

## BEAM DYNAMICS IN THE SPIRAL LINE INDUCTION ACCELERATOR

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### ABSTRACT

Magnetic transport of the high-current electron beam is an important problem in the design of the spiral line induction accelerator (SLIA) [1]. Here, the beam, which is initially matched to the solenoidal focusing field, must be transported through a 180° achromatic bend between each acceleration. The ELBA 3-D PIC code [2] is used to model the SLIA, including transport through the achromatic bend. We find: a) without careful initialization, the beam envelope undergoes growing oscillations that damp once a stable equilibrium is obtained (an improved equilibrium is derived for the intense beam), b) self-field effects in the bend force an increase in the vertical field, but do not significantly change the chromaticity of the bend and c) the beam can be matched to the strong focusing fields in the bends via simple magnetic focusing elements. ELBA runs will be compared to results of ongoing experiments at Pulse Sciences, Inc.

### I. INTRODUCTION

The spiral line induction accelerator (SLIA) is a proposed compact high-current electron beam accelerator which is being developed and studied experimentally at Pulse Sciences, Inc. SLIA uses induction cavities to accelerate the beam and a solenoidal magnetic field,  $B_z$ , for transport in the straight sections. Transport around the bends is accomplished using strong focusing stellarator windings (i.e., twisted quadrupoles) augmented with a vertical magnetic field. The beam line is shown in Fig. 1. The transport lines are isolated from each other except in the acceleration region. In the Proof-of-Concept Experiment (POCE) [3] a 10 kA, 35 ns beam will be accelerated to 9.5 MeV from 3.5 MeV at injection in steps of 1.5 MeV. Recent experiments [4] have used an 850 keV, 200 A injector to study the capture of the beam onto a solenoid, and the matching from the solenoid to a twisted quadrupole (without a bend).

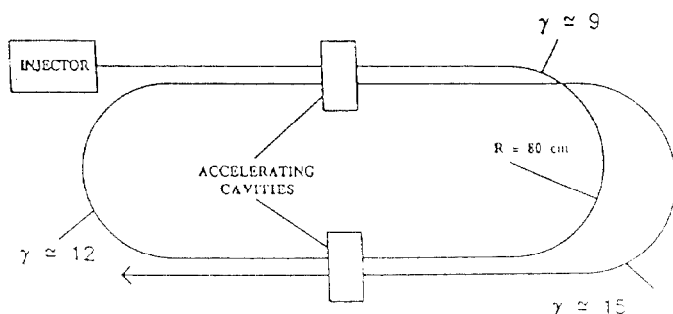


Figure 1. SLIA Proof-of-Concept Experiment.

### II. EQUILIBRIA FOR SOLENOIDAL TRANSPORT

A convenient assumption made in studying beam transport in accelerators is that the beam has a K-V distribution [5] in the rotating frame in the directions  $(x,y)$  transverse to the propagation direction  $(z)$ . Particle simulations of the POCE beam matched in the 5.492 kG solenoidal guide field show a small instability if it is injected with a transverse K-V distribution. (The diode is free of magnetic fields, so the canonical angular momentum of the beam is zero.) The beam radius exhibits growing oscillations at the envelope frequency which typically reach amplitudes of the order of 7% to 10% of the equilibrium radius (Figure 2). Eventually (over a scale length of several meters), the oscillation amplitude saturates. The instability is a kinetic effect which does not occur in envelope code calculations, and it is not suppressed by simply initiating the beam with carefully matched envelope parameters. This behavior tends to mask the features we wish to study in transport calculations, such as the envelope oscillations due to imperfect matching at stellarator-solenoid transitions. It also raises the question of whether such oscillations are inevitable in a real beam, where we do not have perfect control over the beam distribution.

The K-V distribution can be derived from the Vlasov equilibrium given by  $f(r,p) = n_0/2\pi\gamma_0 m \delta(H - \omega L - \gamma_0 mc^2) \delta(P - \beta_0 \gamma_0 mc)$  in the limit  $1/\gamma_0 \rightarrow 0$  [5]. The constants of motion are the energy,  $H$ , the canonical angular momentum,  $L$ , and the canonical axial momentum,  $P$ .  $\omega$ ,  $\gamma_0$  and  $\beta_0$  are parameters. We have derived an improved distribution by retaining the first order terms in  $1/\gamma_0$ , which result in corrections to the K-V including the self-generated diamagnetic axial magnetic field and the kinetic energy variations due to the space charge depression. Although

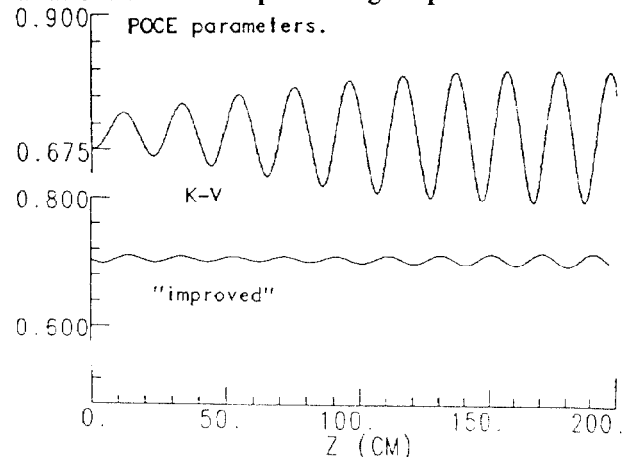


Figure 2. Edge radius of beam in solenoid.

the charge distribution is not radially flat in this approximation, it has a sharp cutoff as does the K-V. When the simulation is initialized with this particle distribution the growing oscillations are not observed (Figure 2). For the POCE parameters the K-V distribution corrected for space charge also gives acceptable results.

The actual beam almost certainly does not have a K-V distribution. Simulations of the injector, for example, result in beams with diffuse edges [6]. We have used the predictions of these simulations as initial conditions to study transport and matching. In the best case, we observe envelope oscillations of about  $\pm 0.5$  mm about the equilibrium radius of 4.3 mm. In 3 m of transport, these oscillations damped by a factor of 2 with little increase in emittance. Recent experimental measurements using a 850 keV, 200 A beam show a spatial profile which is well fit by a parabola [4].

### III. FIELD CORRECTIONS IN THE BENDS

Hoop forces occur in the bends because a given point in the curved beam experiences a net transverse electro- and magnetostatic force due to the remainder of the curved beam. For the case of the POCE, in which the major radius of each bend is 80 cm, these forces can be substantial. For example, theory [7] predicts that the vertical field on the first bend, which is set to 186.9 G in the present design, must be increased by a correction of between 49.3 G to 61.7 G. The results are uncertain to this extent because of uncertainties in the details of the beam equilibrium profile: the lower figure is for the assumption of constant current, whereas the higher is for constant circulation frequency. There are some additional uncertainties due to the fact that the POCE beam traverses a  $180^\circ$  bend with entrance and exit points, whereas the published calculation applies to a complete circular turn.

Simulation results for the POCE first bend ( $\gamma=8.83$ ) show that without correcting for the hoop forces, the beam centroid displacement within the curve will exceed 1 cm (wall radius is 3 cm). Even in this case, the achromatic design brings the centroid back to within 2 mm of the axis on exit. However, this maximum displacement is unacceptably large. Further runs show that the optimal corrected value of the vertical field is 233 G, somewhat smaller than predicted [7]. With the optimal value, the beam centroid remains within 0.06 cm of the geometric axis at all times.

The 25% increase in the design value of the vertical field may invalidate the remaining achromat parameters (i.e., the axial and quadrupole field values). We have checked this by performing simulations of the first bend using the previously calculated achromat parameters, except for the vertical field which is corrected for the hoop forces. We find that for 10% mismatches in the beam energy, the transport is still reasonably achromatic, with the beam centroid displacement on exit less than 2 mm in all cases.

The intense beam gives rise to diamagnetic effects which reduce the vertical field index to less than the design value of 0.5 over most of the beam cross section. The vertical field index is responsible for weak focusing in betatrons, and appears in the equations which determine the achromat parameters. We have performed additional simulations for the POCE in the first bend with the nominal vertical field at the hoop-corrected value and various values of the field index. These calculations showed no effect whatsoever, either on transport at the design energy or the achromatic properties of the first bend. This result might well have been anticipated, at least with respect to transport at the design energy, inasmuch as the strong focusing due to the stellarator fields dominates the weak focusing arising from the betatron field.

### IV. MATCHING FROM THE SOLENOID TO THE TWISTED QUADRUPOLE

Of major concern in the design of the POCE are the transitions between the straight sections where the beam is transported in purely solenoidal fields and the  $180^\circ$  bends which have, in addition, the strong focusing stellarator fields and the vertical bending fields. In the bends a perfectly matched beam assumes an elliptic cross section with constant major and minor radii, and orientation rotating with the pitch of the stellarator windings. If the beam is mismatched, the lengths of the radii will oscillate. The emittance may grow and, in extreme cases, the beam may scrape the wall.

An attractive approach to the matching is to use a single thick quadrupole lens at each transition. Simulations show that a beam propagating in a solenoidal field can be transformed from a standard equilibrium with circular cross section to a new equilibrium with a slowly rotating elliptic cross section, by passage through a thick quadrupole lens. The eccentricity of the ellipse is determined by the strength of the lens. The idea of the matching scheme is to adjust the strength of the lens to match the eccentricity needed in the bend, and to adjust the orientation of the lens so that the angle of the ellipse is in the correct phase as the beam enters the twisted quadrupole in the bends. The phase depends on the details of the entrance windings. A single quadrupole lens can also be used at the exit of the bend to bring the beam back to nearly circular cross section, if desired. Simulations indicate there will be no significant emittance increase if matching is done carefully with two thick quadrupoles. Recent experiments [4] indicate that the beam will return to circular cross section in the solenoid, even without matching lenses. Simulations confirm that this will occur, if the beam distribution is not K-V. However, a significant increase in emittance (10% in the POCE first bend) is seen in the absence of matching elements (Figure 3). Some of the increase occurs in the bend, but most occurs in the following solenoid.

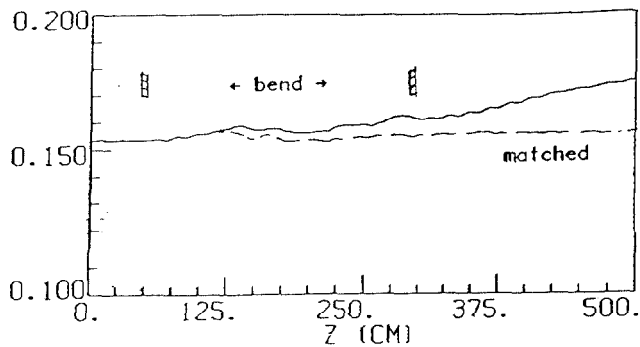


Figure 3. Normalized RMS emittance for POCE beam.

Recent measurements [4] of the beam's radial profile, integrated over the pulse, have been made for an 850 keV, 200 A beam injected without matching from a 1.5 kG solenoid into a straight stellarator which has a quadrupole gradient of 68.5 G/cm. Its length is 251.3 cm which encompasses four complete rotations of its poles. Figure 4 shows the beam profile experimental measurements at several propagation distances near the end of the quadrupole, and also results of simulations which were initiated in the solenoid with a matched K-V distribution. Large oscillations of the major radius, due to the mismatched transition, are seen in both the experiment and the simulation. In the experiment, these oscillations damp, and an equilibrium is reached, with elliptic cross section, that is not much different from that which results from correct matching. The simulations with K-V distributions show envelope oscillations which are undamped. However, damping is observed in the simulations if beams are injected with non-K-V distributions. We speculate that this damping for non-K-V distributions is due to phase mixing. The space charge forces are linear for the K-V.

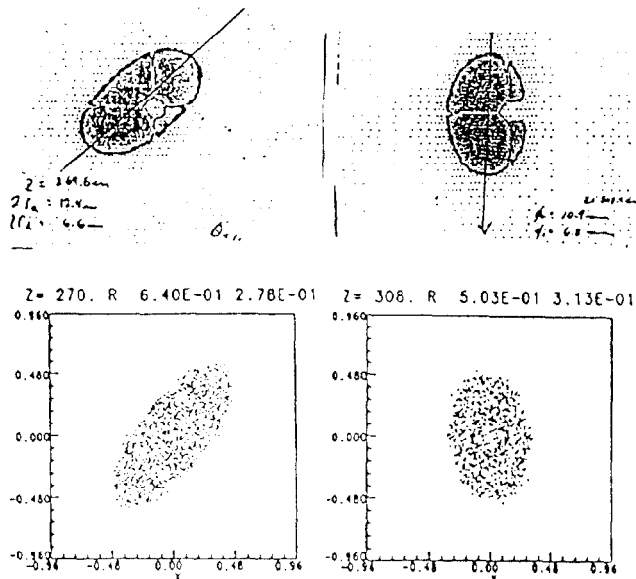


Figure 4. Experimental measurements [4] and simulation results for beam profile. The stellarator starts at  $z=89$ . The major and minor radii at  $z=270$  are 6.2 and 3.3 mm, measured, (6.4,2.8 simulation); at  $z=308$  they are 5.4 and 3.4 (5.0,3.1).

## V. SUMMARY

Simulation studies of the SLIA experiments have been performed. Equilibrium in the solenoidal field has been studied and a generalization of the K-V distribution has been found which gives a "better" equilibrium for the SLIA parameters, though these equilibria are not likely to be produced experimentally. Hoop force corrections in the bends of the SLIA have been calculated and it is shown that the achromatic properties of the design are preserved. Diamagnetic effects on the vertical field index are not important. A scheme for matching the beam on and off the stellarator windings has been devised which uses a single thick quadrupole lens on each end of the stellarator, greatly reducing emittance growth. Simulations reproduce the major features observed in recent experiments.

## ACKNOWLEDGMENTS

We wish to acknowledge many helpful discussions with Dr. V. Bailey of Pulse Sciences, Inc.

This work was supported by Defense Advanced Research Projects Agency under ARPA Order 7781 and monitored by the Naval Surface Warfare Center.

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