THE FIXED-FIELD, ROTATING-MAGNET SYNCHROTRON OR: WHY PULSE?

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Introduction

Superconducting magnets seem to offer great promise for the construction of future very high energy accelerators. However, the problem of pulsing superconducting magnets is not an easy one and, even if "solved", will presumably still result in large heat losses at liquid He temperature, requiring refrigerators which handle, in turn, still much larger amounts of energy at room temperature. In addition, there must be the usual multi-megawatt pulsed power supply, and the usual problem of mechanical fatigue of the coils after long periods of pulsed operation. It therefore is interesting to inquire whether pulsing is even necessary. Superconducting magnets can be short-circuited, so that the current persists indefinitely. No power supply leads (and no power supply) are needed. In that case, perhaps the magnets can be moved mechanically so as to produce the required time variation in guide and focussing fields. We propose an arrangement in which the motion is a slow, uniform rotation of various dipole and quadrupole magnets about the beam axis. Such a motion seems rather innocuous, giving the feeling that there must be some inexpensive way of achieving it. The most appealing scheme, suggested to me by Dr. R. Britton, is to levitate each rotating magnet on superconducting magnetic bearing and "turn the spit" by synchronous motor action. Magnet and bearings could then be immersed in a common He Dewar, with a fixed ceramic vacuum pipe going down the axis.

The basic scheme we propose is as follows: imagine each dipole magnet of a conventional separated-function AGS to be replaced by 4 shorter, but otherwise similar, magnets (which we shall refer to as magnet segments). The first and third segments remain fixed. The second and fourth rotate slowly (5 to 10 RPM) about the beam, but in opposite directions (Fig. 1). If we take the 4 segments to have parallel fields at $t = 0$, then in thin lens approximation the entire 4-segment magnet acts as a dipole, whose strength varies as $1 + \cos \omega t$, ω being the turn frequency. One injects somewhat past $\omega t = 180°$, and accelerates beam until $\omega t = 360°$, as the mean guide field builds up. In practice a ratio of final to initial momentum of 10 seems reasonable, and we shall use this as an example throughout. We now have a choice; we can inject at $\omega t = \cos^{-1}(-0.8)$, i.e., at 10% of peak guide field in the above arrangement, or we can run the segments with unequal currents $I_1=I_3=D_S$, $I_2=I_4=D_r$. The mean guide field then varies as $D_S+D_r \cos \omega t$, and for $\frac{D_S-D_r}{D_S+D_r} = 0.1$ we can inject at 180°, at a time when the time derivative of the mean guide field is zero. We shall adopt this scheme for purposes of discussion, as it has the advantage that the mean guide field at injection is insensitive to errors in the angular position of the magnet segments. We notice that half of the cycle is available for acceleration, which is not unfavorable.

The focussing is to be provided by quadrupole magnets. Each quadrupole singlet is segmented in exactly the same way as the dipoles (Fig. 2); the first and third segments are fixed, the second and fourth counter-rotate, but now at half the frequency of the dipoles. This produces a thin-lens quadrupole strength $Q_S+Q_r \cos \omega t$. For $\frac{Q_S-Q_r}{Q_S+Q_r} = 0.1$, the quadrupole strengths stay in step with the dipoles, as desired.

If there were no other complications, we would now have an accelerator. Let us assume for the moment that this were the case, and see what its properties would be. First, we would be able to use the highest possible guide field at full energy; there need be no compromises in the magnet design. Presumably, 80-100 kilogauss could be achieved. The magnet would take no power to excite, no cooling power, and essentially no power to rotate. It need not contain any iron. There would be no fatigue problem in the coils. The repetition rate would be limited only by the ability of the RF system to accelerate the particles, since the magnet is not pulsed. As for the magnets, eddy current losses in any conductors exposed to the rotating field must be held to a minimum. It is important to recognize, however, that such losses come out of the energy required to turn the magnet, and not out of the energy stored in the persistent field of the magnet. Thus the current tends to persist. Small losses in the magnet stored energy that may occur (for example, if the joint at the coil ends is normal), can be made up from time to time by the flux-pumping technique.

Since this prospect is a pleasant one, we inquire further into the complications of the orbits, which are many. Let us first state the results as far as we have been able to see them: there appear to be no overwhelming difficulties. The particles wiggle a bit at injection, but are stable. The quad field must deviate from synchronism with the guide field by $\lesssim$ 5% of its peak value, to avoid hitting resonances near injection. The radial and vertical motions are coupled, but rather weakly. The magnet segments must be short, but not unreasonably so ($\sim$ 1 m). No extreme tolerances have been encountered. In other words, so far it looks feasible. On the other hand, the examination of the complicated orbits has not been exhaustive, and detailed investigation may yet show up serious trouble.

Central Orbit

At intermediate energies, and especially so at injection, the central orbit is not circular. At injection the magnets are all horizontal, but half of them are upside-down. The orbit then at least lies in a plane. Let us first convince ourselves that a closed orbit exists at injection: consider a ray starting at some position and angle; trace it around the machine until it returns. Now vary the initial angle until the orbit crosses itself at the starting point. Then vary the starting radius until it crosses at zero slope. This can obviously be done, so a closed orbit exists.

Away from injection, the vertical position and slope can be used to ensure vertical closure, with zero slope. Thus a closed orbit exists at all energies.

The solution of the equations of motion will consist of oscillations about this central orbit, and the oscillations will be the same as they would be if the orbit were circular.

How badly will the scalloping of the orbit waste aperture at injection?

Let L_1 be the dipole segment length, L_2 the drift length between dipole segments. The central orbit will look as in Fig. 3. The unwanted excursion S can be minimized by keeping the magnets short and the injection energy high. To be specific, for H = 80 kg, L_1 = 100 cm, L_2 = 25 cm, and p_o = 150 GeV (injection energy), S = 0.3 cm. This does not seem to be serious. Clearly, however, the sagitta would be prohibitive for a sufficiently low energy machine. The above choice corresponds, with p_o/p_{max} = 1/10, to a full energy of 1500 GeV. (For an injection energy as low as 30 GeV, L_1=50cm, L_2=12.5 cm yields S=0.375 cm)

Focussing

We shall discuss one focussing term at a time, even though they are coupled, in an attempt to gain understanding.

A.G. Focussing Terms

The configuration we have set up provides, to first order, conventional AG focussing, in which the strength of each element keeps step with the mean guide field. To higher order, there are unwanted terms, which we now discuss, rather unmathematically.

A rotating quadrupole field can be expanded as a time-dependent linear combination of stationary quadrupole fields of two types, whose magnetostatic potentials are of form xy and x^2-y^2 respectively. The xy term (0° quad) is the one we want. In the arrangement proposed, the x^2-y^2 term (45° quad) cancels to first order, as a result of the opposite sense of rotation of the two rotating elements (Fig. 1). However, because the nearly cancelling segments are separated in space, they do not cancel exactly. There will be a net focussing action, in both $\frac{x+y}{\sqrt{2}}$ and $\frac{x-y}{\sqrt{2}}$, essentially equal in in both. The two-thin lens formula

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

will suffice for our purposes. If $|f_1| = |f_2| = f$ and if $\frac{1}{F'} \simeq \frac{2}{f}$ can be taken as the overall strength of the segmented lens, then

$$\frac{\delta F}{F'} = \frac{(d/f_1 f_2)}{2/f} = \frac{d/f^2}{2/f} = \frac{d}{2f} = \frac{d}{4F'}$$

(This is its strength at ωt = 90°, when the 45° quad strength is at a maximum.)

For our purposes take d ≃ 250 cm, F' ≃ 50 meter, ≃ spacing between quads, which gives

$$\frac{\delta F}{F'} \simeq \frac{2.5}{200} \sim 1.25\ \%.$$

This is then 0.62% of the peak strength, hence ∿6% of the strength at injection. This small perturbation will vary as sin $\omega t/p^2$.

There is a similar effect due to lack of exact cancellation of the xy quadrupole turns near injection. This one will vary as cos $\omega t/p^2$ and seems to be somewhat larger, about 20% of the injection quad strength, or 2% of the full focussing field.

These two calculable changes in tune can be corrected by introducing a corresponding change in the main guide field, or by special correcting quadrupoles. We will defer further discussion of how this should be achieved until after we have discussed the edge focussing in the dipoles.

Edge Focussing in the Dipoles

Each dipole segment is essentially a parallel-edge bending magnet, with a focussing strength

$$\frac{1}{f} = \frac{\int B^2 dZ}{(H\rho)^2} \sim \frac{\text{CONST}}{\rho^2}$$

in the plane parallel to the field, and no focussing in the plane perpendicular to the field.

At full energy the magnets are all parallel and the focussing strength per unit length is

$$-\frac{y''}{y} = \frac{d}{dZ}\left(\frac{1}{f}\right) = \frac{B^2}{(H\rho)^2} = \frac{1}{\rho^2} \sim \frac{1}{R^2}$$

where R is the machine radius,

$$y \sim \cos\frac{Z}{R}$$

which gives, by itself, one cycle per turn. This is a small effect, since we have many cycles per turn in an AGS. Now consider the effect at injection. Half the magnets are upside down; but since the edge focussing is a B^2 effect, it is unchanged by this. Each edge still produces a positive focussing action. The effect also goes as $\frac{1}{p^2}$, so at injection (at p_o/p_{max} = 1/10) we have 100 times this focussing strength, and so 10 cycles per turn if this were the only focussing. Now one wants perhaps 30 cycles per turn in a big machine, so this is a fairly large effect. It seems discreet, therefore, to add a "D.C. gradient" component at injection, so as to cut this vertical focussing in half and provide an equal horizontal focussing. These would then be of such a strength as to produce $\frac{10}{\sqrt{2}} \cong 7$ cycles per turn alone. This is of itself not harmful; but, since the effect drops off as $\frac{1}{p^2}$, if it were not corrected for one would cross several resonances on the way up.

One must therefore program the alternating gradient focussing strength so as to be weaker near injection (by $\lesssim$ 30%) so as to allow for the edge focussing and also the two unwanted higher order alternating gradient terms mentioned in the preceding section. (All three effects are positive; they do not cancel.)

One way to do this would be by programmed non-superconducting pulsed quadrupoles. Another would be by pulsing slightly the non-rotating superconducting quad segments. Since the effects are of order 30% of the injection quad strength (or $\sim$ 3% of full strength), this should not be difficult.

The main point is that these effects are calculable, and that the overall requirement in avoiding resonances is the same here as in any other machine, namely the horizontal and vertical tunes must be kept constant to $\sim$ 1% for certain, and ideally to 0.1%.

Coupling of Horizontal and Vertical Motion

This is brought about by two effects, the x^2-y^2 quadrupole term mentioned before (45° quad) and by the edge focussing of tilted dipole magnets. It tends to be small, because the "45° quad" effect focusses almost equally in $(x+y)/\sqrt{2}$ and $(x-y)/\sqrt{2}$, and hence almost equally in x and in y. The "tilted dipole" effect is small because of the alternation of sense of rotation from one rotating segment to the next.

These couplings between vertical and horizontal motion, while annoying, are probably not of any fundamental importance.

Alignment Sensitivity

One has, first, all the usual sensitivities of an AGS. The frequency must track the guide field, for example. This would be solved in the usual way, by a feedback loop driven from beam pickup electrodes. The focussing field error must be small compared to the spacing between resonances. A precision of 1 in 10^3 should be more than adequate, for example. But here the focussing fields at injection are the small difference between two ten-fold larger fields (in the individual quad segments). Thus a precision of 1 in 10^4 is called for on the currents. But these are persistent currents, which are absolutely stable once set up. Presumably they can be monitored initially to arbitrary precision.

The angular position of the rotating quads must be right to $\sim$ 1 in 10^3, or a milliradian. This would seem a trivial control task, rather like asking a clock not to lose more than five minutes per day, provided the magnets are sufficiently isolated from one another so that forces between them are unimportant. A light signal reflected from a small mirror pasted to the magnet would be a good monitor of the angular position, and could be fed back to the drive motors to control it.

There would be an enormous number of magnets ($\sim$ 10,000). One must ask the question whether statistical buildup of random misalignments will exceed tolerances. The answer is reassuring. Let us take errors in magnet currents as a typical problem of this class, and relate the change in tune ΔQ to the random error in individual magnet segment currents, ΔI.

First we have

$$\Delta \bar{I} = \frac{1}{N^{1/2}} \Delta I$$

where $\bar{I}$ is the average current.

Thus

$$\frac{\Delta Q}{Q} \simeq \frac{\Delta \bar{I}}{\bar{I}} = \frac{1}{N^{1/2}} \frac{\Delta I}{I}$$

Now $\Delta Q << 1$ is required, to avoid resonances.

This means

$$\frac{\Delta I}{I} << \frac{N^{1/2}}{Q} \approx \frac{(10^4)^{1/2}}{30} \sim 3$$

The large number of participating magnets has been nullified by the fact that each has only a tiny effect. (This was pointed out to me by G. Finocchiaro.) Similar considerations are presumed to apply to other random errors.

The enormous number of magnet ends encountered would probably require a careful control of the sextupole moment associated with fringing at the ends.

There is a large sensitivity to radial field components caused by magnet rotation angle errors. This can be minimized by shorted coils enclosing the vacuum pipe whose function is to guarantee that the net radial flux is zero at all times.

Momentum Compaction, Transition Energy, and Flattopping

The momentum compaction is the same as for an AGS of the same Q; the effect of the scalloped orbit is nil. Similarly, one is always well above transition energy, despite the scalloped orbits. We obtain

$$\frac{\delta T}{T} = \left(\frac{1}{Q^2} - \frac{\theta^2}{12} - \frac{1}{\gamma^2}\right) \frac{\delta p}{p}$$

$(\theta \simeq 1^\circ \simeq \frac{1}{60}$ for the example given)

T = period of revolution
$\gamma = (1-\beta^2)^{-1/2}$
Q = tune
θ = bend in one dipole segment

As for flattopping, it may not be necessary. One can leave the RF on, and perform slow extraction while accelerating over the top of the cosine curve. If the external beam magnets are built and rotated in the same way as the machine magnets, one gets a slightly variable energy slow extracted proton beam, RF bunched, on the external target. Since secondary yields are insensitive to slight variations in beam energy, one can do most experiments perfectly well, without a flattop (but with a slow spill). The disadvantage is the RF bunching. The effect of this can perhaps be minimized by decreasing the amplitude of the RF, since the rate of energy gain (or loss) is very low near the top of the cosine curve. The decreased RF amplitude will allow broader swings in phase, thus spreading out the bunches considerably.

The other possibility is to halt the magnet rotation, and thus produce a genuine flattop. The feasibility of this depends very much on the engineering of the rotating magnet supports and drive, and so we defer discussion of it until a later time.

A third possibility is to add some third harmonic to the main cosine curve, (both dipoles and quads) thus flattening the top, by having a small fraction of the magnets rotating at three times the speed.

Conclusion

The method proposed looks reasonably promising. Injection energies as low as 30 GeV look feasible, as long as one does not exceed a momentum ratio of about ten. The natural realm of the scheme is, however, the ultra high energy range. Much will depend on developing an inexpensive, mass-producible magnet and mounting, with successive magnets isolated from interacting with one another.

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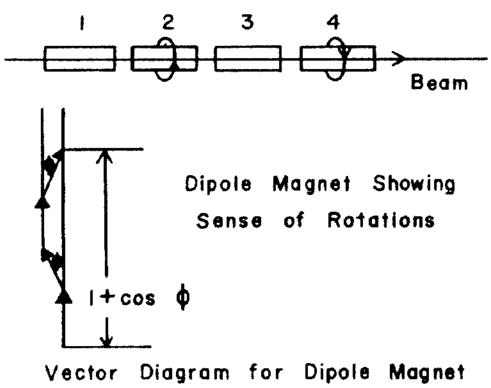


Figure 1.

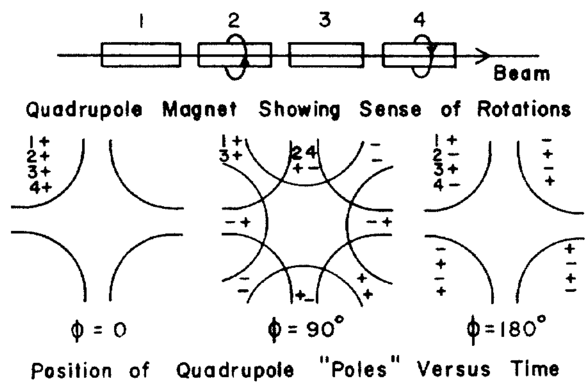
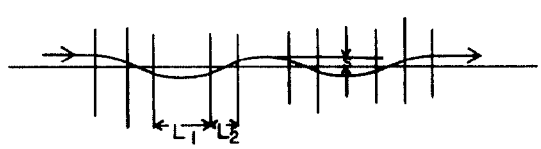


Figure 2.



Central Orbits Viewed From Above

$$S = \frac{L_1^2}{8\rho} + \frac{L_1}{4\rho} L_2$$

Example: 150 Gev, 80 kgauss, L_1= 100 cm. L_2= 25cm.

S = 0.3cm.

Figure 3.