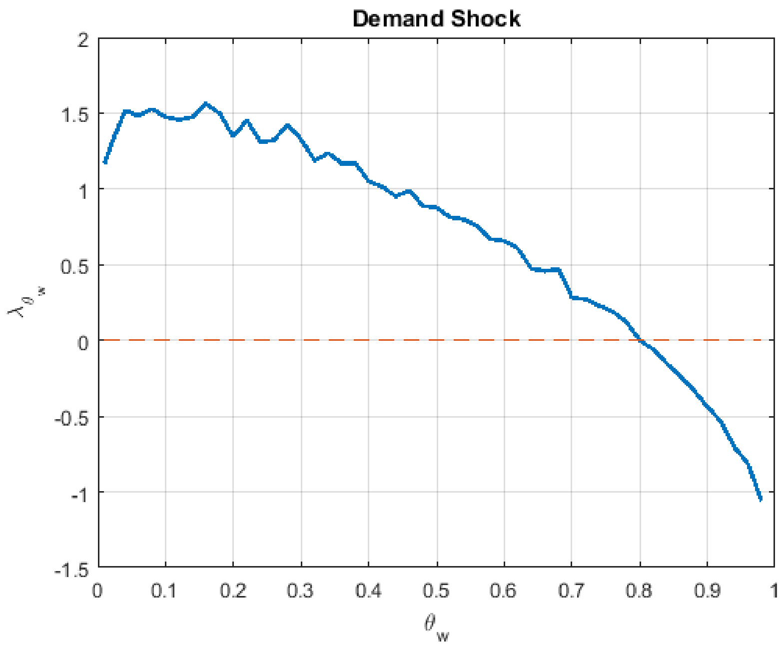

Figure 4: Wage rigidity and welfare loss conditional on a two-standard deviation negative realization of a domestic demand shock. The welfare loss is measured by the consumption compensating variation λ_{θ_w} (in % units) necessary to make the household as well off under a generic θ_w as under the baseline $\overline{\theta}_w$= 0.8.

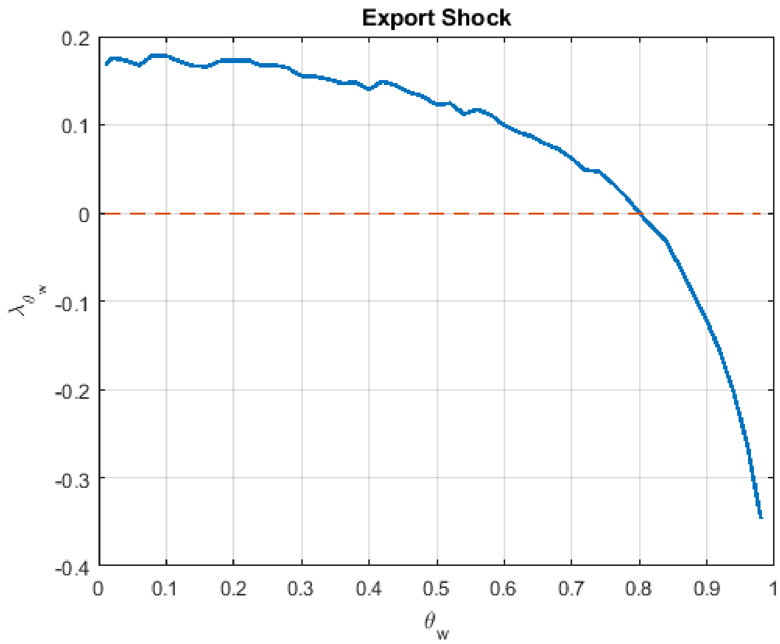

Figure 5: Wage rigidity and welfare loss conditional on a two-standard deviation negative realization of an export demand shock. The welfare loss is measured by the consumption compensating variation λ_{θ_w} (in % units) necessary to make the household as well off under a generic θ_w as under the baseline $\overline{\theta}_w = 0.8$.

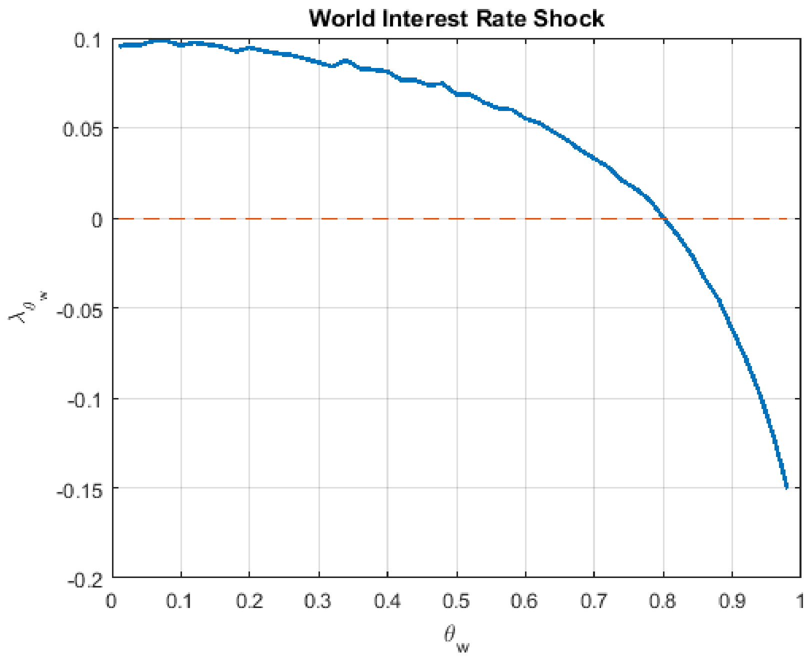

Figure 6: Wage rigidity and welfare loss conditional on a two-standard deviation negative realization of a world interest rate shock. The welfare loss is measured by the consumption compensating variation λ_{θ_w} (in % units) necessary to make the household as well off under a generic θ_w as under the baseline $\overline{\theta}_w$= 0.8.

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