Construction Features and Analysis of Warfare Information Model with Impulse Perturbations under Poisson Approximation Conditions

Ihor Samoilenko[a], Anatolii Nikitin[b,c], Ganna Verowkina[a], and Tetiana Nikitina[d]

[a] *Taras Shevchenko National University of Kyiv, 60 Volodymyrska str., Kyiv, 01033, Ukraine*
[b] *National University of Life and Environmental Science, 15 Heroiv Oborony str., Kyiv, 03041, Ukraine*
[c] *Jan Kochanowski University of Kielce, 5 Zeromskiego str., Kielce, 25-369, Poland*
[d] *Dmytro Motornyi Tavria State Agrotechnological University, 18b Khmelnytsky ave., Melitopol, 32312, Ukraine*

Abstract

We study a continuous model that describes the conflict interaction for two complex systems. External conflict interaction is modeled by the additional influence of chance. The dynamics of internal conflict are similar to the Lotka-Volterra model. We interpret the new model of information warfare as the influence of rare events that rapidly change certain ideas of a large number of people. As a result, the number of supporters of different ideas makes stochastic jumps that we can see using the Poison approximation scheme. We suggest that such a model could be more natural, as important news now has a quick and powerful impact on audiences through television and the Internet.

Keywords

Random evolution, information warfare model, Markov switch, Lotka-Volterra equations, Poison approximation scheme

1. Introduction

The information warfare models with an additional interaction between them may be interpreted as some kind of correlation between the habitants of different regions. In general, the Lotka-Volterra model of prey-predator interaction is one of the main models for simulation of similar processes in applied mathematics, social sciences, and economics [3], [8–11], [17]. Application of this approach to the information warfare model was proposed in [4]. Authors regard some social community of quantity N_0, potentially exposed some information threat of two types, that is, for example, the threat of a negative change in its state by transmitting some information relevant to this group by information two different channels. The values $N_1(t)$, $N_2(t)$ are the numbers of “adherents” depending on time t who accepted the new information, ideas, norms, etc. of type 1 and 2 respectively. These are the main current characteristics of the degree of prevalence of information threats. We propose some results where the generator of the limit process is constructed in explicit form. We also give some interpretations of our model.

2. Classical Information Warfare Model

The main model assumptions are:

1. Both information threats are distributed among the community through the two information channels:

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EMAIL: isamoil@i.ua (A.1); nikitin2505@univ.kiev.ua (B.2); avv@univ.kiev.ua (A.3); nikitina0217@gmail.com (D.4)
ORCID: 0000-0001-6858-8224 (A.1); 0000-0001-5137-0114 (B.2); — (A.3); 0000-0002-6091-0597 (D.4)

- The first one is “external” in relation to the community, for example, advertising media campaigns. Its intensity is characterized by the parameters $\alpha_1 > 0$ and $\alpha_2 > 0$ respectively, both are considered to be independent of time.
- The second, “internal” channel is interpersonal communication between members of the social community (its intensity, that is, the number of equivalent informational contacts, characterized by the parameters $\beta_1 > 0$ and $\beta_2 > 0$ respectively, that are also independent of time). As a result, the adherents of the first idea that has been already “recruited” (their number is equal to $N_1(t)$), make their contribution to the recruitment process by affecting non-recruited members (their number is equal to the value of $N_0 - N_1(t) - N_2(t)$. The same is for the adherents of the second idea.

2. The rate of change of the number of adherents $N_1(t)$ and $N_2(t)$ (that is, the number recruited into the unit time) consists of:

- External recruitment rate (it is proportional to the product of the intensities α_1 and α_2 and on the number of individuals who are not yet recruited $N_0 - N_1(t) - N_2(t)$), that is, $\alpha_1(N_0 - N_1(t) - N_2(t))$ and $\alpha_2(N_0 - N_1(t) - N_2(t))$ respectively.
- Internal recruitment rate (it is proportional to the product of intensities β_1 and β_2, on the corresponding number of active adherents $N_1(t)$, $N_2(t)$ and on the number of non-recruited $N_0 - N_1(t) - N_2(t)$), that is, $\beta_1 N_1(t)(N_0 - N_1(t) - N_2(t))$ and $\beta_2 N_2(t)(N_0 - N_1(t) - N_2(t))$ respectively.

Consequently, the model is described by Lotka-Volterra-type equations [4]:

$$dN_1(t)/dt = (\alpha_1 + \beta_1 N_1(t))(N_0 - N_1(t) - N_2(t)),$$
$$dN_2(t)/dt = (\alpha_2 + \beta_2 N_2(t))(N_0 - N_1(t) - N_2(t)), \quad t > 0.$$

3. Information warfare model with impulsive influence

As we know, the outside world speaks with us the language of probability theory, so the deterministic model is only part of the real situation. That is why we are building a model that describes a model of information warfare that has not yet been explored, that is, a model based on contingencies, and contingencies of different types:

$$dN^\varepsilon(t) = C(N^\varepsilon(t), x(t/\varepsilon^2))dt + d\eta^\varepsilon(t), \tag{1}$$

where

$$C\left(N^\varepsilon(t), x\left(\frac{t}{\varepsilon^2}\right)\right) =$$
$$= \begin{pmatrix} -\alpha_1(x) + \beta_1(x)N_0(x) - \beta_2(x)N_1^\varepsilon(t) & -\alpha_1(x) - \beta_1(x)N_1^\varepsilon(t) \\ -\alpha_2(x) - \beta_2(x)N_2^\varepsilon(t) & -\alpha_2(x) + \beta_2(x)N_0(x) - \beta_1(x)N_2^\varepsilon(t) \end{pmatrix} \begin{pmatrix} N_1^\varepsilon(t) \\ N_2^\varepsilon(t) \end{pmatrix}, \tag{2}$$

ε is a small series parameter; $N^\varepsilon(t)$ is a two-dimensional vector of solutions, components of which are the quantities of the adherents of different ideas; $x(t/\varepsilon^2)$ is uniformly ergodic Markov process in standard phase space $(X, \boldsymbol{X})$, is defined by the generator [1], [2], [6], [13]

$$\boldsymbol{Q}\varphi(x) = q(x)\int_X P(x, dy)[\varphi(y) - \varphi(x)]$$

on the Banach space $B(X)$ of real-valued bounded functions $\varphi(x)$ with the supremum norm

$$||\varphi|| = \max_{x\in X}|\varphi(x)|.$$

The stochastic kernel $P(x, B)$, $x \in X$, $B \in \boldsymbol{X}$, uniformly ergodic embedded Markov chain

$$x_n = x(\tau_n), n \geq 0,$$

with stationary distribution $\rho(B)$, $B \in \boldsymbol{X}$. Stationary distribution $\pi(B)$, $B \in \mathbf{X}$, of the Markov process $x(t), t \geq 0$ is defined by the relation

$$\pi(dx)q(x) = q\rho(dx),$$

where

$$q = \int_X \pi(dx) q(x).$$

Denote by R_0 the potential operator of the generator $\boldsymbol{Q}$, which is defined by the equality

$$R_0 = \Pi - (\Pi + \boldsymbol{Q})^{-1},$$

where $\Pi\varphi(x) = \int_X \pi(dy)\varphi(y)\, \mathbf{1}(x)$ is the projector of zeroes of generator $\boldsymbol{Q}$ onto the subspace

$$N_Q = \{\varphi : \boldsymbol{Q}\varphi = 0\};$$

$\eta^\varepsilon(t)$ is the impulse perturbation process [1], [5], [6], [7], [16] defined by the relation

$$\eta^\varepsilon(t) = \int_0^t \eta^\varepsilon\left(ds, x\left(\frac{s}{\varepsilon^2}\right)\right),$$

where the family of processes with independent increments

$$\boldsymbol{\Gamma}^\varepsilon(x)\varphi(\omega) = \varepsilon^{-2}\int_R (\varphi(\omega + v) - \varphi(\omega))\, \Gamma^\varepsilon(d\, v, x), x \in X$$

and satisfies the properties of Poisson approximation;

P1. The approximation of averages

$$\int_R v\Gamma^\varepsilon(dv, x) = \varepsilon(a(x) + \theta_a(x)), \theta_a(x) \to 0, \varepsilon \to 0,$$

and

$$\int_R v^2\, \Gamma^\varepsilon\, (d\, v, x) = \varepsilon^2(b(x) + \theta_b(x)), \theta_b(x) \to 0, \varepsilon \to 0.$$

P2. The condition imposed on the distribution function

$$\int_R g(v)\, \Gamma^\varepsilon(d\, v, x) = \varepsilon^2(\Gamma_g(x) + \theta_g(x)), \theta_g(x) \to 0, \varepsilon \to 0$$

for all $g(\mathrm{v}) \in C_3(R)$, and $C_3(R)$ is the space of real-valued bounded functions such that

$$g(v)\, / |\, v\, |^2 \to 0, |\, v\, | \to 0,$$

where measure $\Gamma_g(x)$ is bounded for all $g(\mathrm{v}) \in C_3(R)$ and is defined by the relation (functions from the space $C_3(R)$ separate the measures):

$$\Gamma_g(x) = \int_R g(v)\Gamma_0(dv, x),\, g(v) \in C_3(R).$$

P3. The uniform quadratic integrability

$$\lim_{c\to\infty}\int_{|\, v\, |>c} v^2\, \Gamma_0\, (d\, v, x) = 0.$$

P4: Absence of diffusion component

$$b(x) = \int_R v^2\Gamma_0(dv, x).$$

We give a simple example of a random variable ξ that satisfies the conditions of the Poisson approximation:

$$P\{\xi = b\} = \varepsilon p,$$

$$P\{\xi = \varepsilon a\} = 1 - \varepsilon p.$$

The relations for the moments of this random variable are as follows:

$$E\, \xi = \varepsilon(a + bp) + o(\varepsilon),$$

$$E\, \xi^2 = \varepsilon(b^2 p) + o(\varepsilon).$$

4. Asymptotic analysis of the model

Firstly, we consider the asymptotic properties of the perturbation process.

Theorem 1. *Under conditions **P1 – P4**, weak convergence*

$$\eta^{\varepsilon}(t) \to \eta^{0}(t),\ \varepsilon \to 0.$$

holds true for the impulse perturbation process.

The limit process $\eta^0(t)$ is defined by the generator

$$\Gamma\varphi(w) = \hat{a}\varphi'(w) + \int_R [\varphi(w+v) - \varphi(v)]\hat{\Gamma}_0(dv),$$

where

$$\hat{a} = \int_X \pi\,(dx)a(x),$$

$$\hat{\Gamma}_0(v) = \int_X \pi\,(dx)\Gamma_0(v,x).$$

Thus, we deal in a limit with a random process, which has deterministic drift and Poisson jumping component.

Further, we investigate the asymptotic properties of the original evolutionary system (1), in particular, using the approaches proposed in [14], [15].

Theorem 2. *If conditions **P1 – P4** are satisfied, the weak convergence in the sense of generators convergence*

$$(u^{\varepsilon}(t), \eta^{\varepsilon}(t)) \to (u^{0}(t), \eta^{0}(t)),\ \varepsilon \to 0.$$

holds true.

The limiting coupled process is defined by the generator

$$\boldsymbol{L}\varphi(w,v) = \hat{C}(u)\varphi'(w,\cdot) + \Gamma_w\varphi(w,\cdot),$$

where generator Γ_w are the same as defined in Theorem 1, but acting on argument w of the vector-function $\varphi(w,v)$, corresponding to a coupled process.

The averaged function has a form

$$\hat{C}(u) = \int_X \pi(dx)C(u,x).$$

The last correlation means that to obtain the limit characteristics that describe the information warfare model, all the functions in (2) that depend on *x* should be averaged by the stationary measure of the switching Markov process.

5. Investigation methods and results

We apply the approaches to the construction and analysis of complex systems proposed in the works of Korolyuk V.S. [1], [2] and his followers, in particular, we apply the following scheme:

1. Construction of the generator of the Markov additive process.
2. Asymptotic form of the generator acting on some special type of test functions.
3. Solving of a singular perturbation problem on test functions in a form

$$\varphi^{\varepsilon}(u,w,v) = \varphi(u,w) + \varepsilon\varphi_1(u,w,v) + \varepsilon^2\varphi_2(u,w,v).$$

The following resumes should be made:

1. The weak convergence of the processes

$$u^{\varepsilon}(t) \Rightarrow u^{0}(t),\ \varepsilon \to 0.$$

follows from the convergence of respective generators when compactness of the prelimiting set of processes $u^{\varepsilon}(t)$ holds true. Weak convergence of stochastic processes is usually proved by checking the two conditions: tightness of the distributions of the converging processes which ensures the existence of a converging subsequence and uniqueness of the weak limit. The passage to the limit can be done on the semigroups which correspond to the converging processes as well as on appropriate generators. While proving convergence of generators a natural question arises concerning the uniqueness of a limit semigroup. It can be answered by representing the process in focus as a unique solution to a martingale problem which is formulated with the help of the limit generator.

2. The limit process $u^0(t)$ can be given by stochastic differential equation

$$d\hat{u}(t) = [\hat{C}(\hat{u}(t)) + \hat{a}]dt + \int_R v\tilde{v}\,(dt, d\,v),$$

where

$$\boldsymbol{E}\tilde{v}(dt, dv) = dt\tilde{\Gamma}_0(dv).$$

3. The limit process $u^0(t)$ has two components. The deterministic drift is defined by the solution of the differential equation

$$d\hat{u}_d(t) = [\hat{C}(\hat{u}_d(t)) + \hat{a}]dt,$$

where the additional term $\hat{a}$ appears due to accumulation with the normalized time $t/\varepsilon^2, \varepsilon \to 0$ of small jumps of the impulse process that happen with probability, close to one. The second component is rare big jumps that take place with nearly zero probability and are defined in terms of an averaged measure of jumps $\tilde{\Gamma}_0(dv)$ by the generator

$$\Gamma\varphi(w) = \int_R [\varphi(w+v) - \varphi(w) - v\varphi'(w)]\tilde{\Gamma}_0(dv)$$

6. Conclusions

The following results of our investigation were obtained:

- A model that is more general than the classical one was proposed.
- The limit generator of the dynamical system was constructed.
- The behavior of the limit process in terms of its components was analyzed.

7. Interpretation

As we can see in many works on mathematical biology and economics the modeling of population dynamics or economical processes is based on Lotka-Volterra type equations.

We propose a new model of information warfare with an additional influence of chance. That may be interpreted as some kind of rare event that rapidly changes some beliefs of large quantities of people. As a result, the quantities of adherents of different ideas make stochastic jumps, which we may see applying the Poisson approximation scheme. We suppose that such a model could be more essential, as soon as now breaking news produce a quick and astonishing influence on the audience through TV and Internet.

The behavior of our model could not be analyzed obviously for any fixed moment as it was done in a classical case. But, as it is usual for stochastic models, we may obtain functional limit theorems that present the behavior on large time intervals. Thus, we have averaged limit characteristics of the process and may use them to construct obvious solutions. We hope to obtain recommendations for prevalence strategies in information warfare fights in the future.

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