Big Bang Nucleosynthesis and Cosmological Constraints on Neutrino Oscillation Parameters

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Abstract

We present a review of cosmological nucleosynthesis (CN) with neutrino oscillations, discussing the different effects of oscillations on CN, namely: increase of the effective degrees of freedom during CN, spectrum distortion of the oscillating neutrinos, neutrino number density depletion, and growth of neutrino–antineutrino asymmetry due to active–sterile oscillations. We discuss the importance of these effects for the primordial yield of helium-4.

Primordially produced ^{4}He value is obtained in a selfconsistent study of the nucleons and the oscillating neutrinos. The effects of spectrum distortion, depletion and neutrino–antineutrino asymmetry growth on helium-4 production are explicitly calculated.

An update of the cosmological constraints on active–sterile neutrino oscillations parameters is presented, giving the values: $\delta m^2 (\sin^2 2\vartheta)^4 \leq 1.5 \times 10^{-9}$ eV^2 for $\delta m^2 > 0$, and $|\delta m^2| < 8.2 \times 10^{-10}$ eV^2 at large mixing angles for $\delta m^2 < 0$. According to these constraints, besides the active–sterile LMA solution, also the active–sterile LOW solution to the solar neutrino problem is almost totally excluded.

1 Introduction

Cosmological nucleosynthesis is traditionally used as a probe of the conditions of the early Universe at the nucleosynthesis epoch ($T \sim$ MeV). Concerning neutrino physics, the requirement for a concordance between the theoretically predicted and the extracted from observations primordial abundances of light elements, constrains neutrino characteristics: mass, number of light species, degeneracy, decay width and mass of eventual heavy decaying neutrinos, neutrino oscillation parameters, possible new interactions, etc.

On the other hand, the topic of neutrino oscillations is with us since Pontecorvo's hypothesis for these [1] i.e. more than 40 years. Neutrino oscillations imply non-zero neutrino masses and mixings, therefore presenting an indication of physics beyond the electroweak standard model. Recently the positive indications of oscillations obtained from the greatest neutrino experiments (SuperKamiokande, SNO, Soudan 2, LSND, etc.) [1] turned the subject of neutrino oscillations into one of the hottest points of astrophysics and neutrino physics. Hence, it looks like appropriate to provide an updated review of the influence of neutrino oscillations on CN and present the most recent cosmological constraints on neutrino oscillations parameters.

In case neutrino oscillations are present in the primordial plasma of the early Universe, they may lead to changes in CN, depending on the oscillation channels and the way they proceed. Namely, the oscillations effect depends on the kind of oscillations (they can be resonant or nonresonant) and also differs for the equilibrium and nonequilibrium cases. Oscillations may influence nucleosynthesis through their effects on: neutrino and antineutrino number densities, spectrum, neutrino-antineutrino asymmetry and the number of neutrino species.

In the next section we will provide a review of CN with neutrino oscillations, discussing mainly oscillations influence on CN. In the last section we will present an update of the cosmological constraints on $\nu_e \leftrightarrow \nu_s$ neutrino oscillation parameters and discuss how they concern the solutions to the solar neutrino problem.

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[1] All recent data of *the solar neutrino* experiments namely Gallium, Chlorine, SuperKamiokande, SNO, sensitive to different neutrino energy, point to a well pronounced deficit of solar neutrinos in contrast with the theoretical predictions of the Standard Solar Model fluxes.

The measured ratio of the muon over electron neutrino flux by Kamiokande, SuperKamiokande, IMB and Soudan 2 is considerably lower than the predicted one for *the atmospheric neutrinos*, resultant from the primary cosmic rays collisions with the nuclei in the upper atmosphere. Moreover, a zenith-angle-dependent deficit of muon neutrinos was observed.

The LSND experiment has observed electron antineutrino appearance in a flux of muon antineutrinos and electron neutrinos in a flux of muon neutrinos.

These three neutrino anomalies are preferably explained in terms of neutrino oscillations, which are able to provide a natural mechanism for energy dependent and neutrino type dependent suppression of neutrino fluxes. There exist several solutions to the solar neutrino problem: Small Mixing Angle (SMA), Large Mixing Angle (LMA), LOW and Vacuum Oscillations (VO) solutions, which imply neutrino mass differences in the range: $10^{-10} - 10^{-4}$ eV^2. The atmospheric neutrino anomaly can be resolved by oscillations with mass differences $10^{-3} - 10^{-2}$ eV^2. While for the LSND neutrino experiment, much bigger mass difference, of the order of eV is necessary.

2 Historical overview of CN with neutrino oscillations

2.1 Standard Big Bang Nucleosynthesis

For a precise analysis of the oscillations effect on CN, the element that is used traditionally is helium-4, as far as the most reliable and abundant data now available are for that element. According to the standard Big Bang Nucleosynthesis (SBBN) theory ^{4}He is a result of a complex network of nuclear reactions, which proceed after the freezing of the neutron-to-proton ratio n/p.

The abundance of the primordially produced mass fraction of helium-4 $Y_p \sim 2(n/p)_f/(1 + (n/p)_f)$ depends mostly on two compelling processes, determining the neutron-to-proton freezing ratio – $(n/p)_f$, namely the Universe's cooling rate, $H(t) \sim \sqrt{g_{eff}}\, T^2$ and the interaction rates of the weak processes, interchanging neutrons and protons: $\Gamma_w \sim G_F^2(g_V^2 + 3g_A^2)E_\nu^2 T^3$. Hence, the produced helium is a strong function of the effective number of relativistic degrees of freedom at the CN epoch, g_{eff}, and the neutron mean lifetime τ_n, which parametrizes ^{2} the weak interactions strength. Besides, primordially produced ^{4}He is a logarithmic function of the baryon-to-photon ratio η, due to the nuclear reactions dependence on nucleon densities, i.e. $Y_p(g_{eff}, \tau_n, \eta)$. Deuterium measurements in pristine environments towards low metallicity quasar absorption systems at very high $z \sim 3$ [2] provide us with the most precision determination of the baryon density, giving the value: $\eta = 5.6 \pm 0.5 \times 10^{-10}$. Recently, the baryon density was also determined from observations of the anisotropy of the cosmic microwave background (CMB) by DASI, BOOMERANG and MAXIMA experiments [3]. The CMB value is in agreement with the one found from deuterium measurements and SBBN.

SBBN assumes three neutrino flavours, zero lepton asymmetry and equilibrium neutrino number densities and spectrum:

$$n_\nu^{eq} = \exp(-E/T)/[1 + \exp(-E/T)]$$

The primordial ^{4}He abundance Y_p, predicted from SBBN, is calculated with great precision. In recent years, corrections to SBBN code accounting for different physical effects reduced the theoretical uncertainty to less than 0.1% ($|\delta Y_p| < 0.0002$) within a wide range of η [4]. The uncertainty of the observational Y_p is few percent. The predicted helium-4 value is in accordance with the contemporary helium values, inferred from observational data: 0.238–0.245 (the systematic errors are supposed to be around 0.007) [5], and is consistent with other light elements abundances.

Given this accuracy, it can be used as a probe of the eventual new neutrino physics – the neutrino oscillations.

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^{2}Where $\tau_n^{-1} \sim G_F^2(g_V^2 + 3g_A^2)$.

2.2   CN with neutrino oscillations

CN with neutrino oscillations was studied in numerous publications [6]–[38]. The basic idea of oscillations is that neutrinos are not degenerate in mass and there is a mixing, so that the mass eigen-states ν_i are distinct from the flavour eigen-states ν_f and are unitary combinations of the latter:

$$\nu_i = U_{if}\, \nu_f \qquad (f = e, \mu, \tau).$$

Then in the simple two-neutrino oscillation case

$$\nu_1 = c\nu_f + s\nu_{f'},$$
$$\nu_2 = -s\nu_f + c\nu_{f'},$$

where $c = \cos(\vartheta)$, $s = \sin(\vartheta)$, $\vartheta \neq 0$ is the mixing angle, ν_1 and ν_2 are the mass eigen-states with masses correspondingly m_1 and m_2 and $\delta m^2 \neq 0$.

The probability to find at a distance l a given neutrino type f' in an initially homogeneous neutrino beam of another neutrino type f is:

$$P_{ff'} = \sin^2 2\vartheta \sin^2(1.267\ \delta m^2 l / E),$$

where δm^2 is the neutrino mass difference in eV^2, E is the neutrino energy in MeV and l is the distance in meters.

2.2.1   Vacuum oscillations

The oscillations effect on CN was historically first considered for vacuum *oscillations between different neutrino flavours* [6, 7]. It was shown that since there is a slight deviation from equilibrium in that case (temperatures of different flavour neutrinos are nearly the same), flavour oscillations have a negligible effect on neutrino number densities and on nucleosynthesis [6]. The possibility of generation of neutrino-antineutrino asymmetry in the electron sector due to CP-violating oscillations and its effect on CN was discussed in ref. [7].

Active–sterile neutrino [3] *oscillations* effect on CN may be noticeably stronger because they can

(a) increase the effective number of light degrees of freedom during nucleosynthesis [6, 8, 9];

(b) lead to a strong distortion of the neutrino spectrum [10];

(c) lead to a considerable decrease of the total number density of active neutrinos [10];

Active-sterile oscillations may keep sterile neutrinos in thermal equilibrium or bring them into equilibrium in case they have already decoupled. CN allows not more than one additional neutrino type, therefore forbidding efficient production of sterile neutrinos due to oscillations ($\Gamma_R < H$). First constraints [6] on oscillation parameters were obtained

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[3]SU_2 singlet neutrino.

from the requirement that the observed primordial values allow less than one additional light neutrino type. It was shown that the simultaneous presence of Dirac and Majorana neutrino masses would contradict the observed helium-4 abundance if $\delta m^2 > 10^{-6}$ eV^2.

In ref. [8] CN constraints were estimated for fast oscillations ($\Gamma_{osc} > H$) from the condition that sterile neutrinos should not be abundantly produced due to oscillations before nucleosynthesis epoch ($\Gamma_R \sim \Gamma_w \sin^2 2\vartheta < H$ at 1 MeV): $(\delta m^2)^2 \sin^2 2\vartheta \leq 1.6 \times 10^{-17} eV^4$.

In ref. [9] a condition for an efficient production of sterile neutrinos at about 1 MeV, accounting for the rescattering of neutrinos from the fermions of the plasma, was calculated and constraints on neutrino oscillation parameters were also obtained. These constraints [8, 9] as will be discussed in more detail in the next subsection, are considerably relaxed, when a proper account of the dispersion effects is provided.

In ref. [10] the nonequilibrium case of active–sterile oscillations, effective after the neutrino decoupling, i.e. the oscillation rate exceeding the expansion rate $\Gamma_{osc} \sim \delta m^2/(4E) > H$, while neutrino weak rates less than expansion rate $\Gamma_w < H$, was discussed.

The effect of such active-sterile neutrino oscillations is due mainly to the fact that sterile neutrinos may have decoupled much earlier than the active ones [4] and at the nucleosynthesis epoch their number densities were negligible in comparison with the active neutrinos: $N_s \ll N_f$, as far as $T_s < T_f$ and $N \sim T^3$.

The accurate kinetic approach of ref. [6] to a description of the oscillating neutrinos in terms of *neutrino density matrix in momentum space*, was used in ref. [10] to calculate analytically the evolution of the number density of electron neutrinos:

$$n_{\nu_e} = \rho_{LL} = \left\{ 1 - 2c^2 s^2 + 2c^2 s^2 \cos[BT/E(T^{-3} - T_0^{-3})] \right\} \times n_{\nu_e}^{eq},$$

where $B = 0.1 M_{Pl} \delta m^2 / \sqrt{g_{eff}}$ and $T_0 = 3$ MeV.

It was explicitly shown, that for large interval of oscillation parameters of the discussed model, oscillations may cause considerable spectrum distortion and/or depletion of the electron neutrino.

At great δm^2, $\cos[BT/E(T^{-3} - T_0^{-3})]$ is frequently oscillating and can be averaged. Then $\rho_{LL} = (1 - 2c^2 s^2) n_{\nu_e}^{eq}$, and active neutrinos number densities are depleted by oscillations. However, for smaller δm^2, the energy distribution of the active neutrinos may be also considerably changed.

Both the electron neutrino depletion and the decrease of the electron neutrino energy due to oscillations into less energetic sterile neutrinos decrease the weak rates $\Gamma_w \sim E_\nu^2 N_\nu$, leading to higher freezing temperature of the nucleons. The total effect is an overproduction of He-4.

Helium production was numerically calculated for the full range of parameters of this oscillation model. It was shown that, the effect of depletion and spectrum distortion due to oscillations on He-4 may be much stronger than the effect due to excitation of an additional degree of freedom. Assuming that allowed overproduction should be less than

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[4]Therefore, they were not heated by the following after their decoupling processes leading to entropy increase.

5%, the following constraints on the oscillation parameters were obtained: $\delta m^2 < 10^{-9}$ at $\vartheta > \pi/15$ The overproduction of helium due to sterile neutrino species excited by oscillations was calculated.

2.2.2 Matter oscillations

In the following years, it was realized that for a large interval of masses and mixing angles the thermal background in the pre-nucleosynthesis epoch may strongly affect the propagation of neutrinos and, has to be taken into account. Differences in the interactions with the particles from the plasma lead to different average potentials for different neutrino types [11]. In the adiabatic case the effect of the medium can be formally hidden in the oscillation parameters, by introducing matter oscillation parameters that are expressed through the vacuum ones and through the characteristics of the medium. The matter mixing angle is then [11, 39]

$$\sin^2 \vartheta_m = \sin^2 \vartheta / [\sin^2 \vartheta + (Q \mp L - \cos 2\vartheta)^2],$$

where $Q = -bE^2T^4/(\delta m^2 M_W^2)$, $L = -aET^3L^\alpha/(\delta m^2)$, L^α is expressed through the fermion asymmetries of the plasma, a and b are positive constants different for the different neutrino types, $-L$ corresponds to the neutrino and $+L$ to the antineutrino case.

It was realized that although most often the medium suppresses oscillations (by decreasing their amplitude), there also exists a possibility of enhanced oscillation transfer in case a resonant condition between the parameters of the medium and the oscillation parameters holds:

$$Q \mp L = \cos 2\vartheta.$$

Then the mixing in matter becomes maximal, independently of the value of the vacuum mixing angle. Besides, as can be easily judged from the resonant condition, at high temperatures, when $|Q| > |L|$, $\delta m^2 > 0$ corresponds to a nonresonant case, while $\delta m^2 < 0$ corresponds to a resonant case, and the resonance holds in both neutrino and antineutrino sectors. At low temperatures, when $|Q| < |L|$, the resonance is possible either for neutrinos in the case $\delta m^2 > 0$ or for antineutrinos in the case $\delta m^2 < 0$.

Matter oscillations of ordinary flavour neutrinos, occurring before the freeze-out of n/p - ratio were considered first [15]. Two different effects of oscillations were taken into account: the generation of $\nu_e - \bar{\nu}_e$ asymmetry in $\nu_\tau \leftrightarrow \nu_e$ oscillations resonant conversion and the change in the neutrino freeze-out temperature. The effect on primordial production of He-4 was estimated to be very low: $\delta Y_p < 1.3 \times 10^{-3}$.

The effect of *matter active–sterile oscillations* on CN, taking into account the essential processes of neutrino forward scattering of the background particles, was considered first in refs. [16],[17]-[20],[24]. Active–sterile oscillations that are efficient before the nucleosynthesis epoch (corresponding to high mass differences) were discussed there. The production of sterile neutrino states due to oscillations and interactions of the active neutrino with the medium was calculated.

It was shown that matter oscillations, proceeding before the active neutrinos decoupling, are capable of exciting additional degrees of freedom into the plasma. The increase

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in the expansion rate $H \sim \sqrt{g_{eff}}$ and the corresponding overproduction of helium were calculated.

The exclusion regions for the neutrino-mixing parameters were obtained from the requirement that the number of neutrino species is less than a certain value (usually $N_\nu < 3.4$ in the pioneer publications), up to which there is an agreement between the values for light element abundances obtained from theoretical predictions and those extracted from observations. The excluded region of oscillation parameters values was found much smaller than in the case not accounting for the neutrino coherent interactions.

In these works the kinetic evolution of the neutrino ensembles was studied in terms of particle densities, and it was assumed that oscillations slightly shift neutrino density matrix from its diagonal form. First estimates of an eventual depletion of electron neutrinos, due to $\nu_e \leftrightarrow \nu_s$ oscillations, and its corresponding influence on helium-4 were made in [17, 20]. Numerical calculations of the discussed oscillations effects on helium-4 production were made in [24].

The idea that active-sterile oscillations will produce considerable $\nu_e - \bar{\nu}_e$ asymmetry during resonant transfer, which may influence CN was discussed in ref. [16].

2.2.3   CN constraints on active-sterile oscillations

The pioneer works discussing $\nu_e \leftrightarrow \nu_s$ at $\delta m^2 > 0$ excluded the Large Mixing Angle solution (LMA) of the solar neutrino problem.

In the $\nu_\mu \leftrightarrow \nu_s$ oscillations case, the solution to the atmospheric anomaly was found to lie in the cosmologically excluded region. This can be regarded as an indication that the atmospheric neutrino anomaly should be solved by oscillations in the $\nu_\mu \leftrightarrow \nu_\tau$ sector (which is also the experimentally preferred recent solution) [5].

Recently [36] the constraints on active–sterile neutrino oscillations were discussed in connection with the controversy in literature concerning the sterile neutrino production rate: in some works annihilation rate was used when calculating it, while in others the total reaction rate was employed. The problem is still under discussion.

In ref. [25] (see also ref. [26]) a rough estimation of the spectrum distortion by shifting the effective temperature of the neutrino and considering its spectrum equilibrium was provided. However, as was shown in refs. [29, 30, 31, 33] this way the real distortion of the neutrino spectrum cannot be described. Besides, the exclusion plots of refs. [25, 26] for the resonant case are not compatible with the ones for the nonresonant case, they do not coincide at maximal mixing, as they should. Hence, it is preferable to use the results of ref. [24] for the cosmological constraints in the $\nu_\mu \leftrightarrow \nu_s$ and $\nu_\tau \leftrightarrow \nu_s$ as more reliable.

It should be kept in mind also that the constraints on $\nu_{\mu,\tau} \leftrightarrow \nu_s$ have not been updated: they do not take into account the spectrum distortion of the oscillating neutrinos and the growth of the neutrino–antineutrino asymmetry.

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[5]With this in mind, and also the fact that the asymmetry growth due to resonant oscillations still needs some clarification and precision calculations, we will not discuss the numerous works aiming to save the active–sterile solution, by weakening the cosmological constraints due to the large asymmetry production in active–sterile oscillations in the muon or the tau-neutrino sector.

In the resonant case we expect the bounds to become less restrictive in the small mixing angles area, when the estimation of the asymmetry growth due to oscillations is made, because oscillations generated asymmetry at small mixing angles leads to a suppression of oscillations and consequently to a relaxation of the bound on oscillation parameters at small mixings [34]. In both resonant and nonresonant cases we expect less restrictive bounds at large mixings due to spectrum distortion effect.

2.3 Update of oscillations effects on CN

In the last years the oscillations effects on CN were updated [29]–[33],[37], by providing a precise numerical account for the neutrino spectrum distortion, depletion and growth of neutrino-antineutrino asymmetry due to neutrino oscillations.

2.3.1 Spectral distortion.

In refs. [29]–[33],[37] exact kinetic equations for the neutrino density matrix in *momentum* space, accounting *simultaneously* for Universe expansion, neutrino oscillations and neutrino forward scattering, were numerically solved for oscillations with small mass differences. The equations for the neutrino density matrix were solved *selfconsistently* with the kinetic equations of the nucleons during the n/p - freezing (see detail description in the next section), which allowed to provide a precise analysis of neutrino depletion and spectrum distortion effect on CN due to oscillations. The spectrum distortion effect was shown to be considerable [29]. In Fig. 1 the net effect of spectrum distortion is given by the difference between the short-dashed curve from ref. [24] and the long-dashed curve from ref. [33]. In the nonequilibrium case of electron-sterile oscillations, it cannot be estimated simply by shifting the effective temperature or introducing chemical potentials.

On the other hand the precise numerical analysis of the spectrum distortion is technically very difficult. It has been provided only for oscillations with small mass differences [29, 30, 33]. For the adequate description of the spectrum distortion in that case 1000 bins in the nonresonant case and up to 5000 bins for the resonant case were needed. Hence, thousands times larger system of coupled integro-differential equations had to be accurately solved, in contrast to the case of momentum averaged calculations when a set of 8 ordinary differential equations are solved. The computational time was strongly increased.

Recently a successful analytical description of the spectrum distortion of neutrino spectrum due to oscillations was proposed in ref. [45] for small mixing angles and large mass differences. It may be very useful in precision calculations of the oscillations effect on CN at large mass differences, which have not been done.

2.3.2 Neutrino-antineutrino asymmetry growth

The idea of generating neutrino-antineutrino asymmetry due to resonant transfer of neutrinos in active-sterile neutrino oscillations was first proposed in ref. [16]. Alas, it was not developed further by the authors and remained unnoticed.
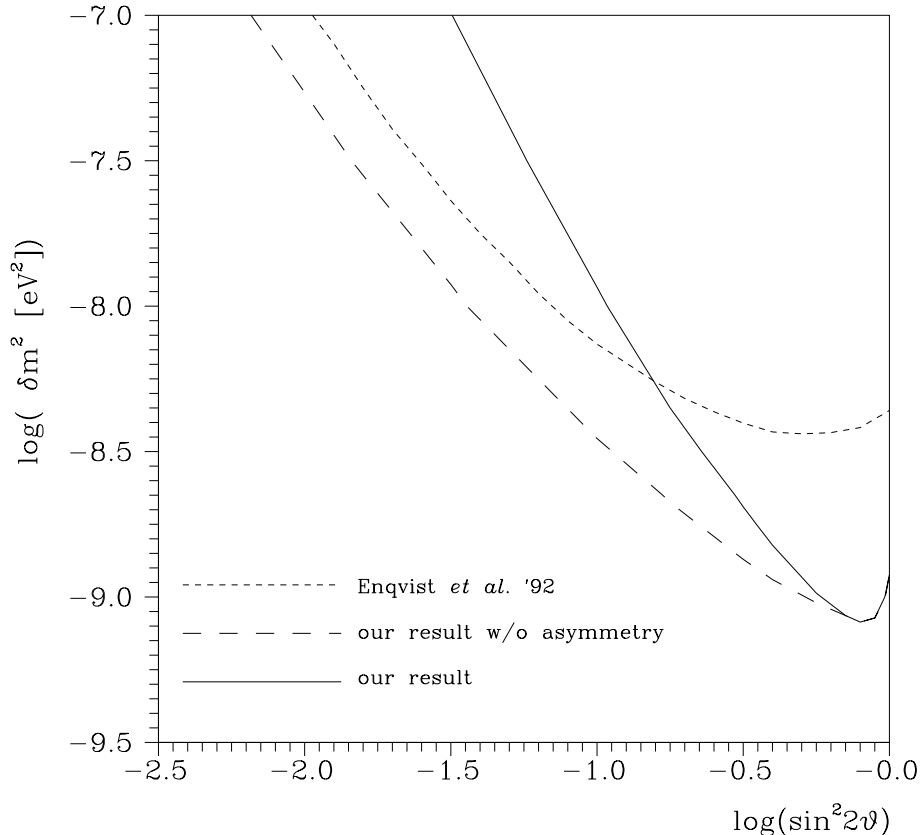
Figure 1: Comparison between the results on primordial helium-4 production obtained in our works [32, 33] and in previous work [24]. The long-dashed curve shows our results when the asymmetry effect is neglected. The short-dashed curve shows the results of Enqvist et al., where both asymmetry and spectrum distortion effects were neglected. The difference between the two curves shows the effect of the proper account of the spectrum distortion of the neutrino. On the other hand, the difference between our curves, the solid and the dashed one, presents the net asymmetry effect.

Estimations of the effect of *asymmetry, generated in flavor oscillations*, on CN of helium-4 were provided in ref. [15]. The effect was shown to be negligible.

Oscillations generated asymmetry was discussed in more detail for *active-sterile oscillations* with large mass differences in refs. [17]-[20], where it was estimated to have negligible effect on CN, and in ref. [27], where asymmetry growth was found possible in case $\delta m^2 > 10^{-5}$ eV^2. In the numerical study, exploring oscillations with small mass differences $\delta m^2 < 10^{-7}$ eV^2 [29], a growth of asymmetry was registered.

The revealment of the asymmetry growth at small mass differences became possible thanks to the exact kinetic approach, namely work with selfconsistent kinetic equations for neutrinos in momentum space. This approach allows to take into account the neutrino spectrum distortion and describe the asymmetry behaviour at each momentum.

In ref. [32] the evolution of the asymmetry was studied selfconsistently with the neu-

trino and nucleon evolution, which allowed to take into account the effect of the asymmetry on CN during *the full evolution of the asymmetry.* It was shown that besides the well-known kinetic effect of the asymmetry on CN, after it has grown up to high values of the order of ± 0.01, very small asymmetries $L < 0.01$ exert a considerable *indirect effect on CN through oscillations.* This is due to the fact that even very small asymmetries change the medium-induced neutrino potential energy and may have a considerable back reaction on the oscillating neutrinos. It has been shown [32, 37], that the results obtained when the indirect asymmetry effect was taken into account differ by many orders of magnitude from results neglecting small asymmetries.

For the case of small mass differences exact kinetic calculations of the asymmetry effect were provided in refs. [29]–[34]. The asymmetry growth up to 5 orders of magnitude was registered in these studies. Hence, these lepton asymmetries experienced *indirect* effect on CN through changing neutrino number densities and the oscillation pattern, which on its turn effected the kinetics of nucleons during n/p - freezing.

The net *indirect* asymmetry effect on CN is illustrated in Fig. 1 by the difference between the solid and the long-dashed curves. The asymmetry growth reduced neutrino oscillations effect and lead up to 10% underproduction of helium in respect to the model of CN with oscillations but neglecting asymmetry. Hence dynamically produced asymmetry leads to a noticeable weakening of the cosmological constraints for small mixing angles [29, 32, 33].

However, the precise numerical description of the asymmetry growth requires more than 5 000 bins for the spectrum distortion, which was found essential for the correct calculation of the asymmetry evolution [33]. This increases enormously the calculational time in the resonant case. Besides, the neutrino evolution equations at resonance have high stiffness and implicit methods should be used to solve the equations numerically. For 5 000 bins of the spectrum a system of 30 000 integro-differential equations describing the neutrino density evolution should be solved simultaneously.

Hopefully, the asymmetry effect on CN for large mass differences may be successfully analytically estimated using the analytical approximations, accounting for the spectrum distortion of neutrino, like the one proposed in [45] for small mixing angles and large mass differences. Such approximations maybe very useful for analyzing oscillations effect on CN, because they simplify the equations and reduce the computational time.

2.3.3   Relic lepton asymmetries.

The case of different *initial asymmetries*, namely $10^{-10} < L < 10^{-4}$, in models of CN with *nonresonant* oscillations, effective after electron neutrino freeze-out, was precisely studied in [30, 31, 34, 37]. It was found that depending on the concrete values of the relic asymmetry, it may suppress, enhance or not influence oscillations, and hence lead to an under-, overproduction of helium or to not changing its abundance at all.

The possibility of lepton asymmetry to enhance oscillations, besides its well-known ability to suppress them, was revealed thanks to the study of neutrino evolution at each momentum:

For the parameters of the model the local term L dominates over the other terms for the mean neutrino momenta, and hence the resonant condition is not fulfilled: $|L| \gg |Q|$, $|L| \gg \cos 2\vartheta$.

However, for neutrinos with a given momentum $p < \bar{p}$ it is fulfilled and these neutrinos suffer a resonant transfer leading to the decrease of their number densities and a decrease of the asymmetry (since the resonance condition is fulfilled only in one sector, either neutrino or neutrino one, the particle densities in the other sector do not change). Then, due to the L decrease, resonant transfer becomes possible for neutrinos with higher momentum, and so on till L changes sign and the running resonance wave similarly passes through the antineutrino ensemble, leading to a rapid increase of L again till the next change of sign of L, etc. i.e. the observed enhancement is "spectrum resonance" effect.

The total effect of the process is enhanced resonance transfer both in neutrino and antineutrino sectors, leading to greater overproduction of helium-4. Oscillation constraints for the case of CN with oscillations and with initial L were obtained [31].

A similar investigation for the resonant case will be more complicated due to technical problems because it will deserve much greater number of bins for the spectrum distortion description because of the asymmetry growth, and hence, longer calculational time. However, this investigation is interesting, as far as such small values of the initial relic asymmetry are not excluded neither from observations nor from some profound theoretical principle.

In conclusion we would like to stress that even a very small asymmetry, either initially present or dynamically generated, thanks to its indirect effect through neutrino oscillations is capable to influence strongly CN. Hence it should be accounted for as precisely as possible.

2.4   Summary of neutrino oscillations effect on CN

In case neutrino oscillations between active and sterile neutrinos proceed in the primordial plasma during the CN epoch, they can effect CN in the following ways:

(a) In case active neutrinos have not decoupled, their oscillations to sterile ones may bring additional degrees of freedom into the primordial heat bath. This will lead to an increase of the Universe expansion rate H(t) and to an earlier freezing of the n/p - ratio, at times when neutrons were more abundant in comparison with the SBBN. Hence, this effect leads to an overproduction of helium-4. This effect was historically the first discussed [8] and on its basis first constraints on the oscillation parameters $\delta m^2, \sin^2(2\vartheta)$ were estimated [9, 17, 18, 19, 20, 24].

(b) Oscillations considerably influence CN by distorting the neutrino and antineutrino spectrum. As far as the oscillation rate is energy dependent $\Gamma_{osc} \sim \delta m^2/E$ the low energy neutrinos start to oscillate first, and later the oscillations become noticeable for the more energetic neutrinos. Due to that, the spectrum of the neutrinos (antineutrinos) may become strongly distorted, especially in the case of oscillations into less abundant sterile neutrinos. This effect was shown considerable both in the vacuum oscillations case [10] and matter oscillations case [29]. Spectrum distortion of the electron neutrinos due to

oscillations leads to overproduction of helium-4, which may be several times larger than He-4 overproduction due to an additional neutrino type.

(c) Oscillations can lead to a substantial depletion of the number densities (N_ν and $N_{\bar{\nu}}$) of active neutrinos, in case they proceed between active and less abundant sterile neutrino states. This slows down the weak rates, $\Gamma_w \sim N_\nu E_\nu^2$, and again leads to an earlier n/p - freezing and a corresponding increase of the helium-4 yield.

This effect was analytically calculated and found to be important for CN first in the vacuum oscillation case [10]. For matter oscillations with great mass differences it was first estimated in ref. [17, 20], where it was described in terms of an effective chemical potential of neutrinos, generated due to active-sterile oscillations. Electron neutrino depletion was numerically calculated, without the account of spectrum distortion and asymmetry growth in ref. [24]. Later these effects were taken into account in ref. [29] for the case of small mass differences ($\delta m^2 \le 10^{-7}$ eV^2).

For oscillations with small mass differences the latter two effects can lead up to 32% overproduction of helium-4 [38].

(d) Oscillations produce neutrino-antineutrino asymmetry, which on its turn influences the evolution of the neutrino and antineutrino ensembles and the oscillation pattern. In the nonresonant case the neutrino-mixing produced asymmetry was shown to have a negligible role on CN. However, in the resonant oscillation case the asymmetry effect on CN was shown to be considerable [29, 33]. For the case of small mass differences it was proven that even very small asymmetries $L \ll 0.01$ considerably influence CN through oscillations, and therefore asymmetry effect on CN should be accounted for during asymmetry's full evolution.

In general, dynamically produced asymmetry at small mixing angles suppresses oscillations, which leads to less overproduction of helium-4 in comparison with the case of CN with oscillations but without the asymmetry account. Hence, the bounds on the oscillation parameters provided without the asymmetry account are alleviated at small mixing angles [33, 34].

Vice versa, the presence of a relic asymmetry in the nonresonant oscillation case leads to an alleviation of the bounds at large mixings due to suppression of oscillations by asymmetry. While at small mixings, due to spectrum resonance enhancement of oscillations caused by the asymmetry, the bounds on oscillation parameters are strengthened [31].

In case of neutrino oscillations, a precise description of the neutrino evolution at each momentum is necessary, inorder to account for the essential effects of spectrum distortion and asymmetry generation due to oscillations. To study the oscillation effects (a)-(d) on the light element production in CN, a selfconsistent study of neutrino and nucleons evolution is to be provided.

3 Updated constraints on oscillation parameters and the solar neutrino problem

Inorder to account for all different effects of oscillations on CN we have made a selfconsistent numerical analysis of the kinetics of the oscillating neutrinos and the nucleons at freeze-out.

3.1 The kinetics

For simplicity we discussed a toy model of oscillations just in the electron sector $\nu_i = U_{il}\,\nu_l$, $l = e, s$:

$$
\begin{aligned}
\nu_1 &= c\nu_e + s\nu_s, \\
\nu_2 &= -s\nu_e + c\nu_s,
\end{aligned}
$$

where ν_s denotes the sterile electron antineutrino, $c = \cos(\vartheta)$, $s = \sin(\vartheta)$ and ϑ is the mixing angle in the electron sector, the mass eigen-states ν_1 and ν_2 are Majorana particles with masses correspondingly m_1 and m_2.

We have solved selfconsistently the set of the following coupled integro differential equations describing the evolution of the neutrino density matrix ρ and neutron number densities n_n:

$$
\begin{aligned}
\frac{\partial\rho(t)}{\partial t} = Hp_\nu\,\frac{\partial\rho(t)}{\partial p_\nu} + \\
+i\left[\mathcal{H}_o, \rho(t)\right] + i\sqrt{2}G_F\left(\pm\mathcal{L} - \mathcal{Q}/M_W^2\right)N_\gamma\left[\alpha, \rho(t)\right],
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
(\partial n_n/\partial t) = Hp_n\ (\partial n_n/\partial p_n) + \\
+ \int d\Omega(e^-, p, \nu)|\mathcal{A}(e^-p \to \nu n)|^2\left[n_{e^-}n_p(1 - \rho_{LL}) - n_n\rho_{LL}(1 - n_{e^-})\right] \\
- \int d\Omega(e^+, p, \tilde{\nu})|\mathcal{A}(e^+n \to p\tilde{\nu})|^2\left[n_{e^+}n_n(1 - \bar{\rho}_{LL}) - n_p\bar{\rho}_{LL}(1 - n_{e^+})\right].
\end{aligned} \tag{2}
$$

where $\alpha_{ij} = U_{ie}^* U_{je}$, p_ν is the momentum of electron neutrino, n stands for the number density of the interacting particles, $d\Omega(i, j, k)$ is a phase space factor and $\mathcal{A}$ is the amplitude of the corresponding process. The sign plus in front of $\mathcal{L}$ corresponds to neutrino, while minus – to antineutrino.

The initial condition for the neutrino ensembles in the interaction basis is assumed of the form:

$$
\rho = n_\nu^{eq}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}
$$

where $n_\nu^{eq} = \exp(-E_\nu/T)/(1 + \exp(-E_\nu/T))$.

$\mathcal{H}_o$ is the free neutrino Hamiltonian. The 'nonlocal' term $\mathcal{Q}$ arises as an W/Z propagator effect, $\mathcal{Q} \sim E_\nu\,T$. $\mathcal{L}$ is proportional to the fermion asymmetry of the plasma and

is essentially expressed through the neutrino asymmetries $\mathcal{L} \sim 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}$, where $L_{\mu,\tau} \sim (N_{\mu,\tau} - N_{\bar{\mu},\bar{\tau}})/N_\gamma$ and $L_{\nu_e} \sim \int d^3 p (\rho_{LL} - \bar{\rho}_{LL})/N_\gamma$. Due to the different temperature dependence an interesting interplay between these two terms during the cooling of the Universe is observed.

The neutron and proton number densities, used in the kinetic equations for neutrinos, are substituted from the numerical calculations of eq. (2). On the other hand, ρ_{LL} and $\bar{\rho}_{LL}$ at each integration step of eq. (2) are taken from the simultaneously performed integration of the set of equations (1).

These equations provide simultaneous account of the different competing processes, namely: neutrino oscillations, Hubble expansion and weak interaction processes.

For the nonequilibrium active–sterile oscillations this is the only acceptable way of description of the asymmetry and oscillating neutrinos evolution. Because, in case of strongly distorted by oscillations neutrino and antineutrino spectrum, the asymmetry cannot be described in a degeneracy terms anymore, and the spectrum may strongly differ from an equilibrium spectrum with shifted effective temperature [29].

The first equation results into a set of coupled nonlinear integro-differential equations with time dependent coefficients for the components of the density matrix of neutrinos. The number of these equations can be reduced to 6 equations for each momentum mode of neutrinos and antineutrinos, due to conservation of the total neutrino number density in the discussed model. The spectrum distortion was described by 1000 bins for the nonresonant case and by up to 5000 bins for the resonant case. In case the spectrum was described by N bins, a system of $6N + 1$ coupled integro differential equations was numerically solved.

The numerical analysis was provided for the characteristic temperature interval [2 MeV, 0.3 MeV] [6] and the full set of oscillation parameters of the active-sterile oscillation model [29], namely $\delta m^2 \leq 10^{-7}$ eV^2 and $\sin^2 \vartheta \geq 0.001$. We calculated precisely the n/p - freezing, which is the essential for the production of helium-4, till temperature 0.3 MeV, and accounted adiabatically for the decays of neutrons till the start of nuclear reactions below 0.1 MeV.

Updated $\nu_e \leftrightarrow \nu_s$ constraints

Spectrum distortion, neutrino depletion and neutrino-antineutrino asymmetry of the oscillating neutrinos, as well as the selfconsistent account of the evolution of neutrinos and nucleons, is essential for estimating oscillations effect on CN. In the last years the constraints in the electron–sterile sector were updated, accounting precisely for these effects [30]–[33],[37].

The combined constraints for the nonresonant and the resonant case of electron–sterile oscillation parameters are shown in Fig. 2 for different values of relative increase of helium-4, $\delta Y_p = (Y_{osc} - Y_p)/Y_p$. On the left-hand side of Fig. 2 the results for the nonresonant case are presented. The selfconsistent account of neutrinos and nucleons evolution and the

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[6]At higher temperatures $T > 2$ MeV the deviations from the standard CN are negligible for the discussed model of oscillations with small mass differences.

precise calculation of neutrino depletion and spectrum distortion, allowed to strengthen the previous constraints by almost an order of magnitude at large mixings.
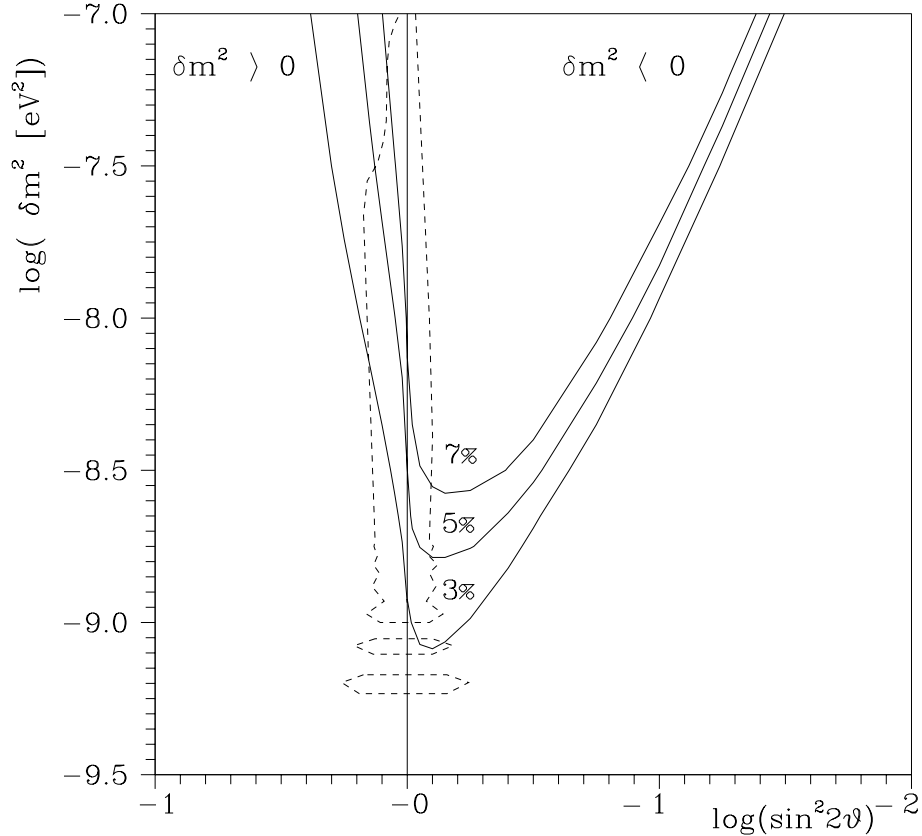

Figure 2: The iso-helium contours corresponding to 3%, 5% and 7% overproduction of primordial helium abundance. LOW sterile solar solution is given by the closed dashed curves.

The analytical fit to the updated constraints, corresponding to $\delta Y_p = 3\%$ is:

$$\delta m^2 (\sin^2 2\vartheta)^4 \leq 1.5 \times 10^{-9} \text{ eV}^2 \quad \text{at} \quad \delta m^2 > 0,$$
$$\delta m^2 \leq 8.2 \times 10^{-10} \text{ eV}^2 \quad \text{at} \quad \delta m^2 < 0 \text{ at large mixing.}$$

In Fig. 3 the plot corresponding to $Y_p = 0.24$ is compared with previous constraints: the pioneer estimates of refs. [17, 20], partially accounting for neutrino depletion; the numerical calculations [24], accounting partially for the neutrino depletion but neglecting the spectrum distortion and the dynamical asymmetry; the results of a recent analytical study [36], estimating the spectrum distortion effect in the approximation of small mixing angles and large mass differences.

In the resonant case, due to the asymmetry growth account, He-4 overproduction is not so strongly expressed, and hence the old bounds on the oscillation parameters [24], provided neglecting the asymmetry, were alleviated at small mixing angles (r.h.s. of
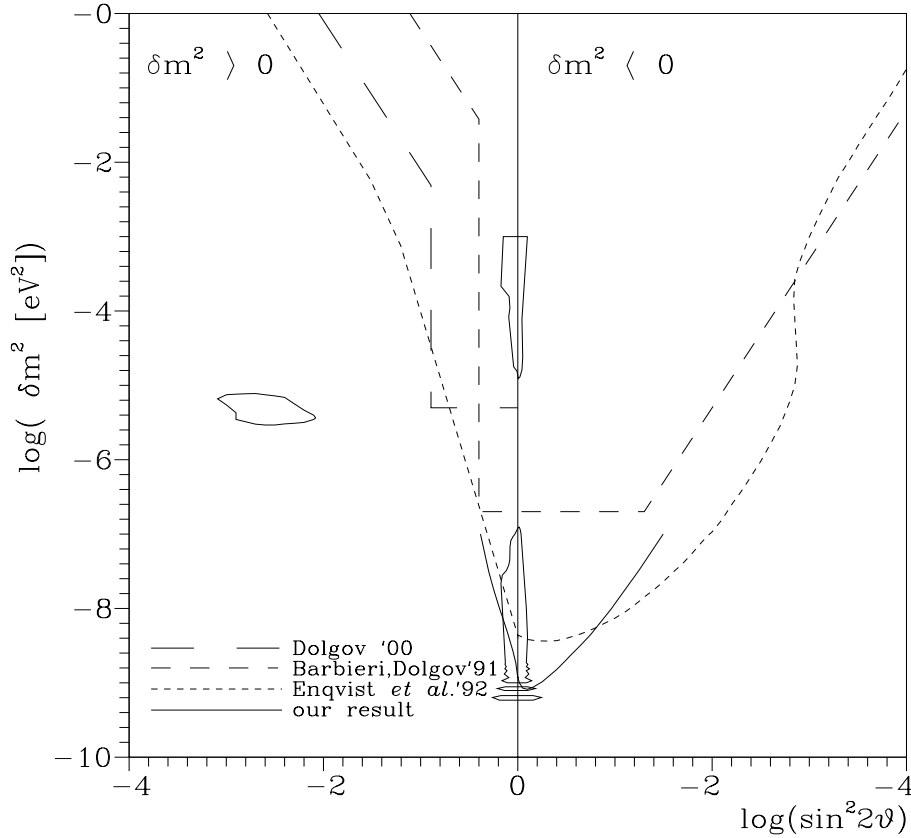
15

Figure 3: Update of the cosmological constraints for the electron–sterile oscillations. For comparison, the dashed curves show the results from previous studies neglecting the spectrum distortion and the growth of asymmetry [17, 20, 24], as well as the recent analytical constraints[36] in the nonresonant case. The precise analysis, accounting for the spectrum distortion and the asymmetry growth [30, 33] allows an almost complete exclusion of the LOW solution of the solar neutrino puzzle.

Fig. 3). However, the precise account of the spectrum distortion of the oscillating neutrino, and the exact kinetic approach to both the neutrino evolution and to the nucleons freeze-out, strengthen the cosmological constraints at large mixing by an order of magnitude.

It will be appropriate to provide similar investigations for large mass differences in the electron–sterile case and also to update the available constraints for the $\nu_\mu \leftrightarrow \nu_s$ and $\nu_\tau \leftrightarrow \nu_s$ cases by taking into account the energy spectrum distortion and the asymmetry growth. We expect that the constraints on the oscillation parameters for the $\nu_{\mu,\tau} \leftrightarrow \nu_s$ cases will be more slightly influenced than in the electron–sterile case, since μ and τ neutrinos do not directly participate in the nucleon kinetics.

16

CN constraints on solar neutrino solutions

These constraints on active–sterile neutrino oscillations exclude the active–sterile LOW solution to the solar neutrino puzzle in addition to the LMA solution, excluded in the pioneer works. LOW electron–sterile solution was obtained from the analysis of the 1258 days SuperKamiokande experimental data on neutrino electron scattering and zenith angle variations of the solar neutrino flux [41, 43]. It is shown in Figs. 2 and 3 by the closed solid line around maximal mixing and $\delta m^2 \sim 10^{-8}$ eV^2.

Assumed $\delta Y_p < 3\%$, electron-sterile LOW solution is almost completely excluded for $\delta m^2 < 0$ and it is completely excluded for $\delta m^2 < 0$ case. It is interesting to note also that even in case of very high primordial helium-4 $\delta Y_p/Y_p = 7\%$, sterile LOW solution still remains partially excluded [37].

According to the global analysis of the solar neutrino data from SuperKamiokande, SAGE, GALLEX + GNO and the Chlorine experiments, the LMA and LOW solutions were found not acceptable for oscillations into sterile neutrinos [40, 41, 42]. The recent global analyses confirmed that LMA and LOW sterile solutions are disfavoured [43, 44].

The conclusions from the global analysis of experimental data in 2000 [40, 42, 41] and in 2001 [43, 44] are in remarkable agreement with the cosmological constraints on LMA (dating from the early 1980's) and on LOW solutions (obtained in 1999).

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