Observable Consequences of Partially Degenerate Leptogenesis

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Abstract

In the context of the seesaw mechanism, it is natural that the large solar and atmospheric neutrino mixing angles originate separately from large 2×2 mixings in the neutrino and charged-lepton sectors, respectively, and large mixing in the neutrino couplings is in turn more plausible if two of the heavy singlet neutrinos are nearly degenerate. We study the phenomenology of this scenario, calculating leptogenesis by solving numerically the set of coupled Boltzmann equations for out-of-equilibrium heavy singlet neutrino decays in the minimal supersymmetric seesaw model. The near-degenerate neutrinos may weigh $\lesssim 10^8$ GeV, avoiding the cosmological gravitino problem. This scenario predicts that $Br(\mu \to e\gamma)$ should be strongly suppressed, because of the small singlet neutrino masses, whilst $Br(\tau \to \mu\gamma)$ may be large enough to be observable in B-factory or LHC experiments. If the light neutrino masses are hierarchical, we predict that the neutrinoless double-β decay parameter $m_{ee} \approx \sqrt{\Delta m_{sol}^2} \sin^2 \theta_{12}$.

Neutrino oscillation data [1, 2] are converging towards unique solutions for both the solar and atmospheric neutrino anomalies [3]. There are two large, almost maximal, mixing angles θ_{12} and θ_{23} in the light neutrino mass matrix, that give rise to the solar and atmospheric neutrino oscillations, respectively, whilst the third mixing angle θ_{13} is constrained to be small [4, 5]. This pattern motivates theoretical approaches based on two 2×2 mixings instead of one general 3×3 mixing.

The smallness of the neutrino masses is generally explained via the seesaw mechanism [6], described by the superpotential [1]:

$$W = N_i^c (Y_\nu)_{ij} L_j H_2 - E_i^c (Y_e)_{ij} L_j H_1 + \frac{1}{2} N^c{}_i (M_N)_{ij} N_j^c + \mu H_2 H_1 \,. \tag{1}$$

Here the indices i, j run over three generations and M_N is the heavy singlet-neutrino mass matrix. We shall work in a basis where $(M_N)_{ij}$ is real and diagonal, $(M_N)_{ij} = M_{N_i} \delta_{ij}$, and we define $M_{N_1} < M_{N_2} < M_{N_3}$. In analogy with the CKM mixing matrix in the quark sector, the mixing matrix V_{MNS} measured in neutrino oscillations is a product of two matrices $V_{MNS} = U_e^\dagger U_\nu$, where U_ν diagonalizes the light-neutrino seesaw mass matrix $\mathcal{M}_\nu$

$$\mathcal{M}_\nu = Y_\nu^T \left(M_N \right)^{-1} Y_\nu v^2 \sin^2 \beta, \tag{2}$$

according to

$$U_\nu^T \mathcal{M}_\nu U_\nu = \mathcal{M}_\nu^D \,, \tag{3}$$

and U_e helps diagonalize the charged-lepton Yukawa coupling matrix Y_e in (1):

$$V_e^\dagger Y_e U_e = Y_e^D. \tag{4}$$

Deriving the observed neutrino mixing angles from the neutrino Dirac Yukawa couplings $(Y_\nu)_{ij}$ and Majorana masses M_{N_i} is problematic. A generic difficulty, in view of the hierarchy $\Delta m_{sol}^2 \ll \Delta m_{atm}^2$, is that large mixing angles for both solar and atmospheric mixings can be obtained only at the price of some fine tuning. Technically speaking, the (23) sub-determinant of the light-neutrino mass matrix must vanish. While Yukawa textures which may accommodate this feature can be obtained from well-motivated physics ideas [7] such as GUTs, Abelian and non-Abelian flavour symmetries, democratic principles, etc., many of these approaches feature unknown model coefficients of order unity that cannot be predicted without further assumptions, failing which they must be tuned *a posteriori*.

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[1]We assume low-energy supersymmetry, which leaves unaltered the flavour parameters, while providing extra low-energy observables.

One way to attack this problem is to note that the observed leptonic mixing pattern can be regarded as the combination of two 2×2 mixings in the neutrino and the charged-lepton Yukawa couplings $(Y_\nu)_{ij}$ and $(Y_e)_{ij}$, respectively. If the matrices U_ν and U_e contain non-trivial mixings only in the (12) and (23) sub-matrices, respectively, large mixing angles θ_{12}, θ_{23} and vanishing θ_{13} follow naturally. Notice, however, that the matrices U_ν and U_e are of different nature. Whilst U_e directly rotates the superpotential couplings in (1), U_ν diagonalizes the effective mass matrix obtained from the superpotential couplings via the seesaw relation (2). Therefore, one may expect some differences between these two matrices U_ν and U_e, which may be the reason why the solar mixing somewhat differs from the atmospheric one [2].

Such a structure for Y_ν and U_ν would arise naturally if there is an (approximate) S_2 symmetry between the first and the second generation fields: $N_1 \leftrightarrow N_2$ and $L_1 \leftrightarrow L_2$, implying

$$Y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad M_N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{5}$$

in the (12) sector. The large mixing angle θ_{12} diagonalizing Y_ν follows immediately from this approximate symmetry, as does the prediction that $M_{N_1} \sim M_{N_2}$, , while M_{N_3} may be different. Larger flavour symmetries would be needed to extend this approach to the third generation, to incorporate the E_i^c fields, and to explain the largeness of θ_{23}. We do not pursue such model-building issues here, but rather pursue the phenomenological implications of the proposed structure (5) for Y_ν, Y_e and M_N, namely that the Dirac Yukawa matrices Y_ν and Y_e have non-trivial 2×2 (12) and (23) sub-matrices and non-vanishing diagonal elements (33) and (11), respectively, whilst the rest of their entries vanish and M_N has a pair of (almost) degenerate eigenvalues. We incorporate the low-energy neutrino data into our parametrization of the Yukawa couplings, and systematically scan over all the remaining free parameters of the seesaw model. In this way we include theoretical models of [7] that predict similar patterns.

We focus our attention on leptogenesis [8] in this framework, finding that the pair of near-degenerate heavy singlet neutrinos enable one to lower the scale of thermal leptogenesis below the gravitino bound on the reheating temperature of the Universe in supergravity [9] and gauge-mediated models [10]. We study implications of this scenario for $\beta\beta_{0\nu}$ decay [11] and on charged-lepton flavour-violating (LFV) decays [12]. A important aspect of this scenario is that there are less free parameters than in the general seesaw model, implying testable phenomenological consequences. We find that $\tau \to \mu\gamma$ may be observable, whilst $\mu \to e\gamma$ is

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[2]The above breakdown is basis-dependent: one could choose to work in the basis in which Y_e^D is diagonal, in which case the mixing in U_e simply moves into Y_ν, while the physics remains unchanged.

suppressed, and make a specific prediction for neutrinoless double-β decay.

We start by counting the physical parameters of the model. In the chosen basis the Yukawa matrix Y_ν contains five complex parameters, and the basis for N_i is completely fixed. Since three phases can be removed by redefining the L_i fields, only two phases are physical. One can parametrize Y_ν as

$$(Y_\nu)_{ij} = Z^\star_{ik} Y^D_{\nu_k} X^\dagger_{kj}, \tag{6}$$

where the unitary matrices X and Z contain only (12) mixing, and hence only one mixing angle. The matrix X is real, while the matrix Z may be written as $Z \equiv P_1 \overline{Z} P_2$, where $\overline{Z}$ is a real matrix. In this 'high-energy' basis, the diagonal matrices $P_{1,2} \equiv \mathrm{diag}(e^{i\theta_{1,2}}, 1, 1)$ contain the two physical CP-violating phases. Diagonalizing Y_e according to (4), the real parameters correspond to three diagonal Yukawa couplings Y^D_e and one mixing angle in each of the rotation matrices U_e and V_e. Again, three out of five phases can be absorbed into redefinition of right-handed fields E^c_i. Thus the basis for E^c_i is now completely fixed and there is one physical phase in the mixing matrix $U_e = \overline{U_e} P_3$, where $\overline{U_e}$ is real and $P_3 \equiv \mathrm{diag}(1, 1, e^{i\theta_3})$. The mixing and one phase in V_e are unobservable. This implies that, in the basis in which the charged lepton masses are diagonal, we have a total of 9 physical parameters in the neutrino Yukawa couplings, which together with the 3 unknown heavy masses M_{N_i} make a total of 12 parameters in this version of the minimal seesaw model. This should be compared with the 18 physical parameters in the general case, implying more predictivity and hence better possibilities to test the scenario at high and low energies.

We recall there are three types of leptonic observables in supersymmetric seesaw models: (i) leptogenesis via out-of-equilibrium decays of heavy singlet neutrinos and (ii) renormalization-induced mixings in the slepton mass matrix due to the off-diagonal Y_ν couplings, as well as (iii) the light neutrino masses and mixings. The leptogenesis CP asymmetry produced in the out-of-equilibrium decays of each of the N_i is given by [13, 14, 15]

$$\epsilon_i = -\frac{1}{8\pi} \sum_l \frac{\mathrm{Im}\left[\left(Y_\nu Y_\nu^\dagger\right)^{il} \left(Y_\nu Y_\nu^\dagger\right)^{il} \right]}{\sum_j |Y_\nu^{ij}|^2}$$
$$\sqrt{x_l}\left[\mathrm{Log}(1 + 1/x_l) + \frac{2}{(x_l - 1)}\right], \tag{7}$$

where $x_l \equiv (M_{N_l}/M_{N_i})^2$. It is clear from (7) that the generated asymmetry depends only on

$$Y_\nu Y_\nu^\dagger = P_1^\star \overline{Z}^\star (Y_\nu^D)^2 \overline{Z}^T P_1 \tag{8}$$

and on the heavy neutrino masses. Notice that $Y_\nu Y_\nu^\dagger$ is independent of the left-rotations of L_i, and is therefore independent of the Y_e basis. Therefore, in our scenario the baryon asymmetry

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of the Universe depends only on the single high-energy CP phase in P_1. If the CP-conserving parameters are known, this can be calculated from the observed baryon asymmetry.

Renormalization of soft supersymmetry-breaking parameters due to the presence of Y_ν above the heavy-neutrino decoupling scales modifies the left-slepton mass matrix $m_{\tilde{L}}$ and trilinear soft supersymmetry-breaking A_e terms. In the basis of diagonal charged leptons one has in leading-logarithmic order [12]

$$
\begin{aligned}
(\delta m_{\tilde{L}}^2)_{ij} &\simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y^\dagger L Y)_{ij}\,, \\
(\delta A_e)_{ij} &\simeq -\frac{1}{8\pi^2}A_0 Y_{e_i}(Y^\dagger L Y)_{ij}\,,
\end{aligned}
\tag{9}
$$

which are proportional to

$$
Y^\dagger L Y = P_3^* \overline{U}_e^{\,\dagger} X Y^D P_2 \overline{Z}^T L \overline{Z}^* P_2^* Y^D X^\dagger \overline{U}_e P_3,
\tag{10}
$$

where L is a diagonal matrix: $L_{ij} = \ln(M_{GUT}/M_{N_i})\delta_{ij}$. Note that the high-energy CP-violating phases in (10) are those in P_2 and P_3. The CP-violating observables in LFV processes all depend on a single CP-violating invariant $J_\nu = \mathrm{Im}H_{12}H_{23}H_{31}$ [16], where $H = Y_\nu^\dagger L Y_\nu$. This influences slepton physics at colliders and also determines the T-odd asymmetry in $\mu \to 3e$ [17]. Therefore, all low-energy CP-violating observables measure one combination of phases in $P_{2,3}$. Due to the near-degeneracy of the heavy neutrino masses, the lepton EDMs are unobservably small in this scenario [18].

So far, we have considered the high-energy parametrization of the neutrino sector in terms of the diagonal Yukawa couplings and the unitary matrixes X, Z, U_e. Counting physical degrees of freedom is straightforward in this approach, but it has limited applicability to low-energy phenomenology. Therefore we develop a complementary 'low-energy' parametrization. We start by discussing the light neutrino mass matrix (2) in our scheme. Of course, at low energy it is most convenient to work in the basis in which charged leptons are diagonal. After redefinitions of the L_i fields, the diagonalizing matrix for light neutrinos becomes

$$
U_\nu = V_\nu P_0,
\tag{11}
$$

where V_ν is real and contains the mixing angles θ_{12}, θ_{23}, and $P_0 \equiv \mathrm{diag}(1, e^{i\phi_2}, e^{i\phi_3})$ contains two Majorana phases $\phi_{2,3}$, whilst the third angle θ_{13} vanishes and the neutrino oscillation phase δ is absent. Therefore, all the low-energy neutrino observables, such as neutrino oscillations, $\beta\beta_{0\nu}$ decay, etc., depend on the 7 effective low-energy parameters, which are functions of the 12 parameters in (1). We recall that, whilst neutrino oscillations measure

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the mass-squared differences of neutrinos and their mixing angles, $\beta\beta_{0\nu}$ decay measures one particular combination of their masses and mixing matrix elements:

$$|m_{ee}| \equiv \left| \sum_i (U_\nu)^*_{ei} m_{\nu_i} (U_\nu)^\dagger_{ie} \right| = \left| m_{\nu_1} \cos^2 \theta_{12} + m_{\nu_2} \sin^2 \theta_{12} e^{2i\phi_2} \right|. \tag{12}$$

The effect of the Majorana phase ϕ_2 becomes visible only if $m_{\nu_1} \sim m_{\nu_2}$, but its measurement is not straightforward even in this case [11]. In the case of hierarchical neutrino masses $m_{\nu_1} \ll m_{\nu_2}$, we have the definite prediction $m_{ee} = m_{\nu_2} \sin^2 \theta_{12}$.

In order to satisfy automatically the oscillation data, the light neutrino parameters must be an input of the parametrization. We therefore define [19]:

$$Y_\nu = \frac{\sqrt{M_N} R \sqrt{\mathcal{M}^D_\nu} U_\nu^\dagger}{v \sin \beta}, \tag{13}$$

where R is a complex orthogonal matrix. In our case, it has a non-trivial 2×2 (12) submatrix only, and is parametrized by just one complex number. Examining the combination $Y_\nu Y_\nu^\dagger$ using (13), we observe that the single complex phase in R is responsible for leptogenesis. On the other hand, (9) tells that in this parametrization the renormalization-induced low-energy CP violation depends also on the Majorana phases in a non-trivial way. One can exploit this to take a complete bottom-up approach, since in the supersymmetric seesaw model the low-energy degrees of freedom may in principle be used to reconstruct all the high-energy neutrino parameters [20]. A parametrization related to the solutions of the renormalization-group equations (RGEs) for the soft supersymmetry-breaking slepton masses (9) was worked out in [21].

Our central objective in this Letter is to study the phenomenology of leptons in the proposed scheme. In particular, we focus our attention on leptogenesis [3] and its relations to neutrino masses, $\beta\beta_{0\nu}$ decay and LFV observables. A strong motivation is provided by the gravitino problem in generic supergravity theories, which restricts the maximum reheating temperature T_R of the Universe after inflation. In supergravity models with a gravitino mass of order 500 GeV [9] or in some gauge-mediated models [10] the bound is

$$T_R \lesssim 10^8 \text{ GeV}. \tag{14}$$

On the other hand, if the baryon asymmetry of the Universe is due to the decays of the lightest singlet neutrino N_1, there is an M_{N_1}-dependent upper bound on the CP asymmetry ϵ_1 [24]. In the context of thermal leptogenesis, the observed baryon asymmetry implies a lower bound $M_{N_1} \gtrsim 10^{10}$ GeV [25] which is potentially in serious conflict with (14).

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[3]Implications of particular neutrino mass textures on leptogenesis have been studied in [14, 15, 22, 23].

This problem could be overcame by abandoning thermal leptogenesis and considering non-thermally produced neutrinos [26], but such scenarios are still speculative and lack predictivity. Another well-known solution to the problem in the context of thermal leptogenesis is to consider the decays of two heavy neutrinos which are approximately degenerate in mass [13]. Because the self-energy contribution to ϵ_i is enhanced for degenerate heavy neutrino masses, the observed baryon asymmetry can be generated by moderately degenerate heavy neutrinos that are relatively light, which is the option pursued here. Since, in the proposed scenario, the Y_ν couplings spanning the (12) submatrix are separate and distinct from the (33) element, it is natural that the masses of the heavy neutrinos $N_{1,2}$ differ from that of N_3 [4].

In the minimal supersymmetric seesaw model, the baryon asymmetry originates from out-of-equilibrium decays of neutrinos and sneutrinos. Assuming $M_{N_3} > M_{N_2} \approx M_{N_1}$, we need consider only the two lighter neutrino decays. Even if $M_{N_2} \approx M_{N_1}$, the CP asymmetries $\epsilon_{1,2}$ may be very different in magnitude. Although the products of the numerator and mass factors in (7) are identical for $\epsilon_{1,2}$, the denominators $(Y_\nu Y_\nu^\dagger)_{11}$ and $(Y_\nu Y_\nu^\dagger)_{22}$ may be very different [5]. Therefore, $N_{1,2}$ may contribute to the lepton asymmetry and to the reaction rates of the processes involved in very different ways, making the washout processes non-trivial. The dominant processes which determine the order of magnitude of the asymmetry are the $\Delta L = 1$ neutrino and sneutrino decays. The $\Delta L = 2$ scatterings are suppressed by additional powers of small Yukawa couplings, and are completely negligible in the low-M_{N_1} regime we consider here [14]. There are also $\Delta L = 1$ two-to-two scatterings involving top quarks and squarks. Those have additional powers of $y_t^2/(4\pi)$ and make a contribution of relative order unity to the washout processes: we neglect them for simplicity. In supersymmetric models there are also processes transforming leptons into scalar leptons and vice versa, e.g., $e + e \leftrightarrow \widetilde{e} + \widetilde{e}$, which we do take into account.

In this approximation, defining $Y_{j\pm} \equiv Y_{\widetilde{N_j^c}} \pm Y_{\widetilde{N_j^c}^\dagger}$ for the scalar neutrinos and their antiparticles, the Boltzmann equations for the ratios of particle densities divided by the entropy density, and for the lepton asymmetries Y_{L_f} and Y_{L_s} in fermions and scalars, respectively, are given by [15]:

$$\frac{\mathrm{d}Y_{N_j}}{\mathrm{d}z} = \frac{-z}{sH(M_{N_1})}\left(\frac{Y_{N_j}}{Y_{N_j}^{\mathrm{eq}}} - 1\right)\gamma_{N_j} ,$$

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[4]We assume here that $N_{1,2}$ are not exactly degenerate in mass, presumably because the (unspecified) underlying symmetry principle for first two generations, such as the S_2 example mentioned earlier, is weakly broken.

[5]This would require the two-generation symmetry to be broken in the Yukawa couplings.

$$\frac{dY_{j_+}}{dz} = \frac{-z}{sH(M_{N_1})}\left(\frac{Y_{j_+}}{Y_{\widetilde{N}_j^c}^{\text{eq}}} - 2\right)\gamma_{\widetilde{N}_j^c} \, ,$$

$$\frac{dY_{j_-}}{dz} = \frac{-z}{sH(M_{N_1})}\left(\frac{Y_{j_-}}{Y_{\widetilde{N}_j^c}^{\text{eq}}} - \frac{1}{2}\frac{Y_{L_s}}{Y_{\widetilde{l}}^{\text{eq}}} + \frac{1}{2}\frac{Y_{L_f}}{Y_l^{\text{eq}}}\right)\gamma_{\widetilde{N}_j^c} \, ,$$

$$\frac{dY_{L_f}}{dz} = \frac{-z}{sH(M_{N_1})}\left\{\sum_j\left[\left(\frac{1}{2}\frac{Y_{L_f}}{Y_l^{\text{eq}}} + \epsilon_j\right)\left(\frac{1}{2}\gamma_{N_j} + \gamma_{\widetilde{N}_j^c}\right) - \frac{1}{2}\epsilon_j\left(\frac{Y_{N_j}}{Y_{N_j}^{\text{eq}}}\gamma_{N_j} + \frac{Y_{j_+}}{Y_{\widetilde{N}_j^c}^{\text{eq}}}\gamma_{\widetilde{N}_j^c}\right)\right.\right.$$
$$\left.\left. + \frac{1}{2}\frac{Y_{j_-}}{Y_{\widetilde{N}_j^c}^{\text{eq}}}\gamma_{\widetilde{N}_j^c}\right] + \left(\frac{Y_{L_f}}{Y_l^{\text{eq}}} - \frac{Y_{L_s}}{Y_{\widetilde{l}}^{\text{eq}}}\right)\gamma_{\text{MSSM}}\right\} \, ,$$

$$\frac{dY_{L_s}}{dz} = \frac{-z}{sH(M_{N_1})}\left\{\sum_j\left[\left(\frac{1}{2}\frac{Y_{L_s}}{Y_{\widetilde{l}}^{\text{eq}}} + \epsilon_j\right)\left(\frac{1}{2}\gamma_{N_j} + \gamma_{\widetilde{N}_j^c}\right) - \frac{1}{2}\epsilon_j\left(\frac{Y_{N_j}}{Y_{N_j}^{\text{eq}}}\gamma_{N_j} + \frac{Y_{j_+}}{Y_{\widetilde{N}_j^c}^{\text{eq}}}\gamma_{\widetilde{N}_j^c}\right)\right.\right.$$
$$\left.\left. - \frac{1}{2}\frac{Y_{j_-}}{Y_{\widetilde{N}_j^c}^{\text{eq}}}\gamma_{\widetilde{N}_j^c}\right] + \left(\frac{Y_{L_s}}{Y_{\widetilde{l}}^{\text{eq}}} - \frac{Y_{L_f}}{Y_l^{\text{eq}}}\right)\gamma_{\text{MSSM}}\right\} \, , \tag{15}$$

where the temperature dependence is via $z = M_{N_1}/T$, Y_x^{eq} denotes the equilibrium density of the particle denoted by the subscript x. The reaction densities for neutrinos and sneutrinos are given by [15]

$$\gamma_{N_j} = 2\,\gamma_{\widetilde{N}_j^c} = \frac{M_{N_1}^4}{4\pi^3}\left(Y_\nu Y_\nu^\dagger\right)_{jj}\frac{a_j\sqrt{a_j}}{z}\text{K}_1(z\sqrt{a_j}) \, , \tag{16}$$

where $a_j = (M_{N_j}/M_{N_1})^2$, and that for the MSSM processes is

$$\gamma_{\text{MSSM}} \approx \frac{M_{N_1}^4\,\alpha^2}{4\pi^3}\frac{1}{z^4}\left[\ln\left(\frac{4M_{N_1}^2}{z^2 m_{\widetilde{\gamma}}^2}\right) - 2\gamma_{\text{E}} - 3\right] \, , \tag{17}$$

where $m_{\widetilde{\gamma}}^2$ is the photino mass which contributes to $e + e \leftrightarrow \widetilde{e} + \widetilde{e}$ in the t channel. The total lepton asymmetry $Y_L = Y_{L_f} + Y_{L_s}$ is then converted by sphalerons into a baryon asymmetry: $Y_B = C\,Y_L$, where $C = -8/15$ in the MSSM.

In our subsequent numerical analysis, we fix the known light neutrino parameters to be $\Delta m_{32}^2 = 3 \times 10^{-3}$ eV^2, $\Delta m_{21}^2 = 5 \times 10^{-5}$ eV^2, $\tan^2\theta_{23} = 1$ and $\tan^2\theta_{12} = 0.4$, corresponding to the LMA solution for the solar neutrino anomaly. The angle θ_{13} is predicted to vanish, together with the neutrino oscillation phase δ, and the connection between leptogenesis and the oscillation phase proposed in [23] does not hold. We assume the normal mass ordering for light neutrinos, $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$, and generate Y_ν according to (13). All the unknown input parameters are generated randomly as follows. The lightest light neutrino mass m_{ν_1} is generated in the range $(10^{-5} - 1)$ eV, the Majorana phases $\phi_{2,3}$ in the range $(0 - 2\pi)$, the

lightest heavy neutrino mass M_{N_1} in the range (10^6 - just above 10^8) GeV, the degeneracy parameter $\Delta \equiv \ln(M_{N_2}/M_{N_1} - 1)$ in the range $(-15 - 1)$, the heaviest neutrino mass M_{N_3} in the range $(2 \times M_{N_2} - 10^{15})$ GeV and the complex parameter r_{12} of the orthogonal matrix R in the range $|r_{12}| = (0 - 10)$ with an arbitrary phase. Each mass parameter is generated with a flat distribution on a logarithmic scale. We solve the Boltzmann equations (15) numerically, assuming initial thermal abundances for neutrinos and sneutrinos, and require that the induced baryon asymmetry be in the range [28]

$$3 \times 10^{-11} \lesssim Y_B \lesssim 9 \times 10^{-11}. \tag{18}$$

We require that the neutrino decay width satisfy $\Gamma_{N_i} < 10^{-5}(M_{N_2} - M_{N_1})$, so that we are far from the resonant region and can trust our perturbative calculation. Subsequently, we study correlations between the leptogenesis parameters and the light and heavy neutrino masses, the $\beta\beta_{\nu 0}$ decay parameter m_{ee}, and the LFV decays of the charged leptons.
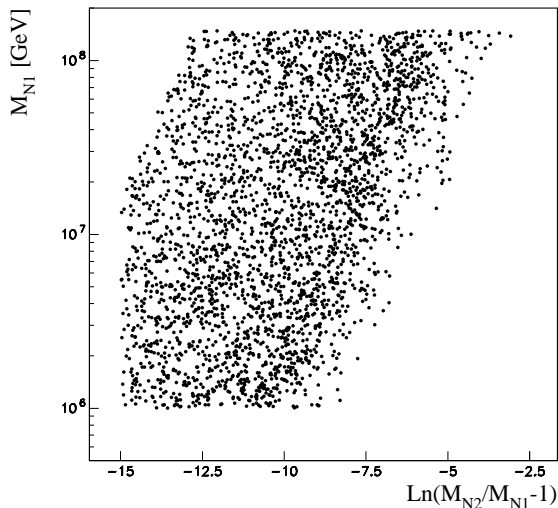

Figure 1: *Scatter plot of the lightest singlet neutrino mass M_{N_1} as a function of the degeneracy parameter $\ln(M_{N_2}/M_{N_1} - 1)$. The baryon asymmetry is required to be in the range (18).*

We find a *weak* correlation of the allowed range of M_{N_1} with $\Delta = \ln(M_{N_2}/M_{N_1} - 1)$, as seen in Fig. 1. We see that successful leptogenesis is possible with light $N_{1,2}$ already if they are degenerate in mass at the level of a few percent. Motivated by the gravitino problem, we limit M_{N_1} to the range 10^6 GeV to about 10^8 GeV in our subsequent numerical examples. We find

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a *stronger* correlation between the leptogenesis parameters $\varepsilon_{1,2}$ and the neutrino degeneracy parameter Δ. In Fig. 2 (a) we plot the CP asymmetries $\epsilon_{1,2}$, denoted by black and green points, respectively, as functions of the degeneracy parameter $\Delta = \ln(M_{N_2}/M_{N_1} - 1)$. There is a Δ-dependent upper bound on $\epsilon_{1,2}$, coming from the mass function in (7). Solutions to the Boltzmann equations (15) are often discussed in the literature in terms of effective mass parameters

$$\tilde{m}_i \equiv \left(Y_\nu Y_\nu^\dagger\right)_{ii} \frac{v^2 \sin^2 \beta}{M_{N_i}}. \tag{19}$$

If leptogenesis originates from N_1 decays, as is the case for hierarchical heavy neutrinos, fixing Y_B implies almost a one-to-one correspondence between ϵ_1 and $\tilde{m}_1$ in the low M_{N_1} region [14, 25]. However, the solutions to (15) are non-trivial, as seen in Fig. 2 (b), where we plot $\epsilon_{1,2}$ versus the parameters $\tilde{m}_{1,2}$, respectively. A direct correlation is observed only for high values of $\tilde{m}_1 \sim \tilde{m}_2 \gtrsim 5 \times 10^{-2}$ eV, where both neutrinos $N_{1,2}$ contribute to Y_B in a similar way. This region corresponds to large and quasi-degenerate light neutrino masses. Because of strong washout effects, $\epsilon_{1,2}$ must be large and, correspondingly, $M_{N_2}/M_{N_1} - 1$ small in this region. For lower values of $\tilde{m}_1$ the non-linear shape of Fig. 2 (b) indicates the non-trivial washout effects of N_2.

To study implications of our neutrino mixing scenario and leptogenesis for low-energy observables, we plot in Fig. 3 (a) the lightest neutrino mass m_{ν_1} versus the $\beta\beta_{0\nu}$ parameter m_{ee}. There is an upper bound on m_{ν_1} coming from the requirement of successful leptogenesis. This is an artifact of our lower bound on the degeneracy parameter Δ and is not physical. If we allow smaller Δ, $\epsilon_{1,2}$ are enhanced and the neutrino mass m_{ν_1} can be higher. At high values of m_{ν_1}, there can be cancellations in m_{ee} (12), because the phases $\phi_{2,3}$ are free parameters not correlated with leptogenesis. However, for small m_{ν_1} there is the definite prediction $m_{ee} = \sqrt{\Delta m_{sol}^2} \sin^2 \theta_{12}$, which is below the sensitivity of the currently proposed $\beta\beta_{0\nu}$ decay experiments [29] [6].

To study LFV in our scenario, we fix the soft supersymmetry-breaking parameters at the GUT scale to coincide with one of the post-LEP benchmark points [30]: $m_{1/2} = 300$ GeV, $m_0 = 100$ GeV, $A_0 = 0$, $\tan\beta = 10$ and $sign(\mu) = +1$. We plot in Fig. 3 (b) the renormalization-induced branching ratio of the decay $\mu \to e\gamma$ versus the branching ratio of $\tau \to \mu\gamma$. Because we require M_{N_1} to be relatively low, the Yukawa couplings Y_ν related to first two families are also very small, suppressing $Br(\mu \to e\gamma)$ below the presently observable level. Since the branching ratios scale as $\tan^2 \beta$, reaching the level 10^{-14} or 10^{-15} planned to

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[6]If θ_{13} is not exactly vanishing, there is an additional contribution to (12) and the prediction for m_{ee} becomes less certain [11].
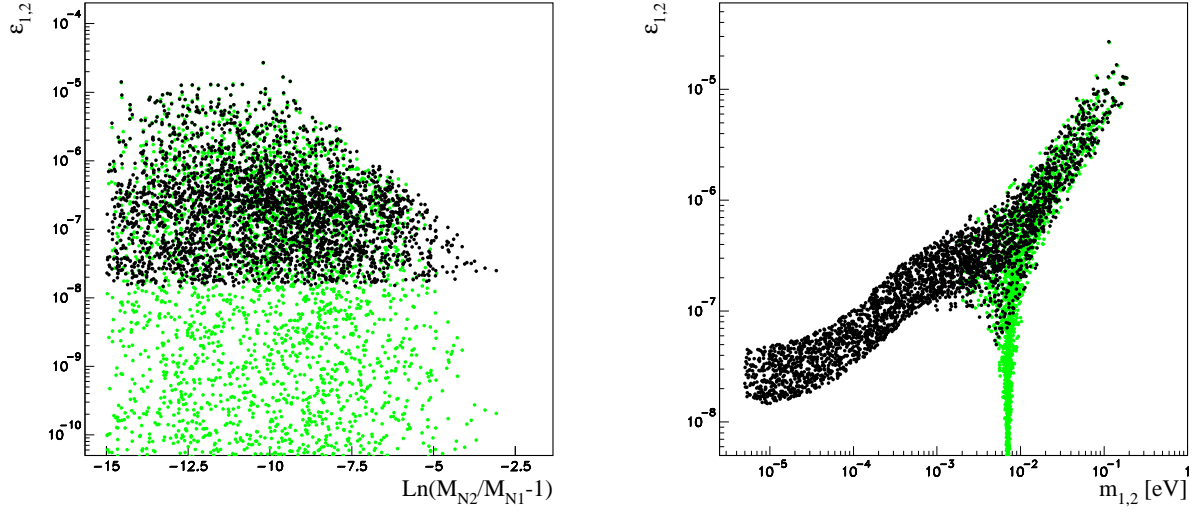
Figure 2: *Scatter plots of the CP-violating asymmetries $\epsilon_{1,2}$ denoted by black and green (grey), respectively, as functions of the degeneracy parameter $\ln(M_{N_2}/M_{N_1}-1)$ and the effective mass parameters $\tilde{m}_{1,2}$. The baryon asymmetry is required to be in the range (18).*
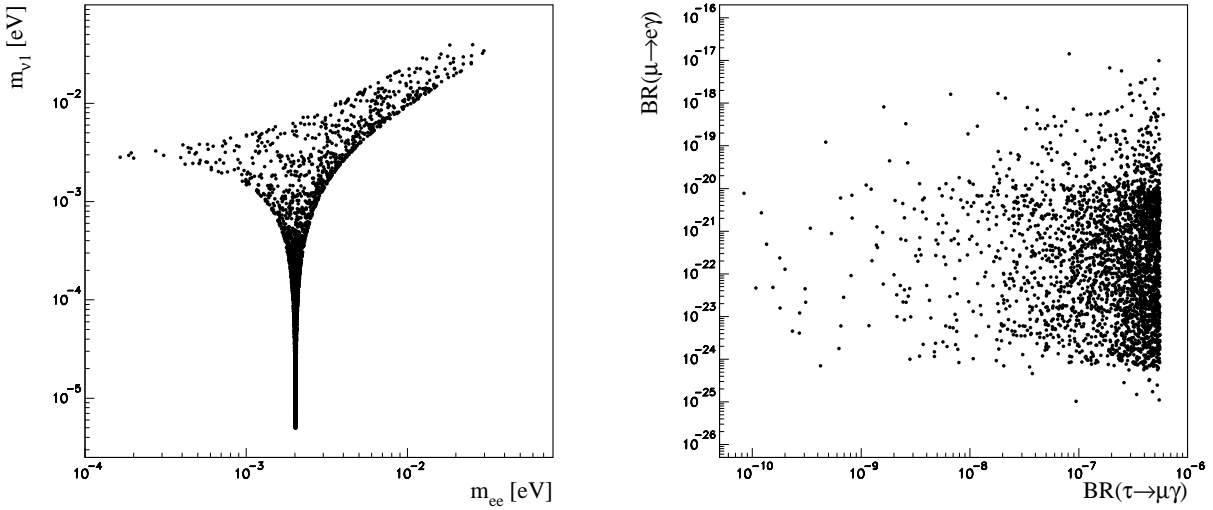

Figure 3: *Scatter plots of the lightest neutrino mass m_{ν_1} versus the $\beta\beta_{0\nu}$-decay parameter m_{ee}, and $Br(\mu \to e\gamma)$ versus $Br(\tau \to \mu\gamma)$. The baryon asymmetry is again required to be in the range (18).*

be achieved at PSI [31] and the neutrino factory experiments [32], respectively, is unlikely for $M_{N_1} \sim M_{N_2} \lesssim 10^8$ GeV. This indicates that the solution to the gravitino problem and the non-observation of $\mu \to e\gamma$ in present experiments may be related. However, $Br(\tau \to \mu\gamma)$ is not necessarily suppressed, because N_3 can be heavy and the related Yukawa couplings large. Therefore $\tau \to \mu\gamma$ may be observable in B-factory or LHC experiments, which should achieve a sensitivity $Br(\tau \to \mu\gamma) \sim 10^{-8}$ to 10^{-9} [33].

In conclusion: we have studied a scenario in which the large leptonic mixing angles θ_{12} and θ_{23} originate from 2×2 mixings in the neutrino and the charged lepton Yukawa matrices, respectively. Using a convenient phenomenological parametrization and scanning over the all free seesaw parameters, and motivated by the gravitino problem, we have focussed on lowering the thermal leptogenesis scale by postulating moderate degeneracy of two lightest heavy neutrinos. This scenario turns out to be quite predictive for $\beta\beta_{0\nu}$ decay and for the LFV decays of charged leptons, since it involves less free parameters than the general seesaw model. We predict suppressed $\mu \to e\gamma$, $m_{ee} = \sqrt{\Delta m_{sol}^2} \sin^2\theta_{12}$ for hierarchical neutrinos and, in general, an observable rate for $\tau \to \mu\gamma$.

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