

# The concept of optimal behavior of a cyber object as a measure of its generalized uncertainty

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## Abstract

In this publication a concept of the generalized cyber object optimal behavior is considered. The situation with two alternatives is highlighted. The measure of the generalized uncertainty of a cyber object (subject of behavior) regarding the set of two alternatives achievable to this subject of behavior is the entropy of preferences with respect to achievable alternatives; that is the problem deals with the case when there are functions related to the alternatives with the properties of the controlled cyber object control and the reverse proportionality measure between the elementary effectiveness function parameter and its control. It is proposed to consider the cyber object effectiveness function of the integral dynamical form implanted into the objective functional analogical to such functional taken in the general view, which was developed in the theory of the entropy of subjective preferences; that theory is also known as Subjective Analysis. It is possible to make parallels with well-known theoretical statistical physics when using this approach. In particular, Jaynes' entropy maximum principle serves as a base. This subjective analysis principle allows one to establish the subjective maximum of entropy in the context of its conditional optimization. The Euler-Lagrange equation made it possible to obtain the best solution in the form of objective functional extrema for preferences as well as for the controlled parameter of the cyber object by identifying the requirements for the existence of the objective functional extremum of the subject of behavior. The maximum value of the functional, proven with variations, is illustrated.

## Keywords

Cyber object, optimal behavior, multi-alternative situation, optimal control, effectiveness function, optimal distribution, alternatives preferences' distribution entropy, entropy maximum principle, objective functional, variational principle

## 1. Introduction

For nowadays it is absolutely obvious the presence of one or another type of a cyber object elements everywhere. Informational boom and information technologies penetrate into any sphere of our life and activity. Engineering is not an exception.

For example, in aviation industry and aircraft operation (including technical operation, maintenance, and aircraft airworthiness support techniques) [23] and [31], it is hard to imagine the absence of such important outcome measures as maintainability, reliability, risk [13] and [15], and so on, which symbolize the technical perfection of the professional activity field. Some aspects of a generalized cyber object behavior can be traced through the prism of the economic utility theory [26] and [27] as well.

## 2. Related works

Recent researches on the cyber conflict management investigation leads to the necessity of introduction and further consideration of a generalized cyber object concept.

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As to the generalized cyber object concept literature survey the key point for the scientific considerations here is at the uncertainty measure applicability [17-20]. This, in a combination with the purely economical categorization [16], gave birth to the theory of subjective preferences entropy [32]. Such composition has already had a prolific circle of applications, for instance, [33] and [4], and will definitely instigate prospective research in the directions of [22; 24; 25; 28; 34].

The developed in references [1-12; 14; 21; 29; 30] ideas should be continued to implement their results into the concept of the generalized cyber object control.

### 3. Problem statement

The formulation of the problem considers an active cyber object, which is controlled by a system of achievable alternative preferences [4; 32; 33].

We consider the distribution of such preferences as optimal. The uncertainty of the distribution of preferences also needs to be taken into account. Therefore, the general postulated functional should be used [17; 32]:

$$\Phi_{\pi} = \alpha H_{\pi} + \beta \varepsilon + \gamma Norm, \quad (1)$$

where  $\alpha, \beta, \gamma$  – structural parameters, which can take the form of the Lagrange coefficient or weighting coefficients depending on the problem statement. We consider them as internal parameters of cyber object control. These parameters make it possible to match the identified "stance" properties of a cyber object with achievable alternatives;  $\pi$  – alternative preferences;  $H_{\pi}$  – entropy of alternative preferences;  $\varepsilon$  – the efficiency function, which determine the optimality conditions for the distribution of achievable alternative preferences. It is used in parallel with the entropy of alternative preferences;  $Norm$  – normalizing condition.

Thus, the solution requires the problem of finding the optimal distribution of achievable alternatives preferences under conditions, which are considered as objective functional (1).

### 4. Main content

First of all let us formulate the general provisions for the problem described with the objective functional (2) that will define in some respect the possible solution of the stated problem.

#### 4.1. Solution to the specific case

For the entropy of the objective functional it is proposed to take the Shannon's view entropy, however modified with the preferences rather than probabilities; it is one of the cornerstones of the theory [32]:

$$H_{\pi} = - \sum_{i=1}^N \pi_i \ln \pi_i, \quad (2)$$

where  $i$  – number of specified achievable alternative;  $N$  – total number of all achievable alternatives.

Time functions  $t$  will play the role of preferences  $\pi_i = \pi_i(t)$  in the dynamic development of the process. Let us present the objective functional from equality (1) in integral form:

$$\Phi_{\pi} = \int_{t_1}^{t_2} \left( - \sum_{i=1}^N \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^N \pi_i(t) F_i + \gamma \sum_{i=1}^N \pi_i(t) \right) dt, \quad (3)$$

where  $\beta, \gamma$  – values of the structural parameter representing the objective functional from expression (1). It takes into account the values of the specified parameters reduced by a value of  $\alpha$ ;  $F_i$  – the effectiveness function of the  $i$ -th achievable alternative of the cyber object.

In (3) term  $\sum_{i=1}^N \pi_i(t) F_i$  taking into account expression (2) within the defined integration period  $[t_1 \dots t_2]$ , is related to the defined average value of the function  $\varepsilon$ . Term  $\sum_{i=1}^N \pi_i(t)$  represents the normalizing condition.

## 4.2. Specific case model construction

It is proposed to consider the cyber object effectiveness function of the integral dynamical form implanted as objective functional:

$$\Phi_{\pi} = \int_{t_1}^{t_2} \left( - \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) + \beta [\pi_1(t) \dot{x}(t) + \alpha \pi_2(t) \dot{x}(t)] + \gamma \sum_{i=1}^{N=2} \pi_i(t) \right) dt, \quad (4)$$

where  $\dot{x}(t) = \frac{dx}{dt}$  – the first time derivative, which is the control function of the parameter to be controlled;  $\alpha$  – in the cyber object effectiveness function, a coefficient for balancing the dimensions;  $x(t)$  – a parameter that is subject to control.

Here, in functional (4), the attention is drawn to the two-alternative situation; that is to the case when the alternatives' properties of the controlled cyber object control have functions connected to them and the reverse proportionality measure between the parameter and its control.

Namely:

$$[\pi_1(t) \dot{x}(t) + \alpha \pi_2(t) x(t) \dot{x}(t)]. \quad (5)$$

## 4.3. Specific case model construction

Let us introduce

$$R^* = - \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) + \beta [\pi_1(t) \dot{x}(t) + \alpha \pi_2(t) x(t) \dot{x}(t)] + \gamma \sum_{i=1}^{N=2} \pi_i(t). \quad (6)$$

Objective functional's (4) solution with the integrand (6) has to be obtained at the extremals:

$$\pi_1^0(t), \quad \pi_2^0(t), \quad x_0(t). \quad (7)$$

The necessary conditions for the extremum of the target functional (4), denoted in the form of Euler-Lagrange equations, allow obtaining the extrema (7):

$$\frac{\partial R^*}{\partial \pi_i} - \frac{d}{dt} \frac{\partial R^*}{\partial \dot{\pi}_i} = 0, \quad \frac{\partial R^*}{\partial x} - \frac{d}{dt} \frac{\partial R^*}{\partial \dot{x}} = 0. \quad (8)$$

In this instance, function (6) is independent of how quickly preferences change over time:

$$\dot{\pi}_i = \frac{d\pi_i}{dt}. \quad (9)$$

Hence,

$$\frac{\partial R^*}{\partial \dot{\pi}_i} \equiv 0, \quad \frac{d}{dt} \frac{\partial R^*}{\partial \dot{\pi}_i} \equiv 0. \quad (10)$$

Thus, the expression (8) will take the form:

$$\frac{\partial R^*}{\partial \pi_i} = 0, \quad \frac{\partial R^*}{\partial x} - \frac{d}{dt} \frac{\partial R^*}{\partial \dot{x}} = 0. \quad (11)$$

Accordingly, conditions (11) yield for preferences

$$\frac{\partial R^*}{\partial \pi_1} = - \ln \pi_1 - 1 + \beta \dot{x} + \gamma = 0, \quad \frac{\partial R^*}{\partial \pi_2} = - \ln \pi_2 - 1 + \alpha \beta x \dot{x} + \gamma = 0, \quad (12)$$

from where

$$\pi_1 = e^{-1+\beta \dot{x} + \gamma} = e^{\gamma-1} e^{\beta \dot{x}}, \quad \pi_2 = e^{-1+\alpha \beta x \dot{x} + \gamma} = e^{\gamma-1} e^{\alpha \beta x \dot{x}}. \quad (13)$$

Then, normalizing condition means

$$\pi_1 + \pi_2 = 1 = e^{\gamma-1} e^{\beta \dot{x}} + e^{\gamma-1} e^{\alpha \beta x \dot{x}} = e^{\gamma-1} (e^{\beta \dot{x}} + e^{\alpha \beta x \dot{x}}) \quad (14)$$

and

$$e^{\gamma-1} = \frac{1}{e^{\beta \dot{x}} + e^{\alpha \beta x \dot{x}}}. \quad (15)$$

For the preferences it gives

$$\pi_1 = \frac{e^{\beta \dot{x}}}{e^{\beta \dot{x}} + e^{\alpha \beta x \dot{x}}}, \quad \pi_2 = \frac{e^{\alpha \beta x \dot{x}}}{e^{\beta \dot{x}} + e^{\alpha \beta x \dot{x}}}. \quad (16)$$

For the extremal of  $x_0(t)$ , at  $\beta \neq 0$ , one can find

$$\frac{\partial R^*}{\partial x} = \alpha\beta\pi_2\dot{x}, \quad \frac{\partial R^*}{\partial \dot{x}} = \beta\pi_1 + \alpha\beta\pi_2x, \quad \frac{d}{dt} \frac{\partial R^*}{\partial \dot{x}} = \beta\dot{\pi}_1 + \alpha\beta(\dot{\pi}_2x + \pi_2\dot{x}) \quad (17)$$

and

$$\alpha\beta\pi_2\dot{x} - \beta\dot{\pi}_1 - \alpha\beta\dot{\pi}_2x - \alpha\beta\pi_2\dot{x} = 0, \dots \dots \dots \dot{\pi}_1 = -\alpha\dot{\pi}_2x. \quad (18)$$

At the presented problem setting

$$\dot{\pi}_1 = \frac{d\pi_1}{dt} = \frac{\partial \pi_1}{\partial x} \dot{x} + \frac{\partial \pi_1}{\partial \dot{x}} \ddot{x} \quad (19)$$

and

$$\dot{\pi}_2 = \frac{d\pi_2}{dt} = \frac{\partial \pi_2}{\partial x} \dot{x} + \frac{\partial \pi_2}{\partial \dot{x}} \ddot{x}. \quad (20)$$

Where partial derivatives are

$$\frac{\partial \pi_1}{\partial x} = -\frac{e^{\beta\dot{x}}\alpha\beta\dot{x}e^{\alpha\beta x\dot{x}}}{(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}})^2} = -\alpha\beta\dot{x}\pi_1\pi_2 \quad (21)$$

and

$$\begin{aligned} \frac{\partial \pi_1}{\partial \dot{x}} &= \frac{\beta e^{\beta\dot{x}}(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}}) - e^{\beta\dot{x}}(\beta e^{\beta\dot{x}} + \alpha\beta x e^{\alpha\beta x\dot{x}})}{(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}})^2} = \\ &= \frac{\beta e^{\beta\dot{x}}(e^{\alpha\beta x\dot{x}}) - e^{\beta\dot{x}}(\alpha\beta x e^{\alpha\beta x\dot{x}})}{(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}})^2} = \beta\pi_1\pi_2(1 - \alpha x). \end{aligned} \quad (22)$$

Then for (19) with (21) and (22)

$$\dot{\pi}_1 = -\alpha\beta\dot{x}\pi_1\pi_2\dot{x} + \beta\pi_1\pi_2(1 - \alpha x)\ddot{x}. \quad (23)$$

For (20)

$$\frac{\partial \pi_2}{\partial x} = \frac{\alpha\beta\dot{x}e^{\alpha\beta x\dot{x}}(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}}) - e^{\alpha\beta x\dot{x}}(\alpha\beta\dot{x}e^{\alpha\beta x\dot{x}})}{(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}})^2} = \frac{\alpha\beta\dot{x}e^{\alpha\beta x\dot{x}}(e^{\beta\dot{x}})}{(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}})^2} = \alpha\beta\dot{x}\pi_1\pi_2 \quad (24)$$

and

$$\begin{aligned} \frac{\partial \pi_2}{\partial \dot{x}} &= \frac{\alpha\beta x e^{\alpha\beta x\dot{x}}(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}}) - e^{\alpha\beta x\dot{x}}(\beta e^{\beta\dot{x}} + \alpha\beta x e^{\alpha\beta x\dot{x}})}{(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}})^2} = \\ &= \frac{\alpha\beta x e^{\alpha\beta x\dot{x}}(e^{\beta\dot{x}}) - e^{\alpha\beta x\dot{x}}(\beta e^{\beta\dot{x}})}{(e^{\beta\dot{x}} + e^{\alpha\beta x\dot{x}})^2} = \beta\pi_1\pi_2(\alpha x - 1). \end{aligned} \quad (25)$$

Substitution of (24) and (25) into (20) yields

$$\dot{\pi}_2 = \alpha\beta\dot{x}\pi_1\pi_2\dot{x} + \beta\pi_1\pi_2(\alpha x - 1)\ddot{x}. \quad (26)$$

Now, with (23) and (26) we have for (20)

$$-\alpha\beta\dot{x}\pi_1\pi_2\dot{x} + \beta\pi_1\pi_2(1 - \alpha x)\ddot{x} = -\alpha x[\alpha\beta\dot{x}\pi_1\pi_2\dot{x} + \beta\pi_1\pi_2(\alpha x - 1)\ddot{x}]. \quad (27)$$

Implying  $\pi_1\pi_2 \neq 0$ , from (27)

$$-\alpha\beta\dot{x}\dot{x} + \beta(1 - \alpha x)\ddot{x} = -\alpha x[\alpha\beta\dot{x}\dot{x} + \beta(\alpha x - 1)\ddot{x}]. \quad (28)$$

From (28)

$$\frac{-\alpha\beta\dot{x}\dot{x} + \beta(1 - \alpha x)\ddot{x}}{-\alpha\beta\dot{x}\dot{x} + \beta(1 - \alpha x)\ddot{x}} = \alpha x = 1. \quad (29)$$

From (29)

$$x_0 = \frac{1}{\alpha}. \quad (30)$$

The same result can be obtained from (18) at  $\pi_2 \neq 0$ :

$$\pi_1 = 1 - \pi_2, \quad \dot{\pi}_1 = \frac{d(1 - \pi_2)}{dt} = \frac{d\pi_2}{dt} = -\dot{\pi}_2, \quad -\dot{\pi}_2 = -\alpha\dot{\pi}_2x, \quad 1 = \alpha x. \quad (31)$$

And, again (30).

Therefore, in the considered case

$$\pi_1^0 = \pi_2^0 = \frac{1}{2}. \quad (32)$$

#### 4.4. Test for extremality

For the normalizing coefficient from the objective functional (4)

$$\gamma_0 = \ln\left(\frac{e}{2}\right) \text{ or } \gamma_0 = 1 - \ln 2. \quad (33)$$

The found value (33) may be not taken into account since it will give just an additive component of “0” to the objective functional (4).

As to the objective functional (4) itself, at the extremal value of the cyber object controlled parameter (30), the expression (4) is reduced to

$$\Phi_\pi = \int_{t_1}^{t_2} \left( - \sum_{i=1}^{N=2} \pi_i(t) \ln \pi_i(t) \right) dt. \quad (34)$$

In order to simulate variations of the preferences numerically it is proposed one of the possible methods.

Consider varying preferences as

$$\pi_i(t) = \frac{\exp[-\beta_i t(t - t_2)]}{\sum_{j=1}^2 \exp[-\beta_j t(t - t_2)]}, \quad (35)$$

where  $\beta_1 = \beta$  and  $\beta_2 = \alpha\beta$ . Here the cyber object controlled parameter and its rate of the change in time are not taken into account.

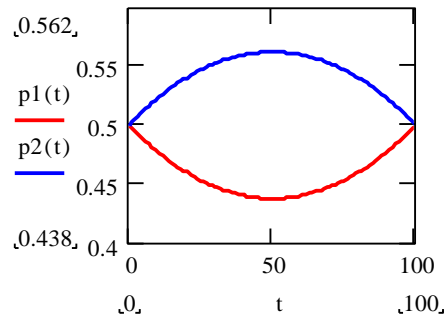
Then, at the accepted values of

$$\beta_1 = 2 \cdot 10^{-4}, \quad \beta_2 = 3 \cdot 10^{-4}, \quad (36)$$

and the time interval of

$$[t_1 = 0, \dots, t_2 = 100], \quad (37)$$

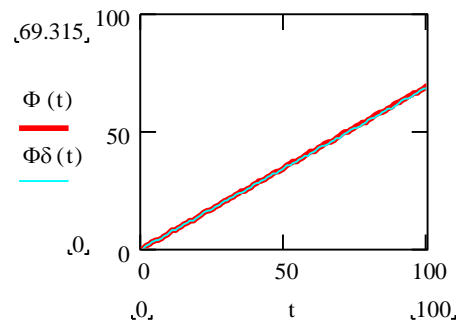
the cyber object preferences functions distribution, calculated by the system of equations (35), will be as it is shown in Fig. 1.



**Figure 1:** A cyber object varying preferences functions distribution composition

In Fig. 1,  $p_1(t)$  stands for  $\pi_1(t)$ ,  $p_2(t)$  stands for  $\pi_2(t)$  obtained by formulas of (35) respectively.

Such small variation of the preferences functions as shown in Fig. 1 results in the tiny exceeding of the cyber object objective functional with the extremal preferences (32), which hardly noticeable in Fig. 2.



**Figure 2:** The maximal value of the cyber object objective functional with the extremal solution preferences functions versus varying preferences functions distribution

In Fig. 2,  $\Phi(t)$  stands for  $\Phi_{\pi}$  calculated by formula (4) with the deliberate omitting of or neglecting normalizing conditions, however they are satisfied, thus, simulation has been conducted by the formula of the entropy integral (34) with (32); and  $\Phi\delta(t)$  stands for the value of the entropy (34), but with the varying preferences functions of the cyber object, obtained by formulas of (35) respectively.

Numerical integrations of the expressions of (4) or (34) have been performed with the varying upper end of the integration in order to obtain the result as a function of time. Both models have been calculated with the data of (36) and (37).

The practically invisible, due to the preferences functions small variation, prevalence of  $\Phi(t)$  over  $\Phi\delta(t)$  becomes more tangible with the use of the tabulated results of the calculation experimentation illustrated in Fig. 3.

$\Phi(t) =$		$\Phi\delta(t) =$	
	0		0
0	0	0	0
1	0.693	1	0.693
2	1.386	2	1.386
3	2.079	3	2.079
4	2.773	4	2.772
5	3.466	5	3.465
6	4.159	6	4.158
7	4.852	7	4.851
8	5.545	8	5.543
9	6.238	9	6.236
10	6.931	10	6.928
11	7.625	11	7.62
12	8.318	12	8.312
13	9.011	13	9.003
14	9.704	14	9.695
15	10.397	15	10.386

**Figure 3:** Prevalence of the maximal value of the cyber object objective functional with the extremal solution preferences functions versus varying preferences functions distribution

From the shown in Fig. 3 numerical results of simulation, now it is visible the extremality of the optimal solution.

In accordance with the values demonstrated in Fig. 3, starting from the time point of “4” and on, the optimal solution prevails delivering the maximal value to the cyber object objective functional. Of course, that dominance would be the greater the larger the variation of the preferences functions were.

## 5. Discussion

Thus, the result of the approach expressed with the procedures of (1) – (37) proven to be optimal.

Although the simulation of (32) – (37) was conducted without the cyber object controlled parameter varying, nonetheless the result would have been the same for any constant controlled parameter.

In the case of absence of the controlled parameter and its change in time rate (control) in the varying preferences there is one more remark (35). The point is that the variations are not the extremal solutions (16). The variations (35) and extremals (16) basically coincide at the boundary points of integration for the tested objective functional (4).

However it looks prospective to try varying the controlled parameter function as well in order to ensure of its optimality too.

## 5.1. Analysis of comparability with previously reported findings

Presented herewith research helps create a formulation of the general theoretical provisions of the generalized cyber object study.

In general terms, this study is to formulate the problem of managing the operation of an active cybernetic element and control of the support of its safety in terms of the multi-alternativeness and uncertainty. Such kind of operation inevitably happens in the vast majority of situations; moreover it occurs in the presence of possible conflicts [32]. This problem is formulated in terms of subjective analysis. Its solution is performed by applying the extremality principles, for example, as it was illustrated in the given research likewise for the individual preferences' subjective entropy of the active elements (subjects) of a control system. The proposed approach involves a compilation of the appropriate objective functional of a cyber object in order to solve the variational problem of the operational management process.

The basic conceptual framework is very similar to [4; 32; 33]. At the same time, work [32] talks about the controllability of an active system through an active element and an active element of an active system as one of its features. For example, in the aviation sector, uncertainties in operational situations can lead to some dangerous situations; that is why the active element (the decision-maker) is forced to act in a multi-alternative situation and in a conflict-prone way. Activity in conditions of scarcity of time, altitude, distance and other resources is the cause of such cases.

In aviation, safety is without a doubt of the utmost importance. And the study of the influence of the human factor on all types of safety is an extremely relevant scientific and practical problem. This problem is urgent and requires the expression of individual preferences through a certain function in an explicit form.

A systematic and fundamental study of multi-alternativeness within the approach to the development of problem-resource conditions is represented in [32]. Mainly two related problems of the general form are considered:

1. Acquire the distributions of subjective functions of the preferences of the active element (subject); on the set of alternatives achievable for him, which delivers the extreme value of the subjective entropy under the conditions of a certain "isoperimetric constraints". Here, the obtained solution, where there is the number of two alternatives; follows that developed delivering the extreme value of subjective entropy under the conditions of a specified view cyber object effectiveness function, which plays the role of those "isoperimetric constraints".

This function is close in description to the subjective effectiveness' function or the function of subjective utility, which can be the "isoperimetric constraint" for the subjective effectiveness' function.

2. Acquire the distributions that deliver the extreme value to the function of subjective efficiency in the presence of the "isoperimetric" conditions for the subjective entropy. Such problem also seems very important, however it was not considered in the presented paper.

## 5.2. Evidently promising investigations

The postulated view cyber object objective functional has the optimal solution illustrated in computer simulation results (see Fig. 1 – 3). Nonetheless, there was no answer to the question about which alternative of the two is better and how much.

Such problem is solved with the use of the notion of the degree of confidence that can be presented as hybrid model of the subjective preferences' combined pseudo-entropy function which was proposed in reference [8].

That measure of certainty is a composition on one hand of the ratio of the traditional view Shannon's entropy exceed by the maximum entropy to the entropy maximum value, which makes an index of a relative certainty, and on the other hand it relates with the factor/index of the preferences prevalence/domination, which gives to the expression the sign of that certainty.

Thus, the required measure combines the certainty and uncertainty values, and channeling certainty and uncertainty in the "right" or "wrong" ("good" or "bad") direction.

For the study herein, this value was not applicable on the reason that optimal distribution of the cybernetic object preferences happens to be equal, see the result of the extremals in the equations (34).

Therefore, at this special case, the maximum of the preferences entropy is realized. Any of the alternatives is equally preferable.

The elementary effectiveness functions are the first complete derivative of the cyber object controlled parameter with respect to time (its control) and the product of the parameter and its control. This, at the constant value in time of the cyber object controlled parameter, gives the “zero” magnitudes to both elementary effectiveness functions. Therefore, both preferences are equal and the preferences entropy undergoes its maximum.

Nevertheless, such situation of the complete indifference of the cyber object concerning achievable for its goals alternatives may be not ever, and in that case the described relative combined pseudo-entropy function could be of a great help in determination of the cyber object preferences uncertainty/certainty perfection. Thus, that powerful tool can be used for the next step in the research dealing with the multi-alternativeness and uncertainty in order to evaluate and decrease the potential conflict-proneness of the operational situation.

Also, a disputable portion of the study remains a model of the cyber object effectiveness function view itself. Since every specific case requires its own unique approach, there is a necessity of the attempt for the general concept implementation.

## 6. Conclusions

The calculus of variations' basic problem is presented in research. It has the optimal solution in the interpretation of the cyber object preferences for the attainable set of two alternatives.

Necessary conditions for the extremum existence provide an opportunity to obtain results that delivers the maximal value to objective functional measure.

In further investigations it seems rational also to vary the controlled parameter function for verification of the cyber object's optimality with respect to it as well.

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