Method for Processing and Assessing the Degree of Digital Image Compression Based on Haar Transformation

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Abstract

The article presents the specifics of the developed method for processing and assessing the degree of digital image compression based on Haar transformation. In order to ensure a sufficient level of quality for the presentation of digital images of different formats and classes in the process of their multi-threaded processing in real time, it is proposed to implement a number of orthogonal transformations. The mathematical representation of the created method reflects the stages of restoration of individual image indicators in the process of its reception and transmission. The proposed method is based on the principles of complex adaptive compression using an estimate of the specific global and local sensitivity of the Haar transform functions. The developed method can be used for adaptive compression of digital images of different saturation with support for dynamic scaling of their sizes based on the estimation of the energy index in the analysis of the image transform, providing detailing of the degree of its saturation.

Keywords

Digital image processing, images compression, energy indicator, image saturation, Haar transformation.

1. Introduction

Currently, the flow of various data of various formats from distributed information and technical systems is actively increasing. Graphic data has a high priority in this specificity, in particular, images of various resolutions and formats, which allow us to detail various aspects of the state of the studied or analyzed components of dynamic systems through their automated intelligent recognition using applied methods of pattern recognition theory [1]. Meeting the performance requirement for processing and transmitting digital images in real time is based on reducing the amount of data associated with images by compressing them. However, the efficiency of image compression significantly affects the quality of the reproduced video information. The use of currently known digital image compression methods allows: reduce the time required to transfer images; to increase the number of images transmitted over the communication channel per unit of time; to reduce the requirements for the speed of information transmission devices; reduce the storage capacity of images [2]. Remote sensing of the earth's surface using unmanned aerial systems (UAC) is widely used, for example, to monitor vegetation and environmental parameters, aimed in particular at optimizing activities in agriculture and forestry. In this context, UACs have become useful for assessing crop health by acquiring large amounts of raw data that require processing to further support applications such as moisture, biomass, and others [3]. The analysis of known works indicates a constant increase in the amount of information received from the UAC during remote sensing of the earth's surface, which outstrips the growth rate of the capabilities of technical means for its processing [4].

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In the process of conducting remote sensing of the earth's surface, one of the ways to solve the problem of operational processing and transmission of images in real time is to reduce the amount of data by compressing images.

The image compression method is understood as a set of actions that allows us to unambiguously match a set of compressed data to the original dataset [5].

Currently, many methods have been developed for compressing digital images, which can reduce this or that redundancy. Nevertheless, for this purpose, intensive development of new methods of compression of digital images continues [4].

Analysis of digital image compression methods shows that most of them are complex, consisting of stages: change of color model, image conversion, image encoding [5].

An effective and simple way to increase the compression ratio provided by image compression algorithms is to select the optimal color model in which the image data is represented. Compression algorithms occupy an essential place in the theory of digital image processing.

This is due to the fact that images presented in digital form require a large amount of memory to store, and when transferring images through communication channels, it takes a long time [6].

2. Literature review and description of problem

To transform a digital image in modern practice according to a number of literary sources [7-9], an orthogonal or wavelet transform is used. Orthogonal transformations consist of direct and inverse transformations [4-6]. Discrete forward and backward transformations in general:

$$Y(k) = <\frac{1}{N}> \cdot \sum_{m=0}^{N-1} X(m) \cdot W(k,m), \ k = \overline{0, N-1} \quad (1)$$

$$X(m) = \sum_{k=0}^{N-1} Y(k) \cdot W(k,m), \ m = \overline{0, N-1} \quad (2)$$

where:

- $X(m)$ – *m*-th sample of the original discrete signal;
- $Y(k)$ – *k*-th conversion coefficient;
- $W(k,m)$ – *m*-th sample of the *k*-th basis function of the orthogonal transformation;
- N – the number of samples in the original signal or in the block of the original signal during block processing;
- $\langle 1/N \rangle$ – normalization coefficient, which may be absent in some orthogonal transformations (for example, a discrete-cosine transform).

The entire image or its fragments is subjected to conversion. A typical image fragment consists of 8 × 8, 16 × 16 or 32 × 32 samples. A fragment size selection is due to the fact that the correlation interval for images does not exceed 8 ... 32 samples.

Analysis of existing varieties of image compression methods shows that most modern compression methods consist of several stages.

The first step in most compression methods involves changing the color model. Currently, various color models are used, which are described in sufficient detail, for example, in publications [7]. The high efficiency of using a color model change as a stage of image compression is confirmed by the experience of operating existing compression methods [8].

However, the use of this stage leads to a significant increase in the computational complexity of the compression algorithm (by 20 ÷ 40%), which is associated with the need to carry out about 9 × n real multiplication operations and 6 × n real addition operations (where n is the number of samples in the image).

The second stage - transformation of the image (for example, orthogonal or wavelet) allows you to translate the original signal from the space-time domain into the spectral-frequency domain. In this case, the original signal (function of time) can be represented through an orthonormal set of spectral functions in the form of a series.

Representation of signals in the form of a set of spectral functions allows their spectral analysis, convolution of complex signals, processing in the spectral domain with decorrelated spectral signal elements, etc.

In addition, the use of transforming the incoming sequence of image samples allows redistributing the image energy.

Using the transform of the processed image provides further compression procedures with decorrelated transform coefficients.

To assess the effectiveness of transformations in [9], the following particular indicators are used:

- $K_{sum/sub}$, $K^{(-1)}_{sum/sub}$ – the number of addition / subtraction operations in direct and inverse transformations, respectively;
- $K_{mul/div}$, $K^{(-1)}_{mul/div}$ – the number of multiplication / division operations for direct and inverse transformations, respectively;
- T_{op} – the type of operations (operands) used in the conversion procedure;
- S_{coef} – sensitivity of conversion factors;
- D_{tr} – dynamic range of transformation ratio values;
- σ – root-mean-square deviation during conversion.

The standard deviation (SD) is a quantitative indicator and characterizes the error of the reconstructed image relative to the original one. SD is determined by the following expression

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} \sum_{j=1}^{M} \left(x_{i,j}^{(в)} - x_{i,j}^{(u)} \right)^2}{N \cdot M}}, \tag{3}$$

where

- $x_{i,j}^{(в)}$, $x_{i,j}^{(u)}$ – i, j-th sample of the reconstructed and original images, respectively;
- N, M – the dimension of the image horizontally and vertically.

Conversion coefficient sensitivity S_{coef} divided into local and global [10]. With global sensitivity, the transformation coefficient is a function of all coordinates of the input sequence space, and for local sensitivity, only a certain part of the coordinates. Accordingly, to calculate the conversion factor with global sensitivity, it is necessary to perform more arithmetic operations than for the coefficient with local sensitivity. The sensitivity of the conversion coefficients is determined by the used orthogonal basis.

Currently, the following bases are widely used: discrete cosine transform (DCT), discrete Walsh transform (DWT), Haar transform (HT) [11] and various wavelet transforms (WT).

The complexity of transformations is greatly reduced when using separable orthogonal transformations having fast computational algorithms. These include Fourier transforms, Haar transforms, and cosine transforms, which are most useful for compressing image data.

The main property of the discrete cosine transform is that its basis vectors approximate very well the eigenvectors of Telltz matrices.

In its decorrelating properties, it approaches the Karunen-Loeve transform, while retaining the capabilities of fast Fourier algorithms. This explains the use of discrete cosine transform in JPEG-formats of representation and transmission of images.

The most efficient, from the point of view of computational costs per one transformation ordinate, is the orthonormal system of piecewise constant functions of the Haar basis. The Haar basis possesses the locality property, which underlies the modern theory of wavelet transforms.

When converting an image, the standard deviation (SD) is used with a quantitative indicator (σ) corresponding to the errors of the reconstructed image in relation to the original image. In fig. 1 shows the corresponding values of the transformation efficiency indicators for the orthogonal basis [6,14-16].

Thus, Fig. 1 illustrates the dependence of the root-mean-square deviation σ when using the analyzed transformations for digital images of three classes: $k=1$ - monotonic, $k=2$ - of the "portrait" type; $k=3$ - with a large number of contours.

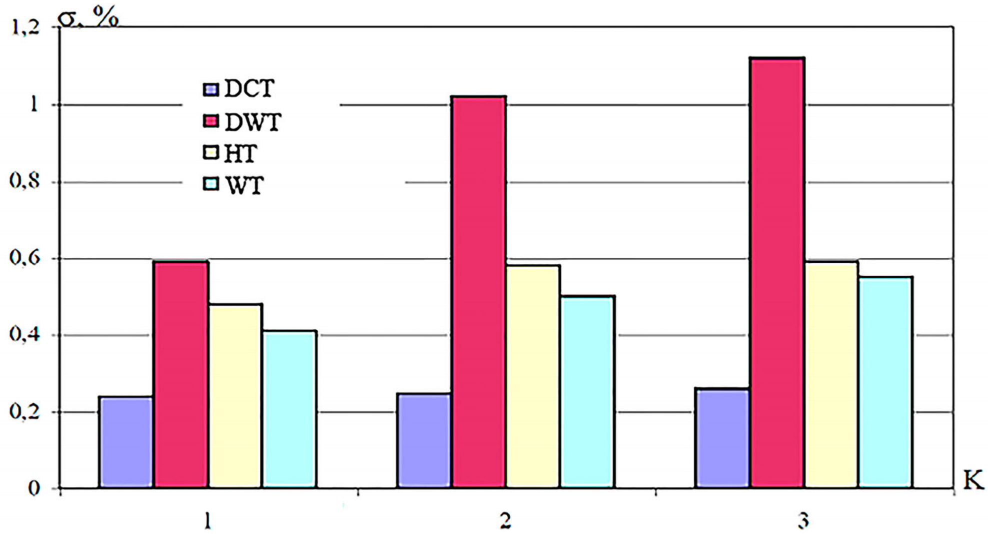

Figure 1: SD dependence when converting images samples with different saturation degrees

As follows from Fig. 1, the Haar transforms have the best values of efficiency indicators. Despite the small SD values provided by DCT and WT, their significant disadvantage is the increased dynamic range.

This leads to the need to perform quantization, as a result of which the SD is increased. One of the stages of digital image compression is image encoding.

As a result of encoding a digital image, a compressed sequence is formed using fixed - length encoding methods.

When compressing digital images, for which the amount of information plays an important role, variable length codes are used.

A distinctive feature of optimal codes is that they do not lead to expansion of the compressible data.

According to the data in Fig. 2, the results of optimal coding (shown by the dotted line) and Huffman coding (shown by the solid line) can be compared. $C(f,S)$ is the source S coding cost f, $p(s)$ is the relative symbol coding frequency.

It can be seen from the graphs that the Huffman coding redundancy approaches optimal only when the relative symbol coding rates are multiples of two.

The advantages of the orthogonal transformation of the original image according to the Haar basis include low computational complexity with a relatively low SD.

Analysis of the efficiency of two-dimensional transformations: DCT, DWT, HT and WT in the Haar basis (Table 1) allows us to single out the HT and DWT transformations as the most promising in terms of the presented indicators, provided that the requirements for the efficiency of image processing are met.

Thus, from the analysis of digital image compression procedures, it follows that in order to meet the requirements for the efficiency of image processing and transmission when implementing methods for compressing them with an acceptable level of distortion, it is necessary to carry out an orthogonal transformation of the original image. Table 1 also shows the values of the proposed performance indicators for the considered transformations [10,12-14]. The use of WT according to the Haar basis provides the smallest increase in the dynamic range (1.5 times), when using other bases, the increase will be greater.

Despite the small SDs that DCT and WT provide, their significant drawback is the increased dynamic range, which leads to the need to perform quantization and, as a result, to an increase in SD.

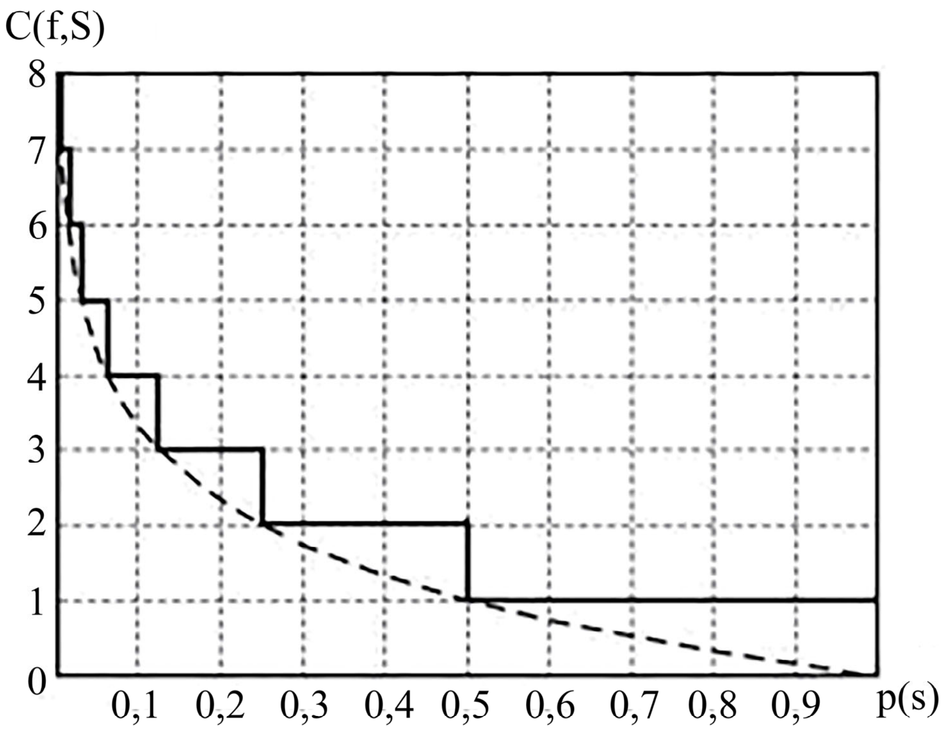

Figure 2: Dependence of coding redundancy for optimal coding - - and Huffman coding -

Table 1. Two-dimensional transform efficiency parameters: DCT, DWT, HT and WT in the Haar basis

		Conversion			
		DCT	DWT	HT	WT (L steps)
Performance indicator	$K_{sum/sub} / K^{(-1)}_{sum/sub}$	$4N^2 \log_2 N$	$2N^2 \log_2 N$	$2(N-1)$	$N^2 \sum_{k=0}^{L-1}(\frac{N}{2^k}-1)$
	$K_{mul/div} / K^{(-1)}_{mul/div}$	$4N^2 \log_2 N$	N	N	$N^2 \sum_{k=0}^{L-1}(\frac{N}{2^k}-1)$
	T_{op}	real	integer	partially real	real
	D_{tr}	increases by 2 - 3 times	not increasing	not increasing	increases by 1,5 times
	S_{coef}	global	global	global and local	local (with the Haar basis)

Analysis of the Table 1 shows that the Haar transforms have the best values of efficiency indicators. Existing lossy compression techniques such as JPEG, JPEG-2000, TIFF, WI and others use conversion as a preparatory step for quantization or selection.

At this stage, the search and elimination of transformation coefficients is performed, the contribution of which to the formation of the restored image is minimal. In this case, it is taken into account that to obtain an accurate approximation of the initial data, it is not required to use all the transformation coefficients. To do this, the JPEG method uses a procedure called quantization. To quantize, simply divide the conversion factors by another number and round to the nearest integer. To determine the quantum value in JPEG, an array of 64 elements is used - a quantization table, in which values for each coefficient of the processed 8 × 8 image block are defined.

Another approach to finding and eliminating transform coefficients can be the selection of transform coefficients obtained as a result of the transformation transform. Selection is based on the zonal, threshold and zonal-threshold principle [4,15,16]. In zonal selection, only those coefficients are selected that are in a predetermined zone (usually in the lower spatial frequency region). In the case of threshold selection, coefficients are selected that exceed a certain threshold level [16]. Analysis of modern works [17,18] shows that zonal-threshold selection is a combination of zonal and threshold selection and allows you to effectively use the coding procedure in the future.

The final step in most compression methods is encoding, which results in a compressed sequence. It is known that character sets in digital image processing are almost always natively represented by the use of encoding methods like EBCDIC or ASCII, which can't implement the counting characters frequency. Moreover, each symbol is encoded with a constant number of bits, which ensures a high computation speed. In image compression applications, where one of the main criteria is information amount, variable length codes are used.

The analysis of works [4,19] showed that the development and widespread use of existing compression methods received statistical coding algorithms. The processing mode for generating such codes for a list of values, based on their frequency's values, is simple and may include two stages: modeling and coding. If the elements probability distribution $p(s)$ generated by the source is known, then the length of the character codes of the alphabet is proportional $-\log \ p(s)$. However, in most cases, the statistical source model is not known, therefore, it is necessary to build a source model that would allow us to estimate the probability of each element occurrence at each input sequence position.

There are some algorithms [20], which form the source model as the data stream is processed or use a fixed model based on a priori ideas about the data nature. The encoding procedure effectiveness depends on the following parameters: entropy $H(S)$ coding source S (for example, according to the Bernoulli scheme $H(S) = \sum_{i=1}^{n} p(s_i) \log_2 p(s_i))$; cost $C(f,S)$ source S coding f; coding redundancy $R(f,S)$. A hallmark of optimal codes f_0 is that they do not expand the compressed data in the worst case and provide such an encoding cost $C(f_0,S)$ at which the inequality $R(f_0,S) \leq R(f,S)$ holds for any coding f [21]. By Shannon's theorem, the best compression when represented in binary form can be obtained by encoding characters with a relative frequency $p(s)$ using the - $\log_2 p(s)$ bit [22]. The redundancy of Huffman coding approaches optimal only when the relative frequencies are multiples of two. It was shown in [19,20,23] that the use of arithmetic coding gives results close to optimal results. Thus, the existing compression methods are a set of procedures, the implementation of which is aimed at reducing various types of redundancy in the representation of images. The analysis of the considered compression methods showed that ensuring the efficiency of processing and transmission of images of the earth's surface with UAC at an acceptable level of their distortion is an urgent task. To solve it, it is necessary to carry out: orthogonal transformation of the original image according to the Haar basis, the advantage of which is low computational complexity with a relatively low SD; carrying out zonal or zonal-threshold selection of conversion coefficients, which allows satisfying the requirements for adaptability and interactivity of the developed image compression method; coding of the selected sequence using optimal codes.

3. Method for assessing the degree of images saturation

To achieve our goal, we will use a direct two-dimensional transformation of images on the Haar basis.

Direct and inverse two-dimensional transformation of images on the Haar basis:

$$y_{k,p} = K_{norm} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} x_{i,j} h_{k,p}^{(1)}(i,j) \tag{4}$$

$$x_{k,p} = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} y_{i,j} h_{k,p}^{(-1)}(i,j), \tag{5}$$

where:

- $i, j,\ k, p = \overline{0, (N-1)}$ – the coordinates of the image reference location or its transform coefficient in the block $N \times N$;
- $x_{k,p}$ - image block count $X(n)$;
- $K_{norm} = 1/N^2$ - normalization coefficient;
- $y_{i,j}$ - Haar transform coefficient (transform element $Y(n)$);
- $h_{k,p}^{(1)}(i,j)$, $h_{k,p}^{(-1)}(i,j)$ - i, j-th element of k, p-th submatrix $H_{i,j}^{(1)}$ direct transformation matrix $H_{пр}^{(2)}(n)$ and submatrix $H_{i,j}^{(-1)}$ inverse transformation matrix $H_{обр}^{(2)}(n)$ respectively.

The performed analysis of the transformation transformants showed that to reduce the dynamic range of the transformant, it is advisable to use subquantization of the coefficients in the direct transformation. Conversion factors should be grouped into selection zones that include factors with the same sensitivity.

According to the selection rule, the configuration of the selection zones of the Haar transformants has the form shown in Fig. 3. (the solid line shows the boundaries of the transformant zones, and the dashed line shows the transformation coefficients in the zones).

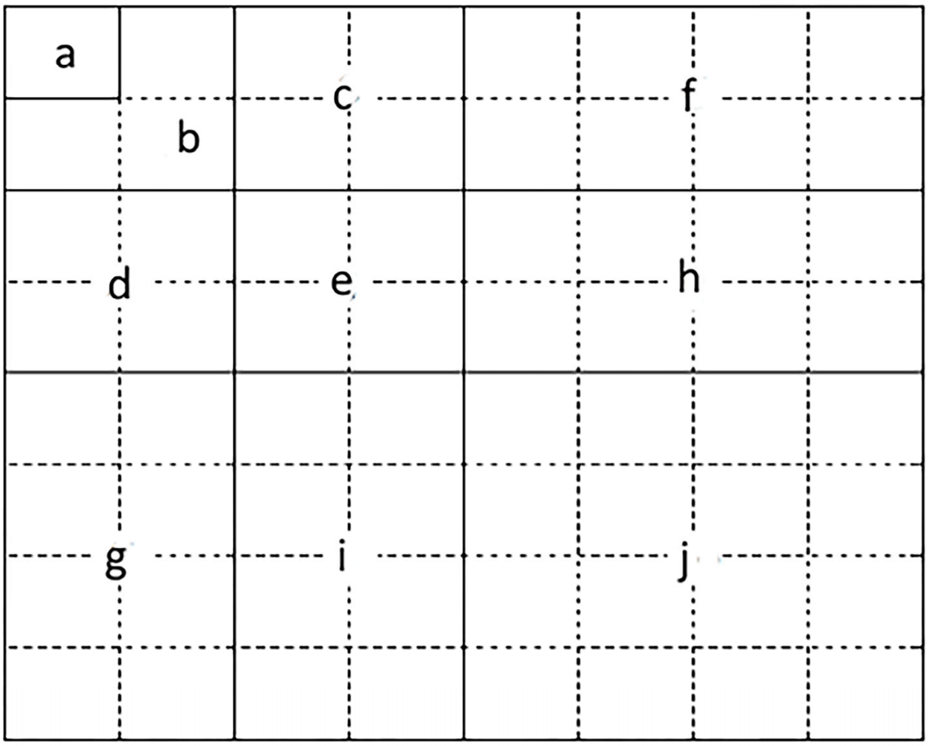

Figure 3: SD dependence when converting images samples with different saturation degrees

The coefficients of the S -th ($s \in a, j$) selection zone of all transformants of N blocks forms the S -th zonal sequence (ZQ). So, for example, f zone ZQ consists of the coefficients f of the selection zones of transformants of all image blocks. Conversion coefficients are grouped into selection zones containing coefficients with the same sensitivity. During subquantization, the value of the transformation coefficients is quantized with less accuracy in relation to the samples of the original digital image.

4. Experiments and results of original image orthogonal transformation research

When conducting a study using the method of adaptive compression of digital images in real time, the distribution of the number of repetitions of the values of the coefficients ZQ $N(y_i)$ was considered using the example of histograms of 3, 5 and 10 zones for test images shown in Fig. 4. The significant differences in the ZQ histograms, which are composed of the all blocks selection zones, for different images suggest the possibility of classifying images according to the energy distributed in them.

$$E = \sum_{s=1}^{Z} E_S = \sum_{s=1}^{Z} \sum_{k=1}^{K_S} C_{k,S}^2 , \tag{6}$$

where E_S – energy S -th transform selection zones; Z – number of breeding zones; $C_{k,s}$ – k -th coefficient S -th transform zone; K_S – coefficients number $C_{k,s}$ at S -th zone, nonzero. It is proposed to use the weighted energy parameter S -th of the zonal sequence as an indicator that allows us to quantify the image energy distribution over ZQ

$$Q_S = E_S / K_S . \tag{7}$$

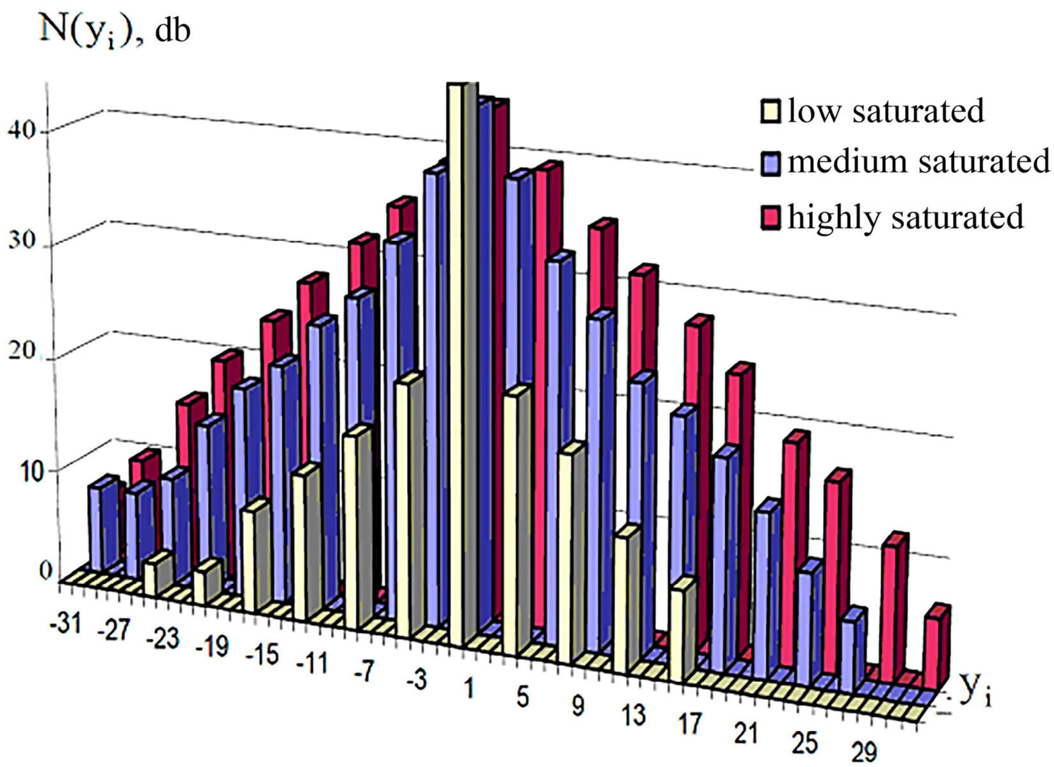

Figure 4: Histograms of the two-dimensional Haar transformation coefficients values distribution ZQ: 10th ZQ

The analysis of ZQ properties showed that their energy depends on the number of nonzero samples of the Haar basis function, which take part in the ZQ coefficients formation, and on the correlation value of the original image samples. Therefore, with an increase in the zone number, zonal energy decreases, and the magnitude of this energy reflects which particular details prevail in the image. An experiment was conducted to accumulate statistics on the energy distribution over zonal sequences for different images.

The original 300 images were used in BMP (bitmap) format with a visualization parameter of 24 bits per pixel.

The results of the zonal sequences average energy estimation $\overline{E} = \sum_{i=1}^{300} E_{s,i} / 300$ are presented in fig. 5. It is proposed to exclude zonal sequences according to the following rules: at $E_S < \overline{E}_S$ S-th ZQ considered priority for exclusion.

For example, when processing images with blocks 8 × 8, W ZQ exceptions are proposed (W is the number of ZQ that must be excluded in order to obtain the necessary compression ratio K_{com}, starting from the z-th) in the following order: 10-th, 9-th and 8-th, 5-th, 7-th and 6-th, 4-th and 3-rd.

The choice of this order is due to the zonal sequence influence, which is excluded, on the change $\Delta\sigma$ restored image; if necessary, eliminate ZQ, whose energy $E_S > \overline{E}_S$, the exception is in the same order.

Because of the values of the coefficients of the selection zone reflect the presence of details in the image of the corresponding size, then, for example, according to the histogram of the 10th ZQ of the third test image, it can be argued that there are more small details in comparison with other test images. According to the histograms of the first image, it can be assumed that there are large areas in the image with a smooth color change. It can be seen that with ZQ number increasing the dynamic range decreases, and the percentage of coefficients with a zero value reaches 40–95%, depending on the processed digital image.

Significant differences in the ZQ histograms, compiled from the selection zones of all blocks, for different images allow for the classification of images. Fig. 6 shows the compression ratio dependence K_{com} from the number of excluded selection zones W.

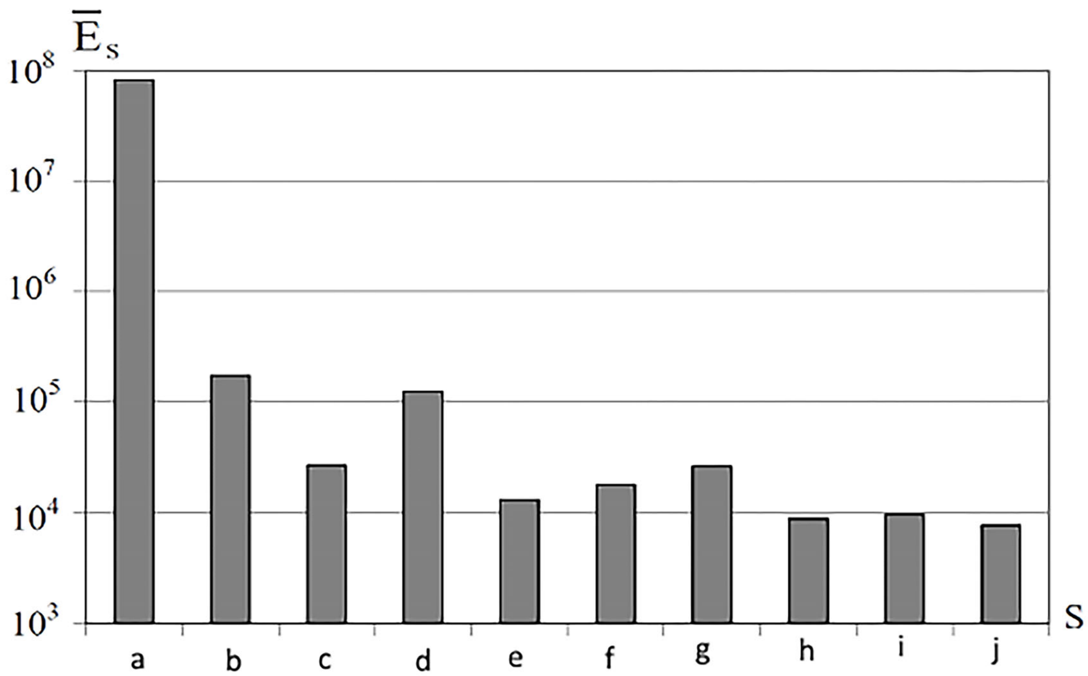

Figure 5: Histogram of arithmetic mean values of energy $\overline{E}_S$ S-th areas of analyzed images

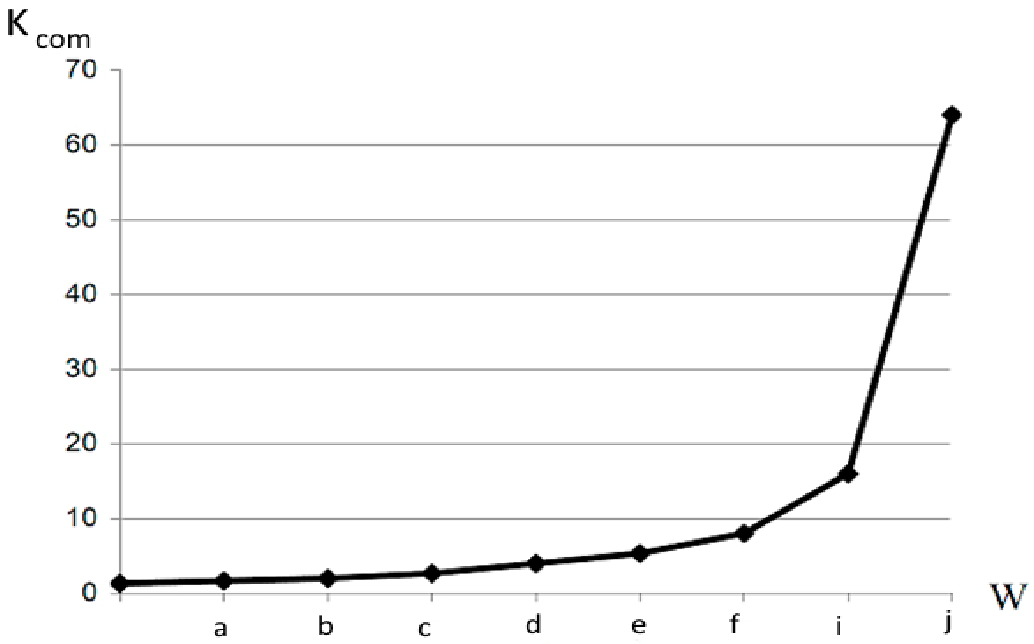

Figure 6: Dependence of the compression coefficients K_{com} on the number of excluded selection zones W, starting from the f zone

It can be seen that when “senior” zones are excluded from processing (at $W=1$, the tenth selection zone, at $W=2$, the tenth and ninth selection zones, etc.). the compression ratio increases slightly (from 1.4 to 8.8 for $W=7$).

5. Results discussion

A significant increase in the compression ratio becomes possible with the "junior" zones (second and third) exclusion. It is not advisable to exclude the first ZQ, because at least 90% of the image energy is distributed in it, and the coefficients have a global sensitivity, therefore, its exclusion will introduce the greatest distortions.

From the analysis of the graph it follows that during the execution of the lossy image compression algorithm: to minimize distortions of the reconstructed image, it should be excluded from the processing of the selection zone with the lowest energy; to obtain significant compression ratios, it is necessary to "discard" ZQ with a large number of conversion ratios.

It can be seen from the figs. 5 - 6 that the most appropriate seems to be an exception from processing j, i, h and e zones, since they have the lowest average energy, and their “dropping” provides a compression ratio $K_{com}=5$.

However, it should be noted that the root-mean-square deviation for high- and medium-resharpened images exceeds 1.5% already with the exclusion of j, i and f ZQ, which are responsible for fine details.

Wherein K_{com} does not exceed more than 4-5 times, and distortions occur throughout the image, which does not allow us to choose the level of detail that can be neglected in the given processing conditions.

Methods of zonal and zonal-boundary selection of orthogonal transformation coefficients have been obtained, which differ from the known ones in that: in the case of zonal selection, the features of the energy distribution over the processed image zonal sequences are taken into account and only those that introduce minimal distortions into the reconstructed image are excluded, which makes it possible to control the ratio of the compression ratio and the standard deviation; with zonal-boundary selection, the uneven redistribution of energy over zonal sequences of each processed image block is taken into account, which makes it possible to reduce the entropy relative to the original image by an average of 4 ÷ 5, increasing the efficiency of using statistical coding.

6. Conclusion

A method for assessing the degree of saturation of images has been developed, based on taking into account the global and local sensitivity of the basic functions of the Haar transformation using the energy index when analyzing the image transformant, which makes it possible to estimate the degree of saturation of the image with small details from the energy distribution for different images by zonal sequences. A total weighted energy index is proposed, which depends on the energy distribution and the number of orthogonal trans-formation coefficients, forming a zonal sequence.

The scientific novelty of the presented work lies in increasing the accuracy and efficiency of the data transformations carried out during assessing the degree of digital image compression by adapting the Haar transformation for the problem under consideration. The development and application of the method of adaptive compression of digital images will ensure the fulfillment of the requirement for the efficiency of processing and transmission of digital video information in real time at an acceptable level of distortion of the reconstructed image

Using the technique for assessing the degree of saturation of images will allow: more accurately, in comparison with expert judgment, to classify images; select the number of saturation classes required in each specific case; automate the classification process when evaluating an image; use the adaptive compression method as a procedure.

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