mathematics

Article

Research on the Integrated Optimization of Timetable and High-Speed Train Routing Considering the Coordination Between Weekdays and Holidays

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Abstract: In recent years, passenger holiday travel momentum continues to increase, which proposes a challenge to the refined transportation organization of China's high-speed railway. In order to save the cost of transportation organization, this paper proposes a collaborative optimization method using a high-speed railway train diagram and Electric Multiple Unit (EMU) routing considering the coordination of weekdays and holidays. Based on the characteristics of the train diagram and EMU routing, this method optimizes the EMU routing synchronously when compiling the train diagram. By constructing a space–time–state network, considering the constraints of train headway, operation conflict, and EMU maintenance, a collaborative optimization model of the train diagram and EMU routing considering the coordination of weekdays and holidays is established. This research combines the actual operation data to verify the model and algorithm. Based on five consecutive days of holidays, a seven-day transportation plan covering before and after the holidays and during the holidays is designed, and a case study is carried out. The results show that the proposed collaborative optimization theory has practical significance in the application scenarios of high-speed railway holidays.

Keywords: train diagram; EMU routing; collaborative optimization

MSC: 90B06

Citation: Zhang, Z.; Guo, F.; Deng, W.; Chen, J. Research on the Integrated Optimization of Timetable and High-Speed Train Routing Considering the Coordination Between Weekdays and Holidays. *Mathematics* **2024**, *12*, 3776. https://doi.org/10.3390/math12233776

Academic Editor: Andrea Scozzari

Received: 15 October 2024
Revised: 20 November 2024
Accepted: 27 November 2024
Published: 29 November 2024

1. Introduction

In recent years, the scale of China's high-speed railway network has continued to expand rapidly, the level of equipment technology and equipment support capabilities have greatly improved, and the number of Electric Multiple Units (EMUs) has continued to grow. As of the end of 2023, the total mileage of high-speed rail in China has reached 45,000 km, accounting for 28.3% of the country's railway operation history, and the number of EMUs is 4427 standard units. According to China's "Medium- and Long Term Railway Network Plan", by 2030, the scale of the long-term railway network will reach about 200,000 km, including about 45,000 km of high-speed railways, with an average annual growth rate of about 2%. At that time, the high-speed railway market will attract a wider customer base and become a modern and efficient mode of transportation. In addition, in countries such as Japan, France, and Germany, the use of high-speed trains and the compilation of train schedules are also highly valued. However, with the change in the supply and demand relationship in the transportation market, a series of problems and challenges have emerged. First of all, passenger travel demand shows a trend of high quality, diversification, and personalization. Passenger flow is more complex especially during the holidays. High-speed railways must adapt to market changes, meet the overall requirements of railway dispatching modernization and high-quality railway development in the new era, and give full play to its advantages of fast speed, high safety, and high punctuality rate, so as to enhance its competitiveness in passenger travel. Secondly, the rational use of transportation resources has become a key link in the supply-side reform of railway transportation. At the level of transportation organization, transportation costs should be reduced and transportation capacity should be improved. At present, China's high-speed railway generally adopts the operation mode of "one-day one-diagram" to cope with the demand of holiday passenger flow. This operation mode aims to flexibly adjust the train operation plan according to the daily passenger flow characteristics and demand changes, so as to achieve the optimal allocation of high-speed rail transportation resources. By adjusting the train diagram daily, it can better adapt to the peak and fluctuation of passenger flow during the holidays, and ensure the convenience and comfort of passenger travel. Compared with weekdays, the passenger flow during holidays often shows the characteristics of peak concentration, large fluctuation range, and long duration, which brings many challenges to railway transportation scheduling. First of all, the purpose of passengers' travel during the holidays is diversified, involving various types of family visits, tourist vacations, business trips, etc., which leads to large fluctuations in passenger flow and is difficult to accurately predict and control. Secondly, the characteristics of high-speed rail passenger flow during the holidays are different from those on weekdays, which need special consideration. The fluctuation in passenger travel under the background of holidays is more obvious than that under the background of weekdays. There is a peak of passenger flow before holidays or at the beginning of holidays, and the passenger flow is large. After the holidays or at the end of the holidays, the return flow of passengers is obvious, mainly long-distance passenger flow. In addition, there are also great differences in the characteristics of passenger flow during holidays in different regions. Some tourist attractions and popular cities often become the concentration of passenger flow, while other regions may be relatively idle, which puts forward higher requirements for the rational allocation of transportation resources. Therefore, it is of great significance to study the transportation organization during the holidays.

In view of the transportation organization during the holiday period, it is necessary to carry out refined research on the basis of the "one-day one-diagram" transportation organization model. In contrast, the transportation organization considering the coordination of weekdays and holidays is more complex, requiring more flexible strategies and more refined scheduling. In the high-speed railway transportation organization, the realization of fine scheduling can be reflected in the daily passenger flow characteristics or short passenger flow cycle and large changes. In addition, the passenger flow varies greatly in different regions and different time periods, so it is necessary to adopt a differentiated transportation organization strategy to give full play to the benefits of transportation resources and improve the overall operation efficiency of the transportation system. Through the accurate analysis and prediction of daily passenger flow data, combined with the special situation during the holidays, the train operation plan can be adjusted more flexibly to meet the personalized needs of passengers. This refined scheduling model can better cope with the peak and fluctuation of passenger flow during holidays, improve transportation efficiency and service quality, and provide passengers with a more convenient and comfortable travel experience.

At present, train dispatching work and other transportation organization are generally carried out manually by dispatchers, which may lead to problems such as low train efficiency and the quality of technical documents such as train timetables depends on the ability of dispatchers. In order to avoid excessive loss of dispatchers' time and energy and improve passengers' travel satisfaction, this paper aims to develop a method based on reinforcement learning to solve the optimization problem of high-speed railway transportation organization in holiday scenarios.

Driven by the above background, based on the collaborative optimization theory of integrated compilation, this paper establishes an integrated optimization model of train diagrams and EMU routing by constructing time–space–state network and combining the characteristics of high-speed railway transportation organization during holidays. The model takes the minimum total transportation cost as the optimization objective, and comprehensively considers the train stay constraint, the train service uniqueness constraint, the train headway constraint, and the interval operation conflict factors to meet the conditions of EMU operation and maintenance under the premise of ensuring the safe and orderly operation of EMU. A practical case based on a holiday application scenario is designed to verify the applicability of the integrated optimization model of train diagrams and EMU routing.

The novelty of this study is reflected in the following features:

- Integration and synergistic optimization innovation: An integrated optimization method for weekday and holiday operation diagrams and rolling stock interchanges is proposed, which takes into account the mutual influences of the two and optimizes them synchronously, breaking through the limitations of traditional single or hierarchical optimization.
- Fine spatiotemporal–state network modeling: A fine spatiotemporal–state network model is constructed, with precise time discretization, describing each link of train and rolling stock operation in detail with multiple arcs, so as to comprehensively and accurately portray the complex relationship of transportation organization.
- Adaptation strategy for holiday characteristics: The optimization strategy is tailored to the characteristics of holiday passenger flow, such as taking the holiday as a whole in the optimization cycle and flexible maintenance arrangement, which enhances the adaptability of the transport organization to the demand of holidays.
- Practical case-driven method validation: Design cases are based on actual operation data, demonstrating the effectiveness of the model in holiday scenarios through examples, and providing practical examples for the industry.

2. Related Work

2.1. Optimization of Train Diagram Compilation

Existing scholars have conducted extensive research on the optimization of train diagrams. Based on the characteristics of high-speed railway train operation resource occupation, Ref. [1] assigned a value to every train and aimed to maximize the overall value. It constructed a mathematical model of the space–time network resource occupation using the improved Lagrangian relaxation algorithm to simplify the model. Different from the traditional deterministic passenger demand, Ref. [2] proposed a method to design the elastic passenger demand in the train diagram. In Ref. [3], a multibeat train diagram optimization model was proposed, and an optimization method based on the idea of cross entropy designed. The study described in Ref. [4] considered the overtaking strategy of the train in the station, optimized the demand-oriented train schedule, and used the branch and price and cutting algorithm to solve the problem.

In addition, according to the type of optimized train diagram, it can be divided into two aspects: a periodic train diagram and a nonperiodic train diagram. The Periodic Timetabling Problem (PTP) is usually modeled in the form of Periodic Event Scheduling Problem (PESP). To improve the quality of transfer service, Ref. [5] proposed a multiobjective mixed integer programming model for high-speed rail periodic train diagrams by considering transfer frequency, dynamic connection, and transfer time. The authors of Ref. [6] studied the method of reasonably setting the cycle length and cycle level, established an integer linear programming model to solve the train operation plan based on different cycle lengths, and proposed a calculation method for multiple cycle mode evaluation indicators. Based on the operation plan and line conditions of Jingtang Railway and Jingbin Railway, Ref. [7] compiled a periodic operation diagram that standardized the operation of intercity trains and cross-line trains to meet the needs of public transport operation.

In the study of nonperiodic operation diagrams, Ref. [8] introduced the concept of "recovery-to-optimality" into the study in order to minimize the recovery cost caused by the adjustment of the operation diagram when interference occurs. In order to insert additional noncircular train paths into the existing circular train diagram, Ref. [9] proposed a multiobjective model that considers minimizing the travel time of new trains and minimizing the fluctuation of existing trains.

One study [10] discussed the influence of virtual marshalling technology on the transportation organization mode, and put forward the method of compiling balanced train headway diagrams based on virtual marshalling technology. The study reported in Ref. [11] established an optimization model of an energy-saving operation diagram of urban rail transit during an off-peak period considering passenger flow, and designed a genetic algorithm to solve the model. When considering the addition of lines in the diagram adjustment stage and taking the regional intercity high-speed railway network operation as the background, Ref. [12] aimed at minimizing the adjustment range of the existing diagram and maximizing the number of EMU turnover connections. This study further considered the constraints of traffic safety, receiving and departure capacity, EMU turnover connection, etc., and constructed a multiline-diagram collaborative-addition optimization model considering EMU turnover connection. The authors of Ref. [13] analyzed the influence of stop time on the train concession plan and the constraint relationship between station headway. With the goal of minimizing the period in a single-track railway interval operation diagram, the optimization process of a single-track railway interval operation diagram under the different stop times for up and down trains was proposed. Lastly, Ref. [14] constructed a hierarchical space–time network to describe the train operation process at different speeds, and established a linear integer programming model based on space–time arc variables to achieve the optimization goal of minimizing the total train operation cost.

2.2. Optimization of EMU Routing Plan Preparation

In the existing research, the EMU routing plan is usually abstracted as a vehicle routing problem with capacity constraints. The optimization goal is to use fewer EMUs and build a model based on the space–time–state network.

In Ref. [15], a mixed integer programming model is proposed to formulate a rolling stock plan that considers the rolling maintenance of EMUs. In Ref. [16], a 0-1 integer programming model is established to minimize the total connection time of the EMU routing and maximize the cumulative operating mileage before the EMU maintenance. Aiming at the problem that the maintenance capacity of the EMU depot is limited, Ref. [17] established a multiobjective integer programming model considering the constraints of the first-level maintenance capacity of the EMU depot, and took “the minimum number of EMUs” as the main objective and “the least number of maintenance times” as the secondary objective to decompose and solve the problem. Subsequently, Ref. [18] proposed a general mixed integer programming model to optimize the rolling stock plan in order to solve the problem of spatial allocation of EMU depots in the road network. Aiming at the characteristics of high-density operation of intercity railways, Ref. [19] proposed an optimization model of an EMU routing plan based on time-axis network modeling. To solve the problem of multiple types of trains and multiple units of depot operation optimization with maintenance constraints under the background of high-speed railway network operation, Ref. [20] adopted a two-stage optimization decomposition algorithm to establish a relaxation model and a constraint model. The study reported in Ref. [21] proposed arc-based and path-based network flow models when solving the optimization problem of subway train routing plans based on multiple depots. At the same time, flexible train formation modes were considered to match passenger demand that changes with transportation capacity and time. In Ref. [22], an optimization method of an EMU turnover plan suitable for adjusting the train diagram drawing was proposed to solve the problem of uneven passenger seating rate of running lines in each routing.

The report provided in Ref. [23] established an optimization model of the EMU routing plan under the constraint of departure time to avoid long-term stop of some trains due to the influence of maintenance skylights when EMUs are overhauled at night. By defining

EMU routing and taking the minimum number of routes together with the shortest train connection time and the minimum number of rolling stocks as the optimization direction, an optimization model of the EMU routing plan with the minimum number of rolling stocks was constructed by Ref. [24].

2.3. Shortcomings of Existing Research

Throughout the research at home and abroad, many scholars have analyzed and demonstrated the preparation method and theoretical system of train operation plans from different perspectives to provide a reference basis for the preparation of train operation plans for China's high-speed railways. At the same time, extensive research on underfloor turnaround plans have also been conducted, including underfloor turnaround plan preparation theory, optimization methods, model construction, algorithm design, and other aspects, forming a more mature theory. However, there is less research on the synergistic consideration of operation plan preparation and underfloor turnaround. The relevant studies are shown in Table 1.

Table 1. Comparison of selected literature on train diagrams and circulation plans.

References	Model Type *	Objective	Solution Methods	Maintenances of EMUs	Timetable Optimization	Circulation Plan for EMUs
[3]	MILP	Minimize travel time for all trains	Cross-metaheuristic algorithm		√	
[5]	BIP	Maximize direct frequency requirements	CPLEX		√	
[16]	BIP	Minimum connection time and maximum mileage before overhaul	Particle swarm algorithms	√		√
[19]	BIP	Minimum number of EMUs	CPLEX	√		√
[20]	MILP	Minimize the cost of EMU utilization	Two-stage decomposition algorithm + CPLEX	√		√
[17]	BIP	Minimize the number of EMUs operated and maintenance	CPLEX	√		√
[22]	MILP	Minimize the number of EMUs in use and the difference in patronage between operating lines in the interchanges	CPLEX + Hierholzer's algorithms		√	√
[25]	MIP	Minimize total train travel time	Lagrangian heuristic algorithm		√	√
[26]	MILP	Minimize the difference between each train and the average of the actual maximum load factor for all columns of iterative car service	Two-stage iterative solution algorithm		√	√
[27]	BIP	Minimize total operating costs	Lagrange heuristic algorithm		√	√
Our paper	MILP	Minimize transportation costs	Gurobi solution	√	√	√

* BIP: Binary Integer Programming, MILP: Mixed Integer Linear Programming, MINP: Mixed Interger Nonlinear Programming.

From the above summary and analysis of the existing research, we can find that there are shortcomings in the current research, and there is less research that considers in a coordinated manner the overhaul of train sets, the preparation of operation charts, and the turnover of car bottoms. At the same time, there are fewer transport organization optimization methods in the existing research that can be applied to special holiday scenarios.

Whether it is the flexible train operation mode based on different daily passenger demand in foreign countries or the research on the transportation organization mode of “one-day one-diagram” in China, the focus of their attention is to ignore the situation that holidays have very different characteristics from the market demand on weekdays; that is, holidays break the rough division of “daily map” and “peak map” on weekdays according to the characteristics of passenger flow on different days. Therefore, the transportation organization optimization method for holiday transportation demand needs more refined optimization methods.

Secondly, comparing subway and general-speed trains, the turnover problem of their moving trains is not exactly the same as that of high-speed trains. The turnover model of metro trains focuses on the effective evacuation of passenger flow and dynamic adjustment of the running interval during the peak period, while the general-speed train focuses on how to improve the running efficiency and passenger service level under the limited line resources, and the research of cargo trains focuses on the optimization of logistics chain and the reduction in transportation cost. In contrast, the focus of the high-speed railway rolling stock turnover problem is on how to improve the turnover efficiency and operational effectiveness of the vehicles while ensuring high speed, safety, and punctuality. This includes the precise control of train running time, the optimization of the vehicle maintenance cycle, and how to achieve the best utilization of trains through an intelligent dispatching system, highlighting the special requirements of high-speed train sets in the pursuit of a high-speed, high-density, and high-reliability operation mode.

Based on the shortcomings of the current research, this paper starts with the analysis of holiday market demand, discusses the characteristics and influencing factors of holiday demand, and provides guidance for the preparation of train diagrams and EMU routing in holiday scenarios, so as to realize the optimization of transportation plans to meet holiday demand.

3. Collaborative Optimization Model of a Holiday Train Diagram and EMU Routing Plan

3.1. Descripition of the Question

3.1.1. Analysis of Mutual Influence Factors Between Train Diagrams and EMU Routing

In the process of compiling the train diagram and the EMU routing plan, the two will affect each other’s results. The EMU routing is based on the arrangement and combination of the train operation lines drawn on the train diagram, and integrates the information concerning to/from the depots’ location and maintenance of the EMU. The influence of train diagrams on the preparation of EMU routing plans has the following aspects:

(1) The operation diagram has an impact on the number of EMUs.
(2) As shown in Figure 1, there is a double-line interval $A-B$, on which there are trains $G1$ and $G2$; τ_{de}^{k} and τ_{ar}^{k} represent the departure time window and the end time window of train k, respectively, $k \in \{G1, G2\}$. It is assumed that if the preparation of the train diagram does not consider whether it is conducive to the preparation of the EMU routing, there may be the following two scenarios:
(3) Poor train connection will directly lead to the addition of EMUs.
(4) The left side of Figure 1 shows the output of the EMU routing plan. At this time, there are at least two EMU routings. If the operation of the EMU is considered, the running line of the train diagram is slightly adjusted to the right side of Figure 1 within the

time window of departure and arrival. At this time, only one train is needed to complete the transportation task of two trains, which can greatly save on the operating cost of the EMUs.

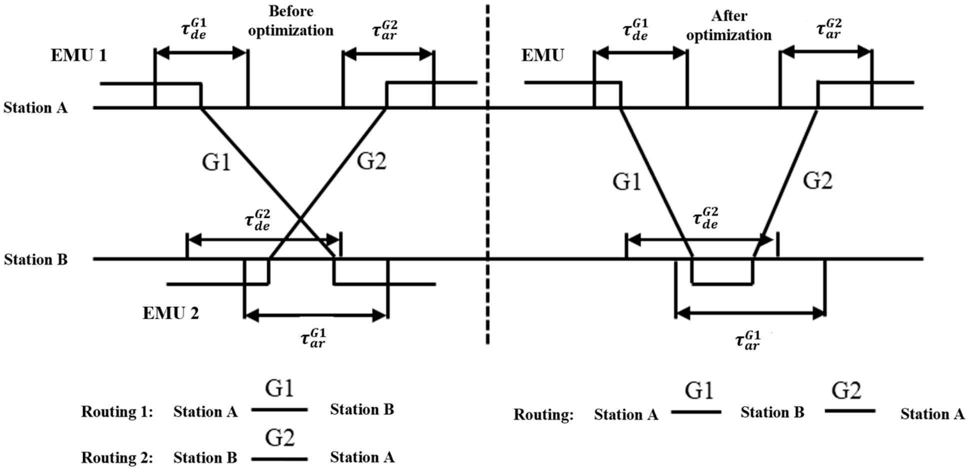

Figure 1. Schematic showing the effect of the train timetable on the EMU circulation.

(5) Poor train connection will affect the robustness of the train diagram and lead to the addition of EMUs.

(6) As shown on the left side of Figure 2, although the connection time of the train in the figure meets the basic arrival–departure headway and the train continuation time standard, because the connection time of the two is too compactly arranged, when the train is delayed and the arrival time of the train $G1$ is delayed, the time of $G1$ connecting $G2$ will not meet the connection time standard. At this time, it is necessary to add a train to perform the transportation task of $G2$, as shown on the right side of Figure 2. Therefore, it is necessary to ensure that the train diagram has sufficient robustness to avoid adverse effects on the EMU operation.

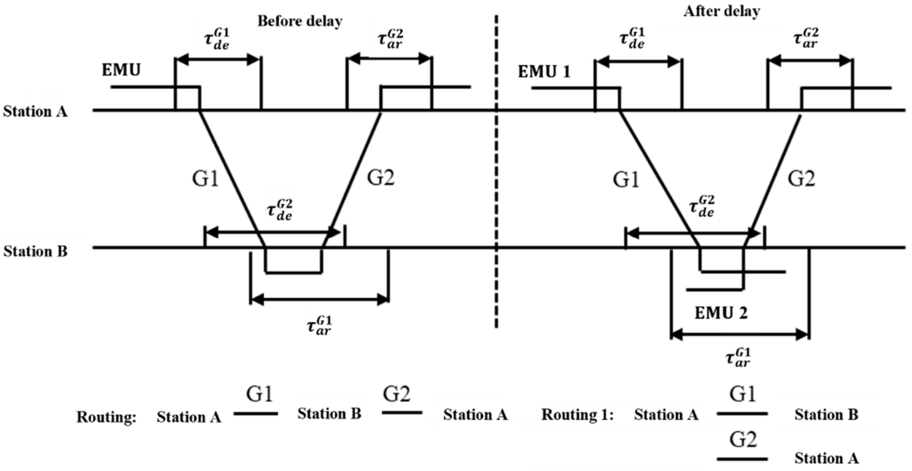

Figure 2. Schematic showing the effect of the train timetable on the EMU circulation in the case of delay.

(7) The train diagram will influence the cost of dispatching empty trains.

If the train diagram is compiled separately, additional empty trains may be generated during the operation stage of the EMU. Taking Figure 1 as an example, assuming that the EMU depot is connected with station A, in the case before optimization, the EMU 2 needs to dispatch the empty train to the station B in the interval $A-B$ and then execute train $G2$. The optimized train diagram avoids the addition of EMUs and eliminates the dispatch of empty trains.

The EMU routing plan also has an impact on the train diagram, which is usually reflected in the feedback adjustment of the train diagram during the drawing process and the adjustment of the train diagram during the operation process. First of all, the preparation of the EMU routing plan will find the abovementioned poor train connection relationship, and then improve the train diagram. Secondly, in the process of operation, it is inevitable to make temporary adjustments to the train diagram. If the route arrangement of the EMU is reasonable, it can effectively alleviate the adverse effects caused by operation accidents such as delayed transmission.

3.1.2. Analysis of Existing Collaborative Optimization Strategies

The existing collaborative optimization strategies generally start from two aspects: hierarchical compilation and integrated compilation. The detailed introduction of the two is as follows:

(1) Layered compilation

Hierarchical compilation refers to a collaborative optimization method that independently compiles a train diagram and then adjusts it according to the feedback of the EMU routing plan. The specific process is to first compile the train diagram without considering whether the train connection conditions are favorable, then compile the EMU routing plan according to the train diagram, calculate the evaluation index of EMU operation, and finally adjust and optimize the train diagram according to the feedback. The optimization method using hierarchical compilation often requires multistep iteration to achieve the global optimum.

(2) Integrated compilation

Integrated preparation refers to a collaborative optimization method that takes the train diagram and the EMU routing as an optimized object, which ensures the rationality of the EMU routing design while optimizing the train operation line. The integrated compilation method needs to comprehensively consider the compilation characteristics and influencing factors of the train diagram and the EMU routing, which can effectively avoid the need to recalculate another tedious iterative process due to the change in the train diagram or EMU routing plan.

In the application scenario of holidays, in the face of rapid distribution of large-scale transportation demand, EMU resources are often in short supply. At this time, if the train diagram is compiled independently and then the EMU routing is compiled for feedback adjustment, a large amount of transportation resources will be wasted, and it is difficult to achieve rapid response and efficient operation. Therefore, it is of great significance to study the collaborative optimization method of the integrated train diagram and EMU routing plan in the context of holidays.

3.2. Modeling Ideas and Assumptions

3.2.1. Modeling Ideas

To abstract this problem into a mathematical problem, a space–time–state network is constructed. The train is abstracted as a point on the space–time–state network to find the shortest space–time–state path that minimizes transportation costs. This problem is essentially a multicommodity flow problem with additional constraints. At the same time, in order to achieve these functions, the model should consider the following issues:

(1) Solving problems in spatiotemporal–state networks

Taking the given train operation plan and EMU resource allocation as the input for the model, the collaborative optimization model will establish a space–time–state network with known conditions, and realize the optimization of the space–time–state path of the EMU through the combination optimization of points and arcs.

(2) Taking the overall duration of the holidays as an optimization cycle

There is a large fluctuation in the operating market during the holidays. To realize the rational allocation of EMU resources and avoid additional empty train allocation, the overall duration of the holidays is selected as the optimization cycle of the model. For example, in the integrated optimization of a three-day long holiday, taking the three-day overall train operation plan as the optimization object is conducive to the continuation of the EMU within three days, avoiding the waste of transportation resources caused by additional empty trains.

(3) EMU flexible maintenance

To improve the robustness of EMU utilization and realize flexible maintenance of EMU, the EMU is not limited to maintenance in this depot during the holidays, and the restrictions on the maintenance location are only within one day before and after the holiday.

3.2.2. Model Assumptions

To facilitate the construction of the model, this paper makes the following assumptions based on the road network environment:

- All lines are double-line lines with sufficient capacity;
- The EMU resources in the EMU depot of the road network are sufficient;
- The train operation plan is known;
- The EMU adopts the operation mode of a nonfixed depot;
- Allow the EMU to carry out maintenance operations in the nonaffiliated depot, and the maintenance capacity of the EMU depot is sufficient;
- Do not consider the distance and time to and from the EMU depot;
- Only consider the first-level preventive maintenance of the EMU.

3.3. Model Construction

3.3.1. Symbol Definition

All parameters involved in this study are shown in Table 2, including the set and parameters of the road network, train, EMU and space–time–state network. The detailed definition of each symbol is as follows:

Table 2. Symbol description of the collaborative optimization model.

Data Type	Symbol	Definition	Unit	Dimension
Sets and parameters of the road network	S	Station set, $s_1, s_2 \in S$	-	-
	D	Set of stations connected to the high-speed train section, $d \in D, D \subset S$	-	-
	I	Interval set， $(s_1, s_2) \in I$	-	-
	T	Discrete time set of optimization period， $t_1, t_2 \in T$	-	-
	h_s^{ar}	Minimum arrival headway of station s	minute	$\lvert S\rvert \times 1$
	h_s^{de}	Minimum departure headway of station s	minute	$\lvert S\rvert \times 1$
	$l_{(s_1,s_2)}$	Mileage of interval (s_1, s_2)	kilometer	$\lvert I\rvert \times 1$
Sets and parameters of the train	K	Train set， $K = G \cup M$	-	-
	G	Single group train， $g \in G, G \subset K$	-	-
	M	Multigroup train $m \in M, M \subset K$	-	-
	E^k	Set of available EMUs for train k	-	-

	o^k	Departure station of train k , $o_k \in S$	-	$\lvert K\rvert \times 1$
	d^k	Arrival station of train $k, d_k \in S$	-	$\lvert K\rvert \times 1$
	τ_{de}^k	Departure time window of train $k, \tau_{de}^k = [t_1, t_2] \subset T$	-	$\lvert K\rvert \times 1$
	τ_{ar}^k	Arrival time window of train $k, \tau_{ar}^k = [t_1', t_2'] \subset T$	-	$\lvert K\rvert \times 1$
	n^m	Group number of multigroup train m	-	$\lvert M\rvert \times 1$
	$\gamma_{(s_1,s_2)}^k$	Minimum running time of train k in interval(s_1, s_2)	minute	$\lvert K\rvert \times \lvert I\rvert$
	S_{inter}^k	Set of intermediate stations where train k needs to stop	-	-
	ω_s^k	Minimum stopping time of train k at station s	minute	$\lvert K\rvert \times \lvert S\rvert$
	Θ^k	Set of EMUs that can be used to execute a train k	-	-
	η^+	Time required for passengers to board the bus	minute	1×1
	η^-	Time required for passengers to exit the train and simple preparation of the train	minute	1×1
Sets and parameters of EMU	E	Set of EMUs, $e \in E$	-	-
	K^e	Set of trains EMU e can bear.	-	-
	d^e	Auxiliary station of EMU, $d^e \in S$	-	$\lvert E\rvert \times 1$
	T^e	Available time range of EMU, $T^e \subset T$	-	$\lvert E\rvert \times 1$
	$\gamma_{(s_1,s_2)}^e$	Minimum empty running time of EMU e in the interval(s_1, s_2)	minute	$\lvert E\rvert \times \lvert I\rvert$
	cr^e	Unit operating cost of EMU e	CNY	$\lvert E\rvert \times 1$
	cs^e	Unit stay cost of EMU e	CNY	$\lvert E\rvert \times 1$
	cf^e	Fixed cost of EMU e executing train k	CNY	$\lvert E\rvert \times \lvert K\rvert$
	μ^+	Time required for a combination of EMUs	minute	1×1
	μ^-	Time required for one disassembly of the EMUs	minute	1×1
	T^1	First-level preventive maintenance time standard of EMUs	minute	1×1
	L^1	First-level preventive Maintenance mileage standard of EMU	kilometer	1×1
	τ_{ma}^e	Minimum duration of the first-level preventive maintenance of the EMU e	minute	$\lvert E\rvert \times 1$
	c_{ma}^e	Fixed cost of the first-level preventive maintenance of the EMU e	CNY	$\lvert E\rvert \times 1$
Sets and parameters of space–time–state network	N	Set of nodes, $(k, s, t) \in N$	-	-
	A	Set of arcs, $a_{(n_1,n_2)} = ((k_1, s_1, t_1), (k_2, s_2, t_2)) \in A$	-	-
	G	Space–time–state network, $G = (N, A)$	-	-

	$\xi^e_{(n_1,n_2)}$	Unit cost coefficient for an EMU e executing arc $a_{(n_1,n_2)}$	-	$\vert E\vert \times \vert A\vert$
	$\tau^e_{(n_1,n_2)}$	Duration for an EMU e executing arc arc $a_{(n_1,n_2)}$	minute	$\vert E\vert \times \vert A\vert$
	$\lambda^e_{(n_1,n_2)}$	Working distance for an EMU e executing arc arc $a_{(n_1,n_2)}$ $a_{(n_1,n_2)}$	kilometer	$\vert E\vert \times \vert A\vert$

3.3.2. Space–Time–State Network Construction

The space–time–state network $G = (N, A)$ contains the set N of points, and the connection between points constitutes the set A of edges. In the network, the train information is only the number of trains performed by the EMU.

In the time dimension, the time unit of the space–time–state network is accurate to 1 "min", and the optimization period is discretized. In the spatial dimension, the station is composed of stations specified by the stop plan. For any station $s \in S$, the spatial information on the station is divided into two spatial positions: starting $p'(s)$ and arriving $p(s)$. The status information reflects whether the EMU undertakes the train or undertakes the number of trains. Nodes can be divided into the following two types:

(1) Virtual starting point and virtual end point

When the EMU undertakes the transportation task, each EMU must start from the virtual starting point and undertake the starting depot of the EMU. Since the time of the EMU to/from the depot is not counted, the EMU depot and the station connected to the EMU depot can be regarded as one. When the transportation task is completed, the EMU returns from the terminal to the attached EMU depot, and then enters the virtual terminal to end the transportation task.

(2) Departure node and arrival node

For any EMU at any station, there are two states of arrival and departure, corresponding to the departure and arrival nodes of the EMU in this state.

Arc a is composed of connections between points. According to the function of the arc, it can be divided into the following types:

(1) Start arc $a_o \in A_o$

For any EMU $e \in E$, at any time $t = 0,1,2,3\ldots T$, if $t \in T^e$, the EMU starts from the virtual starting point, which means that the EMU has been started. This process is the process of specifying the arrival–departure line from the exit depot of the EMU to the departure station, so there is a starting arc: $(o,(0,p(d^e),t))$. The start-up arc cost $\xi^e_{(o,(0,p(d^e),t))} = 0$, the consumption time $\tau^e_{(o,(0,p(d^e),t))} = 0$, and the operating mileage $\lambda^e_{(o,(0,p(d^e),t))} = 0$.

(2) End arc $a_d \in A_d$

For any EMU $e \in E$, at any time $t = 0,1,2,3\ldots T$, if $t \in T^e$, after the EMU completes a cycle of the task of undertaking the train, the EMU will return from the terminal to the attached EMU depot, and the transportation task will end at the virtual end point, so there is an end arc: $((0,p'(d^e),t),d)$. End arc cost $\xi^e_{((0,p'(d^e),t),d)} = 0$, consumption time $\tau^e_{((0,p'(d^e),t),d)} = 0$, operation mileage $\lambda^e_{((0,p'(d^e),t),d)} = 0$.

(3) Preparing arc $a_{pr} \in A_{pr}$

The preparation arc can undertake the starting arc, so that the EMU can undertake the preparation work of the train operation line. For any EMU $e \in E$, if $t_1, t_2 \in T^e$, $t_2 = t_1 + (n^m - 1)\cdot\mu^+ + \eta^+$, there is a train $k \in K$ and $e \in \Theta^k$ that can be assumed. The EMU waits for the train k on the arrival - departure line of its originating station. In this process, if the train is a multigroup train $m \in M$, it needs to wait for multiple sets of EMUs to be connected. The passenger boarding time should be reserved before departure, so there is a preparation arc: $((0,p'(d^e),t_1),(k,p(o^k),t_2))$, where $d^e = o^k$. The application cost of the prepared arc is $\xi^e_{((0,p'(d^e),t_1),(k,p(o^k),t_2))} = cf^e + cr^e \cdot (n^m - 1) \cdot \mu^+ + cs^e \cdot \eta^+$, and the time consumed by the prepared arc is $\tau^e_{((0,p'(d^e),t_1),(k,p(o^k),t_2))} = (n^m - 1) \cdot \mu^+ + \eta^+$, The mileage $\lambda^e_{((0,p'(d^e),t_1),(k,p(o^k),t_2))} = 0$. In addition, there will also be a situation when the EMU is overhauled in the nonaffiliated depot or stays at night in the depot. When the EMU is reloaded after the completion of the train, there is a preparation arc: $((0, d^e, t_1), (k, d^e, t_2))$. There is a situation that the EMU continues to work at the next train departure station after being allocated at the same station or through the empty train. For the train k to be borne by the EMU, there is a preparation arc: $((0, o^k, t_1), (k, o^k, t_2))$. The time, distance, and operation cost of the above preparation arc are the same.

(1) Finish arc $a_{ac} \in A_{ac}$

For any EMU $e \in E$, if $t_1, t_2 \in T^e$, $t_2 = t_1 + (n^m - 1) \cdot \mu^- + \eta^-$, there is an affordable train $k \in K$ and $e \in \Theta^k$. After the EMU completes the train k, it needs to wait for passengers to exit. If the train is a multigroup train $m \in M$ it needs to be disassembled, so there is a completion arc: $((k, p'(d^e), t_1), (0, p(d^k), t_2))$, where the EMU depot and the terminal station of the train k are the same $d^e = d^k$. Then, there is the operating cost of completing the arc $\xi^e_{((k,p'(d^e),t_1),(0,p(d^k),t_2))} = cr^e \cdot (n^m - 1) \cdot \mu^- + cs^e \cdot \eta^-$, the time consumed to complete the arc $\tau^e_{((k,p'(d^e),t_1),(0,p(d^k),t_2))} = (n^m - 1) \cdot \mu^- + \eta^-$, and the running mileage of completing the arc $\lambda^e_{((k,p'(d^e),t_1),(0,p(d^k),t_2))} = 0$. In addition, when the EMU needs to complete the maintenance operation in the nonaffiliated depot, there is also a completion arc: $((k, p'(d^e), t_1), (0, p(d^e), t_2))$, the time, distance, and cost of the above completion arc are the same.

(2) Running arc $a_{op} \in A_{op}$

For any EMU $e \in E$, if $t_1, t_2 \in T^e$, $t_2 = t_1 + max\{\gamma^k_{(s_1,s_2)}, \gamma^e_{(s_1,s_2)}\}$, there is a affordable train $k \in K$ and $e \in \Theta^k$, interval (s_1, s_2) in the running path of train k, then there is a running arc: $((k, p'(s_1), t_1), (k, p(s_2), t_2))$. If it is at the starting station o^k of the train k, that is, $s_1 = o^k$, if $t_1 \in T^e$, then $\xi^e_{((k,p'(o^k),t_1),(k,p(s_2),t_2))} = cr^e \cdot (t_2 - t_1)$, $\tau^e_{((k,p'(o^k),t_1),(k,p(s_2),t_2))} = t_2 - t_1$, $\lambda^e_{((k,p'(o^k),t_1),(k,p(s_2),t_2))} = l_{(o^k,s_2)}$;If $s_2 = d^k$, $t_2 \in T^e$, then $\xi^e_{((k,p'(s_1),t_1),(k,p(d^k),t_2))} = cr^e \cdot (t_2 - t_1)$, $\tau^e_{((k,p'(s_1),t_1),(k,p(d^k),t_2))} = t_2 - t_1$, $\lambda^e_{((k,p'(s_1),t_1),(k,p(d^k),t_2))} = l_{(s_1,d^k)}$; the running arc exists in the intermediate station interval of the train k, then $\xi^e_{((k,p'(s_1),t_1),(k,p(s_2),t_2))} = cr^e \cdot (t_2 - t_1)$, $\tau^e_{((k,p'(s_1),t_1),(k,p(s_2),t_2))} = t_2 - t_1$, $\lambda^e_{((k,p'(s_1),t_1),(k,p(s_2),t_2))} = l_{(s_1,s_2)}$.

(1) The minimum dwell arc $a_{st} \in A_{st}$

For any EMU $e \in E$, if $t_1, t_2 \in T^e$, $t_2 = t_1 + \omega^k_s$, there is a bearable train $k \in K$ and $e \in \Theta^k$, and the station s is in the running path of the train k, then there is a minimum dwell arc: $((k, p(s), t_1), (k, p'(s), t_2))$, then $\xi^e_{((k,p(s),t_1),(k,p'(s),t_2))} = cs^e \cdot (t_2 - t_1)$, $\tau^e_{((k,p(s),t_1),(k,p'(s),t_2))} = t_2 - t_1$, $\lambda^e_{((k,p(s),t_1),(k,p'(s),t_2))} = 0$.

(2) Empty train waiting arc $a_{wa} \in A_{wa}$

For any EMU $e \in E$, if $t_1, t_2 \in T^e$, $t_1 \leq t_2$, the train that has completed the previous arc and if it has the ability to bear the operation task of the next train, it needs to wait at the station in the state of empty train. If there is an affordable train $k \in K$ and $e \in \Theta^k$, then there is an empty train waiting arc: $((0, p'(s), t_1), (0, p'(s), t_2))$, then $\xi^e_{((0,p'(s),t_1),(0,p'(s),t_2))} = cs^e \cdot (t_2 - t_1)$. $\tau^e_{((0,p'(s),t_1),(0,p'(s),t_2))} = t_2 - t_1$, $\lambda^e_{((0,s,t_1),(0,s,t_2))} = 0$.

(1) Empty train dispatch arc $a_{al} \in A_{al}$

For any $e \in E$, there may be dispatch of empty train. If $t_1, t_2 \in T^e$, $t_2 = t_1 + \gamma^e_{(s_1,s_2)}$, there is an empty train dispatch arc: $((0, p'(s_1), t_1), (0, p'(s_2), t_2))$, then $\xi^e_{((0,p'(s_1),t_1),(0,p'(s_2),t_2))} = cr^e \cdot (t_2 - t_1)$. $\tau^e_{((0,p'(s_1),t_1),(0,p'(s_2),t_2))} = t_2 - t_1$, $\lambda^e_{((0,p'(s_1),t_1),(0,p'(s_2),t_2))} = l_{(s_1,s_2)}$.

(2) Overhaul arc $a_{main} \in A_{main}$

For any EMU $e \in E$, when the maintenance requirements are met, the maintenance should be carried out, if $t_1, t_2 \in T^e$, $t_2 = t_1 + \tau^e_{ma}$. When the EMU completes the transportation task of train k at the station where the EMU depot is located, the EMU can be overhauled at the station. There is an overhaul arc: $((0, p(d^e), t_1), (0, p'(d^e), t_2))$, then $\xi^e_{((0,p(d^e),t_1),(0,p\prime(d^e),t_2))} = c^e_{ma} + cs^e \cdot (t_2 - t_1)$, $\tau^e_{((0,p(d^e),t_1),(0,p\prime(d^e),t_2))} = 0$. $\lambda^e_{((0,p(d^e),t_1),(0,p'(d^e),t_2))} = 0$.

3.3.3. Construction of the Collaborative Optimization Model

(1) Decision variables

The decision variable of the model is set to $x^e_{(n_1,n_2)}$. $x^e_{(n_1,n_2)} = 1$ represents the arc $a = (n_1, n_2)$ on the time-space-state path of EMU e. In other cases, $x^e_{(n_1,n_2)} = 0$.

(2) Objective function

The minimum total transportation cost of the used arc is taken as the optimization objective.

$$Z = Min \sum_{e \in E} \sum_{a \in A} \xi^e_a \cdot x^e_a \tag{1}$$

(3) Constraint conditions

(1) Flow balance constraint

To ensure that one train undertakes at most one train, it is necessary to limit the arc executed by the train to be unique.

$$\sum_{\substack{n_2 \neq o, \\ n_2 \in N}} x^e_{(o,n_2)} = 1 \qquad \forall e \in E \tag{2}$$

$$\sum_{\substack{n_1 \neq d, \\ n_1 \in N}} x^e_{(n_1,d)} = 1 \qquad \forall e \in E \tag{3}$$

$$\sum_{\substack{n_1 \in N, \\ n_1 \neq n_2}} x^e_{(n_1,n_2)} = \sum_{\substack{n_3 \in N, \\ n_2 \neq n_3}} x^e_{(n_2,n_3)} \qquad \forall e \in E, n_2 \notin \{o, d\} \tag{4}$$

Equations (2) and (3), respectively, indicate that the starting arc and the ending arc are unique. For any EMU $e \in E$, there must be only one starting and ending arc. Equation (4) is the flow balance constraint. For any EMU $e \in E$, the intermediate nodes in its path need to be in and out.

(2) Train stopping constraints

When the EMU executes the train, it is necessary to strictly follow the stop requirements of the operation plan and meet the minimum stop time standard of the train.

$$\sum_{t_1 \in T} \sum_{\substack{t_2 \in T, \\ t_2 > t_1 + \omega^k_s}} x^e_{((k,p(s),t_1),(k,p'(s),t_2))} = 1 \qquad \forall k \in K^e, s \in S^k_{inter} \tag{5}$$

In the formula, for any EMU $e \in E$, if there is a train $k \in K^e$, the EMU needs to stop at the intermediate station of the running path of the train k, and the stop time is greater than the minimum stop time.

(3) Uniqueness constraint of train service

Whether it is a single-group train or a virtual train that constitutes a multigroup train, it should be borne by at most one EMU. Ensure that there is only one EMU at the starting station of the running line, either from the station itself or from the EMU depot access.

$$\sum_{e\in E^k}\sum_{a\in\Phi_k^e} x^e_{((0,p(o^k),t_1),(k,p'(o^k),t_2))} = 1 \qquad \forall e\in E^k \tag{6}$$

In the formula, the set of connecting arcs Φ_k^e contains all the arcs on the e-path of the EMU e that can be connected with the train k at the same station, which may include the exit arc and the connection arc, and o^k denotes the starting station of the train k.

(4) Train headway constraint

For a station on the road network, continuous arrival and departure need a certain headway between arrival and departure to ensure the safety of train operation. For a running arc or an empty train dispatching arc, it is necessary to ensure the minimum train headway between departure and arrival at both ends.

$$\sum_{e\in E^k}\sum_{a\in\Psi_a^k} x^e_{((k,p'(s_1),t_1),(k,p(s_2),t_2))} = 1 \qquad \begin{array}{r}\forall s_1,s_2\in S, s_1\neq s_2\\ \forall k\in K\cup\{0\}, a\in A_{op}\\ \cup A_{al}\end{array} \tag{7}$$

In the formula, for any train $k\in K$ or any empty EMU $k=0$, any interval running arc $a\in A_{op}$ or empty running arc $a\in A_{al}$ on the running path, there are running arcs or empty running arcs of other trains or empty EMUs in the same interval. If the two do not meet the train headway standard, they are classified as the conflict arc set Ψ_a^k of arc a. $\Psi_a^k=\{((k,s_1,t_3),(k,s_2,t_4))|-h_{s_1}^{de}\le t_1-t_3\le h_{s_1}^{de}, -h_{s_2}^{ar}\le t_2-t_4\le h_{s_2}^{ar}\}$ denotes the set of conflicting arcs in the running arc $a_{op}=((k,p'(s_1),t_1),(k,p(s_2),t_2))$ or the empty running arc $a_{al}=((0,p'(s_1),t_1),(0,p(s_2),t_2))$ of a train k or an EMU e in the interval (s_1,s_2), the minimum departure headway $h_{s_1}^{de}$ adjacent to the departure time t_1 of the station s_1 and the minimum arrival headway $h_{s_2}^{ar}$adjacent to the arrival time t_2 of the station s_2.

(5) Interval operation conflict constraint

Since the railway interval cannot be directly overtaking, the train operation conflict caused by the interval overtaking should be avoided.

$$\begin{array}{l}\displaystyle\sum_{e\in E}\sum_{k\in K\cup\{0\}} x^e_{((k,p'(s_1),t_1),(k,p(s_2),t_4))} + \\ \displaystyle\sum_{e\in E}\sum_{k\in K\cup\{0\}} x^e_{((k,p'(s_1),t_2),(k,p(s_2),t_3))} = 1\end{array} \qquad \begin{array}{r}\forall s_1,s_2\in S, s_1\neq s_2,\\ 0<t_1<t_2<t_3<t_4\\ <T\end{array} \tag{8}$$

In the formula, for any interval (s_1,s_2), there is at most one running arc or empty running arc in the time range $[t_1,t_4]$.

(6) First-level preventive maintenance time constraints

When the cumulative running time of the EMU reaches the first-level preventive maintenance time limit, the EMU needs to be maintained in the depot.

$$t_{n_1}^e\le(1+\alpha)T^1 \qquad \forall e\in E, n_1\in N \tag{9}$$

$$0\le t_{n_2}^e\le M(1-x_{n_1,n_2}^e) \qquad \begin{array}{l}\forall e\in E,(n_1,n_2)\\ \in A_{main}\end{array} \tag{10}$$

$$t_{n_2}^e+M(1-x_{n_1,n_2}^e)\ge t_{n_1}^e+\tau_{n_1,n_2}^e \qquad \begin{array}{l}\forall e\in E,(n_1,n_2)\\ \notin A_{main}\end{array} \tag{11}$$

$$t_{n_2}^e+M(1-x_{n_1,n_2}^e)\le t_{n_1}^e+\tau_{n_1,n_2}^e \qquad \begin{array}{l}\forall e\in E,(n_1,n_2)\\ \notin A_{main}\end{array} \tag{12}$$

In the formula, M represents a very large positive number, and α represents the positive fluctuation range of the cumulative time in the first-level preventive maintenance. For any arc (n_1, n_2), Formula (9) stipulates that the cumulative running time of the EMU shall not exceed the first-level preventive maintenance time limit of the EMU, and $t^e_{n_1}$ represents the cumulative running time of the EMU e reaching the node n_1. When any arc $(n_1, n_2) \in A_{main}$, Formula (10) indicates that the cumulative running time of the EMU is cleared after maintenance. When any arc $(n_1, n_2) \notin A_{main}$, Formulas (11) and (12) will accumulate arc (n_1, n_2) consumption time $\tau^e_{n_1,n_2}$; otherwise, the inequality holds.

(7) First-level preventive maintenance mileage constraint

When the cumulative operating mileage of the EMU reaches the limit of the first-level preventive maintenance mileage, the EMU needs to be maintained in the depot.

$$m^e_{n_1} \le (1+\beta)T^1 \qquad \forall e \in E, n_1 \in N \tag{13}$$

$$0 \le m^e_{n_2} \le M(1 - x^e_{n_1,n_2}) \qquad \forall e \in E, (n_1, n_2) \in A_{main} \tag{14}$$

$$m^e_{n_2} + M(1 - x^e_{n_1,n_2}) \ge m^e_{n_1} + \nu^e_{n_1,n_2} \qquad \forall e \in E, (n_1, n_2) \notin A_{main} \tag{15}$$

$$m^e_{n_2} + M(1 - x^e_{n_1,n_2}) \le m^e_{n_1} + \nu^e_{n_1,n_2} \qquad \forall e \in E, (n_1, n_2) \notin A_{main} \tag{16}$$

In the formula, M represents a very large positive number, and β represents the positive fluctuation range of the accumulated mileage in the first-level preventive maintenance. For any arc (n_1, n_2), Formula (13) stipulates that the cumulative operating mileage of the EMU shall not exceed the time limit for the first-level preventive maintenance of the EMU. $m^e_{n_1}$ represents the cumulative operating mileage of the EMU e to the node n_1. When any arc $(n_1, n_2) \in A_{main}$, Formula (14) indicates that the accumulated running mileage of the EMU is cleared after maintenance. When any arc $(n_1, n_2) \notin A_{main}$, Formulas (15) and (16) will accumulate the mileage $\nu^e_{n_1,n_2}$ of the arc (n_1, n_2), otherwise, the inequality is always true.

(8) Continuation time constraint

For any EMU $e \in E$, if it does not meet the first-level preventive maintenance standard after completing the operation line of the previous train, it is possible to continue the next train at this station or after the dispatch of an empty train. At this time, the EMU needs to meet the minimum connection time standard in the process of continuing the train. This process is called normal continuation operation in this paper. In addition, if the first-level preventive maintenance standard is reached, the connection time of the two trains before and after maintenance needs to meet the minimum connection time standard. This process is called maintenance connection operation in this paper. Normal connection and maintenance connection need to meet different connection time:

$$T_{con} \cdot x^e_a \le \tau^e_a \cdot x^e_a \qquad \forall e \in E, a \in A_{pr} \cup A_{ac} \cup A_{al} \cup A_{wa} \tag{17}$$

$$T_{ma} \cdot x^e_a \le \tau^e_a \cdot x^e_a \qquad \forall e \in E, a \in A_{pr} \cup A_{ac} \cup A_{al} \cup A_{wa} \cup A_{main} \tag{18}$$

Formula (17) represents the minimum continuation time constraint under normal continuation. Equation (18) specifies the shortest connection time under maintenance conditions. In the case of allowing the dispatch of an empty train, T_{con}, T_{ma} represent the minimum time allowed for normal continuous operation and maintenance continuous operation under the condition of allowing an empty train dispatch. The values are divided into the following two cases:

$$T_{con}=\begin{cases}T_{fs}^{con}+\tau^{e}_{(k,p'(s_1),t_1),(k,p(s_2),t_2)} & s_1\neq s_2, a\in A_{al}\\ T_{fs}^{con} & s_1=s_2\end{cases} \tag{19}$$

$$T_{ma}=\begin{cases}T_{fs}^{ma}+\tau^{e}_{(k,p'(s_1),t_1),(k,p(s_2),t_2)} & s_1,s_2\in D, s_1\neq s_2, a\in A_{al}\\ T_{fs}^{ma} & s_1=s_2\end{cases} \tag{20}$$

Formula (19) represents the two cases of EMU under normal connection conditions. T_{fs}^{con} denotes the immediate turn-back time standard at the station for the connection from the last train to the next train. In general, $T_{fs}^{con}=(n^m-1)\cdot\mu^{+}+\eta^{+}+(n^m-1)\cdot\mu^{-}+\eta^{-}$. If the EMU needs to carry out maintenance and connection operation, Formula (20) stipulates its minimum connection time standard. T_{fs}^{ma} denotes the time standard for on-station maintenance operation and turn-back from the last train to the next starting train. In general, $T_{fs}^{ma}=(n^m-1)\cdot\mu^{+}+\eta^{+}+(n^m-1)\cdot\mu^{-}+\eta^{-}+\tau^{e}_{ma}$. The first case is that when the stations at the two ends of the arc are not the same, the empty train dispatch needs to be considered. In the second case, there is no need for empty train dispatch, and the value is equal to the immediate return time standard.

(9) Decision variable constraints

The decision variable is a 0-1 variable.

$$x^{e}_{(n_1,n_2)}=\begin{cases}1 & EMU\ e\ serves\ as\ the\ operation\ of\ arc\ a\\ 0 & other\end{cases} \tag{21}$$

In the formula, when $x^{e}_{(n_1,n_2)}=1$, the EMU e incorporates the arc $a=(n_1,n_2)$ into its own operation path. In other cases, $x^{e}_{(n_1,n_2)}=0$.

4. Case Analysis

4.1. Case Setting and Solution

To test the applicability of the collaborative optimization method proposed in this paper for the 5-day holiday scenario, this section describes a case design based on the market demand characteristics of the 5-day holiday summarized in Section 2. The travel demand during the holiday can be summarized as the characteristics of medium and long distance, and the peak appears at both ends of the holiday. Therefore, this section is based on a high-speed railway in actual operation.

The station, EMU depot configuration, and interval information of a double-track high-speed railway line in actual operation are shown in Figure 3. There are six stations on the line, and the set of stations is $S=\{s_1,s_2,s_3,s_4,s_5,s_6\}$. There are five EMU depots in the line, and the set of EMU depots is $D=\{d_1,d_2,d_3,d_4,d_5\}$. Among them, the EMU depot d_1 is connected to the station s_1, the EMU depot d_2 is connected to the station s_3, the EMU depot d_3 is connected to the station s_4, the EMU depot d_4 is connected to the station s_5, and the EMU depot d_5 is connected to the station s_6.

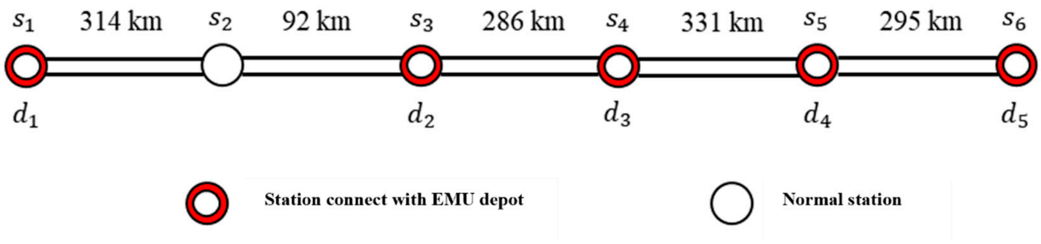

Figure 3. Schematic of the railway line.

This case assumes that the holiday lasts for 5 days. To implement the idea of smooth connection of EMUs, the optimization cycle is set to 7 days, including 1 day before the holiday, 5 days during the holiday, and 1 day after the holiday. Combined with the change rule in holiday passenger flow summarized in Section 2, the two days before and after the holiday adopt the operation plan similar to the "weekend map" in the "on-day one-diagram" mode, and the "peak map" is implemented during the holiday period, and the operation line is added and subtracted according to the characteristics of passenger flow.

The existing train operation on this line at 2 days before and after a holiday and 5 consecutive days during the period is described as follows. The operation mode of "peak map" is implemented in the two days before and after the holiday, and 30 pairs of trains are run every day. Since the first day and the last day of the holiday are the peak period of passenger travel and return, the train operation plan during the holiday period adds "peak operation line" on the basis of "peak map", and an additional 8 pairs of trains are operated to carry the surge of passenger flow during the holiday period as much as possible.

The interval length and interval operation time of the line are shown in Table 3. It is assumed that all the trains involved above are operated in the form of eight carriages. The parameters of train operation and EMU configuration on the line in the case are shown in Table 4. The starting additional time of the train is set to 2 min, and the stopping additional time is set to 3 min; the departure time window and arrival time window are set to 20 min.

Table 3. Line interval distance and pure running time.

Interval	Length of Interval (km)	Operation Time of Interval (min)
$s_1 \rightarrow s_2$	314	86
$s_2 \rightarrow s_3$	92	25
$s_3 \rightarrow s_4$	286	78
$s_4 \rightarrow s_5$	331	90
$s_5 \rightarrow s_6$	295	80

Table 4. Train and EMU-related parameters.

Parameter	Symbol	Taking Value
Minimum dwell time of train	ω_s^k	2 min
Minimum arrival headway of the station	h_s^{ar}	5 min
Minimum departure headway of the station	h_s^{de}	5 min
Shortest duration of the first-level preventive maintenance	τ_{ma}^e	240 min
Time standard of the first-level preventive maintenance	T^1	2880 min
Mileage standard of the first-level preventive maintenance	L^1	4000 km
Fixed cost of the first-level preventive per maintenance	c_{ma}^e	200 CNY
Time required for passengers to board the bus	η^+	5 min
Time required for passengers to exit the train and simple preparation of the train	η^-	6 min
Unit operating cost of EMU	cr^e	1 CNY/min
Unit stay cost of EMU	cs^e	1 CNY /min
Fixed cost of EMU train operation	cf^e	10 CNY

4.2. Analysis of Experimental Results

4.2.1. Calculation Results and Analysis of the Operation Diagram

To intuitively show the composition of the train diagram before and after the holiday and the train diagram during the holiday, the train diagram of the first day before the holiday and the first day of the holiday is drawn, as shown in Figures 4 and 5.

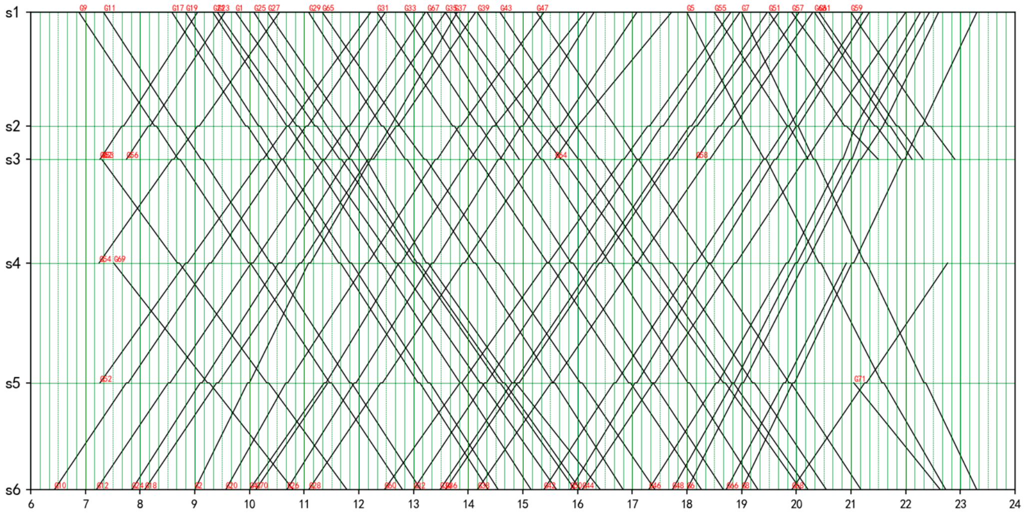

Figure 4. Schematic diagram of the train timetable on the first day before the holiday.

The two days before and after the holiday are the "weekend map", and 30 pairs of trains are drawn; the "peak map" was implemented during the holidays, and 35 pairs of trains were operated. From Figures 4 and 5, it can be seen that the drawing of the train diagram satisfies the constraints of the road network environment on the headway and overtaking conditions. In addition, the "peak map" is more compact than the "weekend map", and the running line is more appropriate, which can better meet the market travel demand of holiday transportation. It can be seen that during the holiday period, medium- and long-distance trains account for a large proportion of the overall trains. The train operation lines drawn in the figure meet the standards of arrival–departure headway and overtaking conditions stipulated by the road network, and there is no conflict.

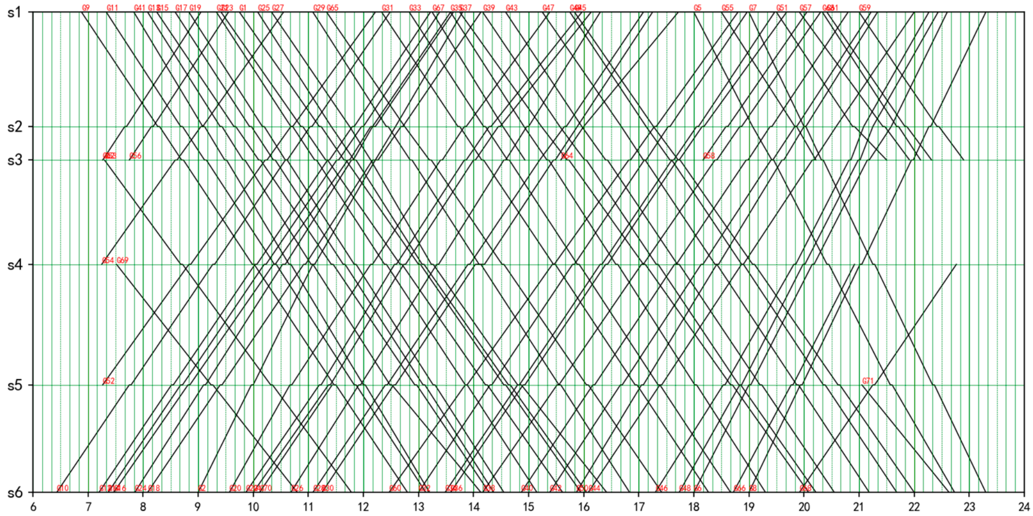

Figure 5. Schematic diagram of the train timetable on the first day of the holiday.

4.2.2. Calculation Results and Analysis of the EMU Routing Plan

By using the model and design method proposed in this paper, the train operation plan in the optimization period is solved and the EMU routing plan for seven days is obtained. Some of the EMU routing plans for seven days are shown in Table 5.

Table 5. Line interval distance and pure running time.

Index	EMU Routing						
	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day7**
1	G10-G33-G8-M *	G12	G62-M *-G34-G45	G54-M *	G71	G14-G34-G9- M *	G36- M *
2	G9-G34-M *	G14-G35- M *	G12-G67-G68	G24-M-G45-G9	G11-G1-G8-M *	G24-G43	G54- M *
3	G12-G37	G66-G31	G32	G9-M *	G62-G39- M *	G63-G68	G14- M *
4	G29-M *	G24-G43	G35- M * -G63	G69-M *	G52- M *	G62	G62- M * -G35
5	G66	G56- M	G66-M *	G52- M *	G41-G43-G57- M *	G17-G37	G53- M * -G39
6	-	G62-G23-G44-M *	G15-G10	G14- M * -G66	G24- M * -G55	G8- M *	G56-G25-G57
7	G53-G7-M *	G54-G37- M *	G69	G56- M * -G66	G69-G32- M *	G41-G39	-
…	…	…	…	…	…	…	…

M *: represents EMU maintenance.

Statistics are made on the number of EMUs used per day and the maintenance operations carried out during the optimization cycle, as shown in Table 6.

Table 6. Statistics of daily operation and maintenance quantity in an optimization cycle.

Index	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Number of EMUs operated	40	46	43	42	42	45	41
Number of maintenances	8	25	24	30	23	22	18

It can be seen from Table 6 that the number of EMUs used in these two days is relatively small because the train operation lines on the first day before and after the holiday are relatively sparse compared with those during the holiday period. According to the model assumptions, the cumulative running time and mileage of the EMU on the first day before the holiday start from 0, so the EMU on the day rarely reaches the time or mileage limit of the first-level preventive maintenance, so the number of EMUs that need to be maintained on the first day before the holiday is not large. During the holidays, due to the sudden increase in passenger travel demand, the transportation tasks on the first day of the holiday increased, the number of EMUs required increased, and the number of EMUs that met the maintenance standards increased. Accordingly, the fifth day of the holiday is the peak of the return journey, and the number of EMUs also reaches the peak. From the second day to the fourth day of the holiday, there are still many train operation lines, and the number of EMUs is stable at a high level. Among them, on the third day of the holiday, there is a peak in the number of EMU maintenance, which is related to the increase in the demand for EMUs during the holidays and the fact that there are many first-level preventive maintenance conditions for EMUs on the previous day.

4.3. Multiscenario Comparison

To better verify the validity of the model, we conducted numerical experiments on the scenarios under different train running densities, with the line conditions and other model parameters kept unchanged, and the specific number of train running pairs as shown in Table 7, where Scenario 1 is the original scenario.

Table 7. The number of train pairs in different scenarios.

Scene Number	Number of Train Pairs	
	Two Days Before and After a Holiday	During Holidays
1	30	35
2	33	38
3	36	42

After modeling, the number of EMUs used and the number of overhauls under different scenarios are obtained, as shown in Table 8.

Table 8. The number of EMUs used and overhauls under different scenarios.

Scene	Index	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	Number of EMUs operated	40	46	43	42	42	45	41
	Number of maintenances	8	25	24	30	23	22	18
2	Number of EMUs operated	44	51	47	46	45	50	45
	Number of maintenances	10	28	27	33	26	25	21
3	Number of EMUs operated	47	56	52	51	50	55	48
	Number of maintenances	11	31	29	36	28	27	23

As can be seen from the Table 8, with the increase in the number of pairs of trains in operation, the number of train sets required to be utilized each day shows an upward trend, both during holidays and before and after holidays. This is because more trains require a corresponding number of train sets to perform transportation tasks to meet passenger travel demand. Scenario 3 shows a significant increase in the number of rolling stock utilized on all days compared to Scenario 2, indicating a significant positive correlation between the increase in transportation demand and the increase in the demand for rolling stock resources. The increase in the number of pairs of trains leads to a rise in the number of rolling stock utilization, which requires the railway department to do a good job of reserve and deployment of rolling stock in advance before the festival, to ensure that there are a sufficient number of suitable types of rolling stock in operation at different stages. The increase in the number of overhauls means that the workload of the overhaul facilities and personnel increases, and it is necessary to reasonably arrange the overhaul time and site to avoid the backlog of rolling stock waiting for overhaul or the impact of untimely overhaul on operation due to the concentration of overhaul tasks. For example, in Scenario 3, the number of overhauls reaches 36 on Day 3, and the overhaul department needs to efficiently organize the overhaul process to ensure that the rolling stock can be overhauled and put into service on time.

5. Conclusions

5.1. Main Research Work

Aiming at the application scenarios of high-speed railways during holidays, this paper analyzes the requirements and internal relations of holiday transportation plans on the basis of summarizing the characteristics of holiday passenger demand, and explores the collaborative optimization method for train diagrams and EMU routing plans. By establishing a space–time–state network, an integrated optimization model of a high-speed railway train diagram and EMU routing plan is constructed. According to the characteristics of the model, a solution algorithm based on reinforcement learning is designed. After example verification, a collaborative optimization framework of high-speed railway transportation organization based on holidays is realized. Based on these research objectives, the main research work of this paper is divided into the following aspects:

Based on the collaborative optimization method of integrated compilation, a collaborative optimization model of high-speed railway holiday operation diagrams and EMU routing is established with time–space–state network as the carrier. Through the optimization of the time–space–state path of the EMU, the operation of the train can be effectively coordinated, so that the train diagram and the routing can cooperate with each other to provide the best transportation effect and achieve the lowest transportation cost of the overall transportation plan.

The effectiveness of the proposed model and algorithm is verified based on the actual operation data of a certain line. Firstly, based on a high-speed railway line in actual operation, for the application scenario of a 5-day holiday, a total of 7 days before and after the holiday are taken as the case optimization cycle design. Secondly, a small case is selected to set and verify the hyperparameters of the deep neural network in the reinforcement learning algorithm, and the hyperparameter set with the highest efficiency of the model is found in the candidate set of hyperparameters. The optimal hyperparameter set is used to solve the original case. The experimental results can verify that the collaborative optimization method for holiday application scenarios based on reinforcement learning proposed in this paper has practical application value.

5.2. Shortcomings and Prospects

The research described in this paper has shortcomings. Future research can be carried out from the following aspects:

(1) Expand the collaborative optimization object of high-speed railway transportation organization.

In this paper, the research on collaborative optimization of transportation organization is limited to two objects: the train diagram and the EMU routing. Necessary for processing a complete high-speed railway transportation organization, it should also include train and crew operation plans. In view of the analysis of the interaction between the train diagram and the EMU routing in this paper, it can be concluded that in order to realize the global optimization of the whole process of the transportation organization, it is difficult to achieve the expected effect only by optimizing a small number of links in the transportation organization. Therefore, it is of great research significance to integrate as many technical documents as possible and even cover the whole process of transportation organization. Future collaborative optimization research can start from the above direction.

(2) Consideration of turnover time for EMUs

In addition to the transportation cost, the connection time between train sets is also an important indicator Ref. [28]. Although compared with the literature, this paper establishes a longer time span and collaborates with the optimization preparation of the operating map, the consideration of the connection time is static and has certain limitations. Therefore, in the future, including the connection time of the moving train set can also be considered in the model.

(3) Consider the assumptions that meet the actual conditions of transportation production.

The collaborative optimization model established in this paper is a simplification of the mathematical model abstraction of the actual transportation problem. In the stage of setting the assumptions of the model, some conditions that should be considered in the actual transportation production are ignored. For example, the limitation of the maintenance capacity of the EMU depot and the time and distance consumed by the EMU access depot are not considered. Considering more assumptions that meet the actual conditions of transportation production is conducive to increasing the practical application value of the model.

6. Discussion

In the actual high-speed railway operation, transportation organization during holidays has been a complex and critical task. The integrated optimization method of operating map and rolling stock interchanges considering the synergy between weekdays and holidays proposed in this paper provides new ideas and effective means to solve this problem.

From the perspective of transportation cost, the traditional transportation organization mode is often difficult to accurately deploy resources when coping with the fluctuation in passenger flow on holidays, and is prone to irrational use of rolling stock and empty driving, which leads to an increase in transportation cost. The method in this paper can effectively avoid these problems by constructing a space–time–state network and synchronously optimizing the traffic routes of the moving train sets when preparing the operation map.

Considering the aspect of passenger service quality, the optimized operation diagram can better adapt to the diversified travel needs of passengers during holidays. Before and after holidays and during holidays there are significant differences in travel purpose and traffic distribution of passengers. Through integrated optimization, the operation chart can be flexibly adjusted according to the characteristics of passenger flow in different periods, such as reasonably arranging the number of medium- and long-distance trains and the time in the middle of and at the two ends of the holiday to ensure that passengers can travel more conveniently. At the same time, the reasonable arrangement of rolling stock interchanges also helps to improve the punctuality and reliability of trains, reducing the inconvenience caused by late trains or poor connections, thus enhancing the overall travel experience of passengers.

The optimization method also has certain advantages in managing unexpected situations. Although it may not be possible to fully cover all the actual contingencies in the model construction, by reasonably considering the constraints of train tracking interval time, zone operation conflicts, and the flexibility of the maintenance arrangement of the train sets, the transportation organization can be more robust in the face of some common disturbing factors.

In addition, from the practical level, although the model involves many complex parameters and constraints, it is designed based on actual operational data and its feasibility is verified through case studies. This provides a reference basis for the railway transportation department in practical application, so that the relevant personnel can adjust and apply the model according to their own line characteristics and operational needs.

Author Contributions: Data curation, F.G.; methodology, Z.Z., F.G., W.D. and J.C.; project administration, J.C.; supervision, J.C.; validation, W.D.; visualization, F.G.; writing—original draft, Z.Z., F.G. and W.D.; writing—review and editing, Z.Z., W.D. and J.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by the Fundamental Research Funds for the Central Universities (Science and Technology Leading Talent Team, project no. 2022JBQY005).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article; further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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