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TM-FEL With a Longitudinal Wiggler and an Annular Beam *

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Abstract

We investigate theoretically the interaction of an electron beam with a TM mode in the presence of a longitudinal wiggler. The electron motion is longitudinal and radial therefore adequate for interaction with the electric field components of the mode. There are two facts which make this device unique: firstly, the equations which describe its dynamics are practically identical to the equations of the traveling wave tube(TWT) rather than the free electron laser(FEL) thus its sensitivity to beam quality is significantly lower, and the efficiency in the absence of tapering is higher. Secondly the interaction is possible only with an annular rather than a pencil beam as in a regular FEL - therefore the amount of current it is possible to inject is larger. Optimal operation at $20.56GHz$ is examined.

I. INTRODUCTION

From basic electrodynamic arguments it can be shown [1] that the total power of the spontaneous radiation emitted by a charge accelerated parallel to its velocity is larger by γ^2 than that obtained if the acceleration perpendicular to the velocity - assuming the same acceleration in both cases. With regard to stimulated (coherent) process the situation is similar. In order to illustrate the problem we shall briefly consider here two devices: first the traveling wave tube (TWT) where the interaction of the electrons is with the TM mode - thus the acceleration of the electron is parallel to its main velocity component. The other device is the free electron laser (FEL) - where the acceleration is transverse to its main velocity component.

The equations which describe the dynamics of both TWT and FEL are summarized in Table I - the details can be found in Refs.(2-3) for TWT's and [4] for FEL's. In both cases a is the normalized amplitude of the electric field, γ_i is the relativistic factor of the i^{th} electron, χ_i is the phase of the i^{th} electron relative to the wave, α is the coupling coefficient, $\Omega = \omega d/c$, $K = kd$ is the normalized radiation wavenumber and $K_w = k_w d$ is the normalized wiggler wavenumber; d is the total length of the system. The coupling coefficient α is completely different in the two cases but for the present purpose the explicit expressions are not important. Furthermore, the amplitude a in the TWT case represents the longitudinal component of the electric field whereas in the FEL it represents the transverse field. In addition we present the spatial growth rate. The major difference between these two sets of equations is the factor $1/(\gamma\beta)^2$ which appears in both the amplitude and particle equations of the FEL - as a result of the transverse oscillation. In the amplitude equation it introduces a different weighting function to the phase term which causes a significantly different behavior of the particles relative to the wave.

Although typically an FEL requires a higher quality beam than a TWT, its inherent advantage of a very simple (smooth waveguide) electromagnetic structure has its own appeal - in particular for high power microwave generation where problems of breakdown become significant. It was exactly this issue which triggered the question of whether it is possible to force the electrons to oscillate primarily in the z direction by means of a magnetic wiggler and to achieve in the upper range of microwave frequencies ($10-30GHz$) operation similar to a TWT based on a slow wave structure. Our study indicates that this is possible. The wiggler is not the regular wiggler of an FEL but a set of periodic magnetic lens - or practically a longitudinal periodic magnetic field which has zero transverse component on axis and it has an exponentially growing radial component off axis.

The interaction in this kind of device has been investigated in the early eighties by McMullin and Bekefi [5-6] in an attempt to achieve higher frequencies ($300-400GHz$) than in regular FEL's for the same electrons energy. Some experimental work was done a few years later [7]. The present approach differs from theirs in four aspects: *(1)* we consider the interaction of the electrons with the TM mode rather than the TEM mode, *(2)* our system can operate only with an <u>annular</u> beam rather than a pencil beam. *(3)* We adopt the single particle equation of motion rather than the fluid approximation to describe the electrons dynamics. *(4)* The frequency range which has motivated this work is much lower ($10-30GHz$) than that considered in the past.

II. SYSTEM DYNAMICS

Let us consider a beam of electrons injected into an uniform pipe of radius R. The pipe is immersed in a magnetic field which in the interior of the pipe is given by

$$B_r(r,z) = \delta B I_1(k_w r) sin(k_w z) \quad ,$$

$$B_z(r,z) = B_0 + \delta B I_0(k_w r) cos(k_w z) \quad ; \qquad (1)$$

B_0 is the guiding magnetic field, δB is the amplitude of the undulator on axis ($r = 0$) and $k_w = 2\pi/L$ where L is the spatial periodicity of the wiggler.

Due to the periodicity of the magnetic field it will be natural to expect that the motion of the electrons will follow a similar periodicity. Furthermore, the presence of

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	TWT	FEL	TM-FEL
Amplitude Dynamics	$\frac{d}{d\varsigma}a = \alpha\langle e^{-j\chi_i}\rangle$	$\frac{d}{d\varsigma}a = \alpha\langle\frac{e^{-j\chi_i}}{(\gamma_i\beta_i)^2}\rangle$	$\frac{d}{d\varsigma}a = \alpha\langle f_i e^{-j\chi_i}\rangle$
Equation of Motion	$\frac{d}{d\varsigma}\gamma_i = -\frac{1}{2}[ae^{j\chi_i} + c.c.]$	$\frac{d}{d\varsigma}\gamma_i = -\frac{1}{2}[a\frac{e^{j\chi_i}}{(\gamma_i\beta_i)^2} + c.c.]$	$\frac{d}{d\varsigma}\gamma_i = -\frac{1}{2}[af_i e^{j\chi_i} + c.c.]$
Phase Equation	$\frac{d}{d\varsigma}\chi_i = \frac{\Omega}{\beta_i} - K$	$\frac{d}{d\varsigma}\chi_i = \frac{\Omega}{\beta_i} - K - K_w$	$\frac{d}{d\varsigma}\chi_i = \frac{\Omega}{\beta_i} - K - K_w$
Global Energy Conservation	$\frac{d}{d\varsigma}[< \gamma > -1 + \frac{1}{2\alpha}\|a\|^2] = 0$	$\frac{d}{d\varsigma}[< \gamma > -1 + \frac{1}{2\alpha}\|a\|^2] = 0$	$\frac{d}{d\varsigma}[< \gamma > -1 + \frac{1}{2\alpha}\|a\|^2] = 0$
Spatial Growth Rate	$q = \frac{\sqrt{3}}{2}\left[\frac{\alpha\Omega}{2}\langle\frac{1}{(\gamma_i\beta_i)^3}\rangle\right]^{\frac{1}{3}}$	$q = \frac{\sqrt{3}}{2}\left[\frac{\alpha\Omega}{2}\langle\frac{1}{(\gamma_i\beta_i)^5}\rangle\right]^{\frac{1}{3}}$	$q = \frac{\sqrt{3}}{2}\left[\frac{\alpha\Omega}{2}\langle\frac{f_i^2}{(\gamma_i\beta_i)^3}\rangle\right]^{\frac{1}{3}}$

Table I: The equations which describe a TWT, FEL and TM-FEL.

the magnetic field (and in particular it's modulation) will cause the electrons to gyrate. If we now assume that this gyration is much faster than all other time variations i.e.

$$\frac{eB}{m\gamma\beta} >> ck_w, ck_{rad} \quad , \qquad (2)$$

then the effect of this motion on the TM_{01} mode averages out to zero. Subject to this condition, the trajectory of the electrons can be approximately described by

$$\bar{r} \approx R_0 + R_1 cos(k_w \bar{z}) \quad ; \qquad (3)$$

in this expression we ignored higher harmonics contributions. The equations which describe the dynamics of such a system (also assuming single mode operation) are presented in the third column of Table I. The filling factor is

$$\bar{f}_i \equiv \frac{1}{2\pi}\int_0^{2\pi} d\theta e^{jp_1\frac{R_{0,i}}{R}cos\theta} J_1(p_1\frac{R_{1,i}}{R}cos\theta) \ . \qquad (4)$$

If the beam is very thin such that the variations of $f(R_0)$ are negligible, then the form factor does no longer depend on the index of the particle: at this stage one can redefine:$a_{new} = a_{old} * f(R_0)$ and $\alpha_{new} = \alpha_{old}f^2(R_0)$. With these newly defined quantities the equations of the $TM-FEL$ are completely identical to those of the TWT.

III. SYSTEM OPERATION

Kinematics: The beam line intersects the electromagnetic mode line in two points. For a given geometry and velocity we conclude from the resonance condition that there could be two resonant frequencies:

$$f_\pm = \frac{c}{L}\gamma^2\beta\left[1 \pm \frac{1}{\gamma}\sqrt{(\gamma\beta)^2 - (\frac{p_1}{k_w R})^2}\right] \qquad (6)$$

subject to the condition $\gamma\beta k_w R > p_1$. At the limit $\gamma\beta k_w R = p_1$ the beam line intersects the EM mode line in a single (tangent) point. In the present study the radius of the pipe is taken to be $R = 1.6cm$ and the periodicity of the wiggler is $L = 2.0cm$. For these parameters the energy of the injected electrons has to be at least $55keV$ in order to have at least one resonant frequency.

Magnetic Field: The ratio between the amplitudes of the wiggler is taken to be $\delta B = 0.107B_0$. The trajectory of the electrons was calculated assuming that $B_0 = 200G$. The two amplitudes R_0 and R_1 of the electron trajectory are correlated for a given magnetic field. Based on this correlation we have numerically calculated $\bar{f}(R_0 = 0.57R) = 0.14$.

Spatial Growth Rate We shall now use the analytic expression for the spatial growth rate presented in Table I in order to determine the optimal regime of operation. In Figs.1 and 2 we present the frequency and the growth rate for the low and high resonant frequencies respectively as a function of the normalized velocity $\beta = V/c$. The coupling as a function of the initial velocity is calculated assuming a $200A$ current and a total interaction length of $d = 30cm$. In these plots we defined $q_\pm(dB/cm) = q_\pm = 20log(e^{Im[q(f_\pm)]})/d$. The large peak in the gain of the low resonant frequency occurs at the point of zero group velocity. Exactly at cutoff our approximations are no longer valid - however the general trend of operation with low group velocities in order to achieve high gain has been identified also in the beam-wave interaction in metallic corrugated waveguides [8]. The gain per unit length for electrons of energies lower than $127keV$ is less than $0.5dB/cm$ therefore it is not attractive. At about the same energies we obtain the maximum gain per unit length for the high resonant frequency waves. The corresponding frequency is $f_+ = 20.56GHz$ and $q_+ = 1.588dB/cm$.

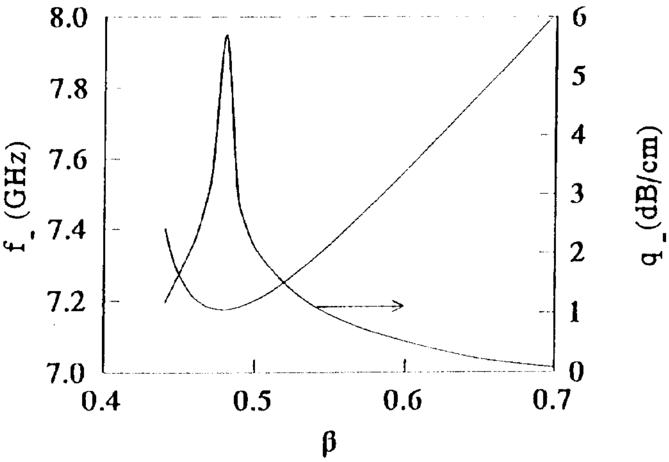

Fig. 1: The lower frequency and the spatial growth rate as a function of the normalized velocity β.

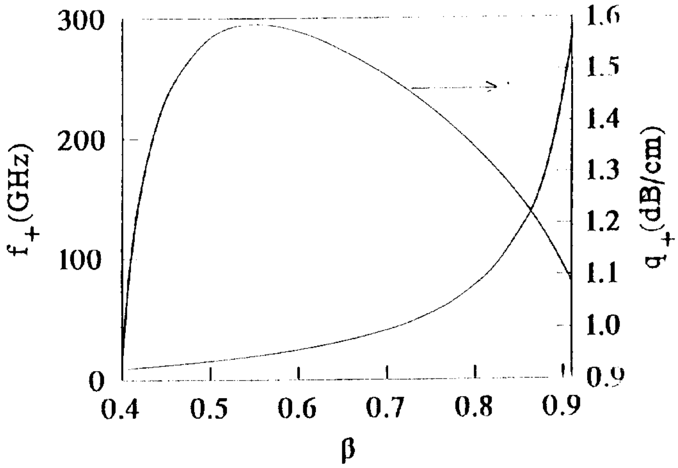

Fig. 2: The higher frequency and the spatial growth rate as a function of the normalized velocity β.

Quasi-Analytical Approach. Based on the set of equations presented in Table I, we can simulate the operation of the device as an amplifier. At $f = 20.56GHz$, for $400A$ and a $128kV$ beam with $6kW$ of power at the input the maximum gain is about $30dB$ corresponding to more than 11% efficency. For a prebunched beam the efficiency can be doubled without tapering the magnetic field.

Particle in Cell Simulations. The analytical and quasi-analytical results presented above have been tested with the particle in cell code MAGIC -excluding the TE modes. Fig. 3 depicts the trajectories of the electrons. Typically these remain unchanged even when the phase space distribution is significantly altered; the density of particles may vary significantly from one location to another. The system was driven at $7.2GHz$ which is, at the input energy, the resonance frequency of the system - close to cutoff ($7.176GHz$). The spectrum of the radiation generated is stable and "clean". The Poynting vector integrated over the waveguide cross section indicates about $2MW$ (25%).

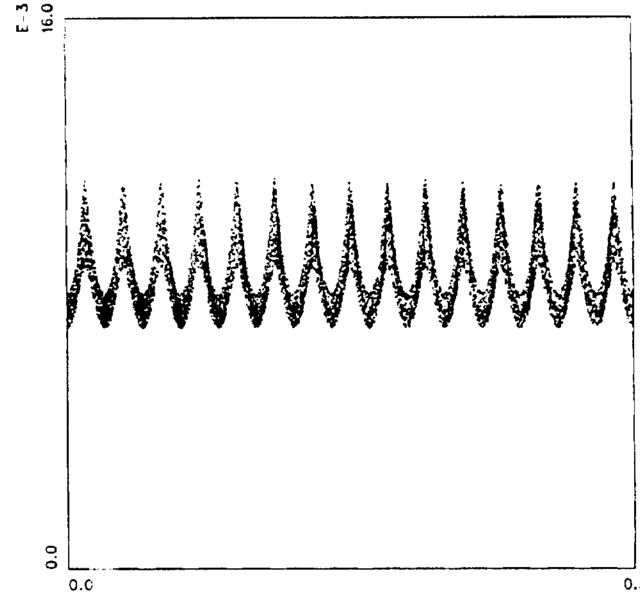

Fig. 3: Electron trajectory as calculated with MAGIC.

IV. DISCUSSION AND SUMMARY

We have presented preliminary results from the study of the interaction of a beam with a TM mode in a longitudinal magnetic wiggler. For a thin beam the simplified set of equations which describe the dynamics of such a system are identical to these of a traveling wave tube and differ from the regular transverse wiggler free electron laser. The quasi-analytical model indicates that the TM-FEL has a low sensitivity to initial spread of the kinetic energy - like the TWT and high (11%) "bare" efficiency (no tapering). With a prebunched beam the efficiency can be increased. The trajectories of the electrons are stable even at saturation and are determined by the wiggler. Combining these results with the very simple geometry required for the electromagnetic structure (smooth waveguide), make the device appealing for high power microwave generation in the range of ($10-30GHz$).

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