Journal of
Risk and Financial Management

MDPI

Article

Application of a Robust Maximum Diversified Portfolio to a Small Economy's Stock Market: An Application to Fiji's South Pacific Stock Exchange

Ronald Ravinesh Kumar [1], Hossein Ghanbari [2] and Peter Josef Stauvermann [3,*]

[1] Department of Economics and Finance, The Business School, RMIT University, Saigon South Campus, Ho Chi Minh City 700000, Vietnam; ronald.kumar@rmit.edu.vn

[2] School of Industrial Engineering, Iran University of Science & Technology, Tehran 13114-16846, Iran; hossein_ghanbari@ind.iust.ac.ir

[3] Department of Global Business and Economics, Changwon National University, Changwon 51140, Republic of Korea

* Correspondence: pjsta@gs.cwnu.ac.kr

Abstract: In this study, we apply a novel approach of portfolio diversification—the robust maximum diversified (RMD)—to a small and developing economy's stock market. Using monthly returns data from August 2019 to May 2024 of 18/19 stocks listed on Fiji's South Pacific Stock Exchange (SPX), we construct the RMD portfolio and simulate with additional constraints. To implement the RMD portfolio, we replace the covariance matrix with a matrix comprising unexplained variations. The RMD procedure diversifies weights, and not risks, hence we need to run a pairwise regression between two assets (stocks) and extract the R-square to create a P-matrix. We compute each asset's beta using the market-weighted price index, and the CAPM to calculate market-adjusted returns. Next, together with other benchmark portfolios (1/N, minimum variance, market portfolio, semi-variance, maximum skewness, and the most diversified portfolio), we examine the expected returns against the risk-free (RF) rate. From the simulations, in terms of expected return, we note that eight portfolios perform up to the RF rate. Specifically, for returns between 4 and 5%, we find that max. RMD with positive Sharpe and Sortino (as constraints) and the most diversified portfolio offer comparable returns, although the latter has slightly lower standard deviation and downside volatility and contains 94% of all the stocks. Portfolios with returns between 5% and the RF rate are the minimum-variance, the semi-variance, and the max. RMD with positive Sharpe; the latter coincides with the RF rate and contains the most (94%) stocks compared to the other two. An investor with a diversification objective, some risk tolerance and return preference up to the RF rate can consider the max. RMD with positive Sharpe. However, depending on the level of risk-averseness, the minimum-variance or the semi-variance portfolio can be considered, with the latter having lower downside volatility. Two portfolios offer returns above the RF rate—the market portfolio (max. Sharpe) and the maximum Sortino. Although the latter has the highest return, this portfolio is the least diversified and has the largest standard deviation and downside volatility. To achieve diversification and returns above the RF rate, the market portfolio should be considered.

Keywords: portfolio allocation; robust maximum diversified and benchmark portfolios; South Pacific Stock Exchange; small market economy; Fiji Islands

check for updates

Citation: Kumar, Ronald Ravinesh, Hossein Ghanbari, and Peter Josef Stauvermann. 2024. Application of a Robust Maximum Diversified Portfolio to a Small Economy's Stock Market: An Application to Fiji's South Pacific Stock Exchange. *Journal of Risk and Financial Management* 17: 388. https://doi.org/10.3390/jrfm17090388

Academic Editor: Thanasis Stengos

Received: 12 July 2024
Revised: 27 August 2024
Accepted: 30 August 2024
Published: 2 September 2024

1. Introduction

Portfolio optimization is a critical component of investment management, focusing on maximizing returns while minimizing risk through the careful selection and weighting of assets. The primary goal of this technique is to create an investment portfolio that achieves the best possible balance between expected returns and risk. Portfolio optimization methods, particularly the concept of mean-variance optimization, were pioneered by Markowitz

(1952a, 1952b). His groundbreaking work laid the foundation for modern portfolio theory, introducing a quantitative framework for constructing portfolios. Following Harry Markowitz's pioneering work in 1952, many researchers expanded upon his foundation, developing a variety of advanced strategies for portfolio optimization, each with its own unique approach and methodology (Ghanbari et al. 2023; Kalayci et al. 2019; Milhomem and Dantas 2020; Righi and Borenstein 2018; Salo et al. 2024). Generally, any portfolio strategy aims to create some level of diversification across assets and/or stocks to manage risk. Several studies have been conducted to examine and rationalize different methods of portfolio diversification, and their findings help investors understand the benefits and limitations of diversification (Bielstein et al. 2023; Koumou 2020). However, as noted in the literature, studies on portfolio diversification and investment analysis are generally focused on large and well-developed financial markets, in developed or large emerging economies, mainly because from an investor's perspective, these countries have a vibrant stock market that offers greater liquidity, hence providing the ease of trading. Moreover, higher liquidity in the market can generate scale effects, thus reducing transaction costs, in addition to supporting faster execution of trades (Al Janabi 2021; Andreev 2019; Baumann and Trautmann 2013).

Markets in small economies are generally less liquid, less volatile, and have lesser numbers of participants (listed companies, retail traders and investors, and brokers) than developed markets. With such characteristics of a small market, the task of managing risk and identifying suitable stocks for investment can appear straightforward. However, fewer investors and low volumes of trade can also contribute to a lack of confidence among potential investors to participate in the stock market. Therefore, to support the development and progress of new and infant stock markets, it is vital to identify the nuances present in the market and examine suitable portfolios to educate and inform potential investors. Hence, a contribution of this study is to focus on the small island economy of Fiji's stock market and extend the existing analysis by incorporating newer methods of portfolio construction that can be considered for investment.

The Sustainable Development Goal 8 (SDG 8) emphasizes the need to "improve access to financial services...", and to "strengthen the capacity of domestic financial institutions to encourage and expand access to banking, insurance and financial services for all".[1] In this regard, the study is relevant and important for both existing and potential investors in small markets, with the view to enhancing financial inclusion and literacy. By focusing on small economies, our research underscores the need for greater financial inclusion for all.

Although Fiji's stock market (South Pacific Stock Exchange—SPX) has been in existence for more than four decades, in a prominent local newspaper, *The Fiji Times*, Singh (2019) documents that from 1979, it operated for some 17 years as Trading Post, with an exchange officer handing trades without any regulatory body, stockbrokers, and dealers. Then, in July 1996, a "open outcry system" or a "call market" was introduced where brokers used a physical trading floor for trading, and later in the same year (December), the first securities market legislation, known as the Capital Markets Development Authority Act (CMDA), was established to formally institute investment advisors, the stock exchange, and the brokers (Whiteside 2016). In 2000, the name of the exchange was changed to the South Pacific Stock Exchange (Saliya 2022), and a Central Share Registry (CSR) was created in 2002. In 2010, SPX launched its electronic trading platform, and in 2016, it launched its web-based online secured portal for shareholders and listed companies. While this progress indicates significant efforts towards development of the stock market in Fiji, the Reserve Bank of Fiji (RBF 2017), in its report on *Financial Sector Development Plan 2016–2025* under Pillar 10, notes that SPX aims to "become the preferred capital market center in the South Pacific". However, the report (RBF 2017, p. 38) also notes that

> "... there are many issues besetting Fiji's capital market. These include small retail investor base, limited financial products, illiquid stock market, high transaction costs, lack of awareness of the capital market and inconsistent application of policy by regulatory agencies".

Against this background, we extend the earlier studies by applying a newly proposed method—the Robust Maximum Diversified (RMD) portfolio (Bielstein et al. 2023)—and compare the results with a set of benchmark portfolios. Moreover, we also examine the RMD portfolio with additional constraints to generate additional insights. Noting the results, we present alternative approaches for potential investors in small markets to consider diversification and risk management. We analyze all portfolios based on market-adjusted returns. This is because in less liquid markets, changes in asset prices can be due to small volumes traded at a higher frequency, although a large amount of assets may remain untraded for a long time. Hence, considering the responsiveness of asset prices with respect to the market is more informative. Moreover, computing market-adjusted returns informs investors on how the respective asset returns correlate with the overall market performance. Therefore, analysis based on average returns can be misleading, as the returns do not reflect an asset's true performance (relative to the market) (Kumar and Stauvermann 2022).

In this study, we focus on the stock market of a small island economy, Fiji, which at the time of study, comprised 19 listed companies. Some recent studies have provided a comprehensive background on Fiji's stock market and considered a few conventional approaches (Kumar et al. 2022; Kumar and Stauvermann 2022). These studies have laid the groundwork for understanding the dynamics of Fiji's stock market, while noting the challenges and opportunities associated with investing in such a small and relatively less liquid market.

Based on the share price movements, we considered 18 stocks and the stock price data from July 2019 to May 2024. We computed monthly returns for each stock for the period from August 2019 to May 2024 and used these data to estimate each stock's beta. For market return, we use the market-weighted price index published by SPX (SPX 2024a). Using the respective asset's beta, we computed the market-adjusted expected returns and constructed a set of benchmark portfolios (mean-variance, minimum-variance, target ratio, semi-variance, and most diversified portfolios). We then constructed RMD portfolios (Bielstein et al. 2023) and compared them with the benchmark portfolios. The importance of the RMD approach stems from its feature that it does not rely on expected returns or volatilities, and hence it is robust to measurement errors. This is because it uses a pairwise regression between two assets to derive the degree of variation, and then considers the portfolio weights that maximize the unexplained variation of the portfolio.

The study contributes to the literature on small economy finance in the following ways: it provides insights that can enhance the understanding and management of investments in small markets; it highlights the importance of equipping investors with knowledge and tools necessary to make informed decisions, thereby encouraging broader participation in the stock market and other financial activities. This increased participation not only democratizes access to financial markets but also strengthens the overall financial ecosystem. Furthermore, our study supports the objective of promoting financial literacy (education) and provides insights to potential investors and businesses in small markets on navigating the complexities of investing, managing risks more effectively, and ultimately achieving sustainable financial outcomes. Well-informed financial strategies and methods creates greater confidence among investors, and hence support the growth of stock markets and broader economic development goals (Voica 2017; Dash and Mohanta 2024; Kandpal 2020; Kara et al. 2021; Ozili 2022).

The remainder of this paper is organized as follows: Section 2 presents the literature review, providing an in-depth exploration of the investment strategies relevant to this study. Section 3 outlines the materials and methods used in the study, detailing the methodologies applied for portfolio construction and diversification, and introduces the preliminary concepts and benchmarks, setting the foundation for the subsequent analysis. Section 4 presents the analysis and results, where we compare the performance of the newly proposed robust maximum diversified (RMD) portfolios against traditional benchmark portfolios, highlighting key findings and statistical significance. Finally, Section 5 offers the conclusion, summarizing the study's contributions, discussing the implications for small market economies like Fiji, and suggesting potential areas for future research.

2. Literature Review

Investors have a range of strategies at their disposal when it comes to allocating wealth in the stock markets, each tailored to different levels of engagement and risk tolerance. One common approach is the meticulous selection of individual companies for investment. This strategy often requires a deep dive into fundamental analysis, which involves evaluating a company's financial health, performance metrics, and market position (Hilkevics and Semakina 2019; Narkunienė and Ulbinaitė 2018). In addition to fundamental analysis, ensuring proper diversification across different sectors and asset classes is crucial to mitigate risk and optimize returns (Ahmed et al. 2018). Alternatively, some investors rely on technical analysis to guide their stock selections. This method focuses on patterns in stock price movements and trading volumes, using historical data to forecast future trends and inform investment decisions (García et al. 2018). Another strategy is to manage a portfolio of companies directly, where the selection of companies is performed simultaneously. For investors interested in managing an entire portfolio rather than selecting individual stocks, portfolio management strategies come into play. In this approach, the selection of companies is conducted simultaneously, allowing for a more holistic view of the portfolio's risk and return profile. The following literature survey provides an in-depth exploration of these investment strategies, their underlying principles, and the methodologies employed in their execution.

Markowitz (1952a, 1952b) introduced the concept of portfolio selection based on the trade-off between risk and return. He argued that an investor should not only focus on the expected return of an individual asset but also consider how the asset's return interacts with the returns of other assets in a portfolio. Subsequently, Markowitz proposed the mean-variance optimization framework, where the goal is to construct a portfolio that maximizes expected return for a given level of risk or minimizes risk for a given level of expected return. This approach leads to the identification of the "efficient frontier", a set of optimal portfolios that offer the highest expected return for a given level of risk. Markowitz's work was groundbreaking because it formalized the process of diversification, showing that the risk of a portfolio is not just the sum of the risks of individual assets, but also depends on the correlations between them. This insight has had a profound impact on investment management and laid the groundwork for the development of various financial theories and practices. Later, Markowitz (1991) addressed the foundational principles of portfolio theory, which he originally developed, aiming to clarify how it differs from traditional microeconomic theory. Specifically, he focused on investor behavior under uncertainty, as opposed to the behavior of firms or consumers. The aim of the study was to examine the evolution and refinement of these principles over time, addressing both the practical challenges and the broader applications of portfolio theory, particularly in guiding institutional investors. Moreover, Markowitz (1991) delved into the ongoing relevance of mean-variance analysis in approximating expected utility, while underscoring the need for further research and refinement to enhance the theory's effectiveness in contemporary financial settings. Sharpe (1994) revisited the performance measure he originally introduced in Sharpe (1996), known as the "reward-to-variability ratio" and now widely referred to as the Sharpe ratio, emphasizing its utility in assessing the risk-adjusted performance of mutual funds and other investment portfolios. By acknowledging the measure's evolution and varied terminology over the years, the paper reinforced the Sharpe ratio's position as an essential tool in modern portfolio management.

Elton and Gruber (1997) offered a comprehensive review of the development and evolution of modern portfolio theory from the 1950s to the late 1990s. The authors trace the origins of portfolio theory to Markowitz's (1952a, 1952b) groundbreaking work of the early 1950s, highlighting the introduction of mean-variance optimization, the efficient frontier, and the core principles of portfolio selection. They also discuss the key inputs necessary for portfolio optimization, such as the estimation of expected returns, variances, and covariances, while addressing the challenges in accurately estimating these parameters. The paper further explored the application of portfolio theory within financial institutions, particularly the complexities of incorporating liabilities and institutional constraints into the analysis, the importance of risk-adjusted returns and the continued relevance of metrics like the Sharpe ratio. The authors concluded that further research is needed to refine the application of portfolio theory in more complex financial environments.

De Athayde and Flôres (2004) addressed the problem of portfolio selection by considering not just the mean and variance of asset returns but also the skewness. The goal of this study was to develop a general methodology for constructing an efficient portfolio set that accounts for the first three moments of asset returns—mean, variance, and skewness. The authors present analytical formulas for determining the efficient surface in a three-dimensional space defined by these moments and explore the geometric properties of this surface. The methodology allows for the computation of optimal portfolio weights, even in cases where the investor is willing to trade off negative skewness for higher expected returns. By expanding on traditional mean-variance analysis, the paper provides a more comprehensive framework for portfolio optimization, addressing situations where asset returns are not normally distributed. The authors also introduced a duality result that plays a crucial role in solving the optimization problem, particularly in cases where the solution may involve multiple local optima.

Markowitz (2010) reflected on the foundational concepts of modern portfolio theory, revisiting key assumptions and their applications in contemporary finance. He discussed the practical use of mean-variance analysis, emphasizing that he never assumed return distributions to be Gaussian or investors' utility functions to be quadratic. Markowitz explored the conditions under which mean-variance analysis is applicable and extended his analysis to newer risk measures like Value at Risk (VaR) and Conditional Value at Risk (CVaR). He also revisits the hypothesis he proposed in 1952 regarding the coexistence of behaviors like buying lottery tickets and insurance by the same economic agents, comparing his utility function to that of prospect theory. Additionally, Markowitz discusses the Capital Asset Pricing Model (CAPM) and critiques some of its foundational assumptions, particularly the unrealistic representation of short positions. Overall, the paper serves as a retrospective and an update on Markowitz's views on portfolio theory, offering insights into both its historical foundations and its relevance to current financial practices.

Maillard et al. (2010) explored an alternative approach to portfolio construction known as the Equally Weighted Risk Contribution portfolio. They analyzed the theoretical properties and practical performance of Equally Weighted Risk Contribution portfolios, which aim to balance the risk contribution of each asset in the portfolio rather than simply equalizing the weights. The Equally Weighted Risk Contribution approach is positioned as a middle ground between the minimum-variance portfolio, which can suffer from concentration issues, and the equally weighted portfolio, which may lack proper risk diversification. The authors delve into the mathematical foundations of Equally Weighted Risk Contribution portfolios, providing both theoretical insights and empirical evidence to support their effectiveness. They compare the Equally Weighted Risk Contribution portfolios with strategies like the 1/n and minimum-variance portfolios, through numerical examples and empirical simulations. The study concluded that Equally Weighted Risk Contribution portfolios offer a more balanced risk profile while maintaining competitive performance, making them a valuable tool for investors seeking robust portfolio diversification without excessive concentration in a few assets.

Clarke et al. (2011) focused on the construction and analysis of minimum-variance portfolios, which aim to achieve the lowest possible risk for a given set of assets. They explored the composition of minimum-variance portfolios, particularly under long-only constraints (where no short-selling is allowed). The authors provide an analytic solution for determining the optimal security weights within these portfolios, emphasizing that such portfolios are primarily composed of stocks with lower market betas. The paper also highlights how high idiosyncratic risk reduces a security's weight but does not eliminate it from the portfolio unless the market beta is high. Furthermore, the study compares these analytic results with empirical data, and finds that the optimization of minimum-variance portfolios often leads to portfolios dominated by low-beta stocks. This finding aligns with the longstanding critique of the Capital Asset Pricing Model (CAPM), which suggests that low-beta stocks can yield returns similar to or even higher than those from high-beta stocks. The authors conclude that minimum-variance portfolio strategies are effective in reducing risk while potentially offering high returns, making them a valuable tool in portfolio management, especially considering the observed anomalies in traditional risk–return relationships.

Boasson et al. (2011) explored the use of a semi-variance framework as an alternative to the traditional mean-variance approach in portfolio optimization. They demonstrated how a semi-variance approach can more effectively measure downside risk in portfolio selection, compared to the traditional mean-variance framework. The authors argued that semi-variance, which only considers return dispersions below the expected value, aligns better with investors' intuitive perceptions of risk, focusing on minimizing losses rather than overall volatility. The study used a sample of exchange-traded index funds (ETFs) to compare portfolios constructed using the mean-semi-variance model with those created using the traditional mean-variance model. The results indicate that portfolios optimized under the mean-semi-variance framework offer potential benefits, such as maintaining or improving expected returns while minimizing downside risk exposure. The findings have practical implications for both individual and institutional investors, particularly those in industries like insurance, where minimizing downside risk is crucial.

Todoni (2015) explored the application of Post-Modern Portfolio Theory (PMPT) in evaluating the risk-adjusted returns of major indices from emerging markets in Central and Eastern Europe over the period 2008–2013. The main goal of this paper is to apply two methods based on PMPT to assess the risk-adjusted returns of five major Central and Eastern European market indices: Romania (BET), Hungary (BUX), Czech Republic (PX), Bulgaria (SOFIX), and Poland (WIG). The study employed the Sortino ratio, a well-known measure to evaluate risk-adjusted return of a portfolio considering only the downside risk. The authors then propose an alternative method that builds on the Sortino ratio by incorporating a "multipliers method" to calculate a global measure of risk, which aims to provide a more nuanced assessment of risk by distinguishing between unrealized return areas and loss areas (negative returns). The analysis was conducted over two sub-periods, 2008–2010 and 2011–2013, highlighting how Central and Eastern European markets responded to the financial crisis. The results show that Hungary consistently outperformed across both methods, while Bulgaria typically exhibited the worst performance. The study concludes by suggesting that the proposed alternative method may offer a more refined measure of risk that better aligns with the realities of investment decision-making, particularly in emerging markets.

Booth and Broussard (2016) explored the application of the Sortino ratio within the context of asset allocation, particularly using Extreme Value Theory (EVT). They assessed the effectiveness of the Sortino ratio, which focuses on downside risk, in optimizing portfolio allocations. The authors compare the performance of portfolios constructed using the Sortino ratio against those optimized with the Sharpe ratio, particularly in environments with significant downside risks, such as during financial crises. The analysis is performed using a dataset that includes U.S. real estate investment trusts (REITs) and the S&P 500 index. The authors also incorporate the EVT to model the extreme returns in the left tail of the distribution, which are crucial in assessing the downside risk more accurately. Their findings indicate that the choice of performance measure (Sortino versus Sharpe) significantly influences the optimal asset allocation, especially under different risk tolerances and market conditions. The results suggest that portfolios optimized using the Sortino ratio, particularly with EVT adjustments, might offer better protection against extreme negative events, making it a valuable tool for investors concerned with downside risk.

Taljaard and Maré (2021) focused on the S&P 500 since 2016 and compared the equally weighted portfolio with the market-capitalization-weighted portfolio. The primary goal of this study was to analyze the reasons behind the underperformance of the equally weighted portfolio relative to the market-cap-weighted portfolio and to propose strategies to mitigate this issue. The authors utilized stochastic portfolio theory to explore the dynamics that contribute to this underperformance. They find that while the equally weighted portfolio typically outperforms over the long term, it has faced significant short-term underperformance due to increased market concentration in the cap-weighted portfolio and reduced diversification benefits. To address these challenges, the study suggested a dynamic approach that involves switching between equal-weighted and cap-weighted portfolios based on a linear regression model that predicts relative performance. This model considers factors such as changes in market concentration and the benefits of diversification, with the aim of optimizing short-term performance while maintaining long-term gains. The findings offer practical insights for improving the performance of equal-weighted portfolios, particularly in volatile or momentum-driven markets.

Surtee and Alagidede (2023) explored an innovative use of the MPT by incorporating different risk–reward ratios beyond the traditional Sharpe and Sortino ratios. They evaluated the effectiveness of the Sterling and Treynor ratios as alternatives to the commonly used Sharpe and Sortino ratios in portfolio optimization. The authors argue that while MPT and the Sharpe ratio have been widely used, these traditional methods may not fully capture the potential for higher returns under varying market conditions. By applying the Sterling and Treynor ratios to MPT, the study finds that these ratios often produce higher-performing portfolios. The paper emphasizes the statistical advantages of using these ratios, particularly in their ability to adapt to different market environments and provide more robust performance metrics compared to traditional methods. The study concludes by suggesting further research into optimizing portfolio performance through the application of these alternative ratios and exploring advanced optimization algorithms to enhance the results.

3. Materials and Methods

3.1. RMD Portfolio

The application of the RMD portfolio is the focus of the study, hence we present the approach as follows (Bielstein et al. 2023). The objective of the RMD portfolio is to maximize the unexplained variation between the asset returns, enhancing diversification by emphasizing assets that behave differently from each other. We consider an asset universe of N assets and compute a sample return over a specific period from $t = 0$ to $t = T$, where t = 0 is the starting point for returns, and t = T is the end point for returns. We can then compile the correlation matrix, denoted as $\boldsymbol{C}$, which presents the pairwise correlations between these asset returns, as follows:

$$\boldsymbol{C} = \begin{bmatrix} 1 & \cdots & \rho_{1,N} \\ \vdots & \ddots & \vdots \\ \rho_{N,1} & \cdots & 1 \end{bmatrix} \quad (1)$$

The matrix **C** can be used to derive the explained variations between any two asset returns (say asset 1 and asset 2), which are essentially the R^2 values computed by squaring the correlation coefficient, $\rho_{1,2}$. We compile the R^2 values in a P-matrix, as follows:

$$\boldsymbol{P} = \begin{bmatrix} 1 & \cdots & \rho_{1,N}^2 \\ \vdots & \ddots & \vdots \\ \rho_{N,1}^2 & \cdots & 1 \end{bmatrix} \quad (2)$$

Alternatively, we can directly compute the R^2 values by taking two asset returns, say assets 1 and 2, and regressing the returns of asset 1 on the returns of asset 2, to determine the proportion of variance of asset 1 explained by asset 2.

Next, we compute the unexplained variation (denoted by the R-matrix), because a diversified portfolio should include assets whose return variations are largely unexplained by other assets. Since the explained variation between any two assets is denoted by R-square values (or matrix P), where $\rho^2 \in [0,\ 1]$, the unexplained variation between any two assets is computed by $1-\rho^2_{1,2}$. We can construct the matrix of unexplained variations between the respective pair of asset returns denoted by the R-matrix given below:

$$\boldsymbol{R} = \begin{bmatrix} 0 & \cdots & 1-\rho^2_{1,N} \\ \vdots & \ddots & \vdots \\ 1-\rho^2_{N,1} & \cdots & 0 \end{bmatrix} \tag{3}$$

This matrix $\boldsymbol{R}$ represents the unexplained variations between the asset pairs, which the optimization function uses to determine the portfolio weights. The goal is to maximize these unexplained variations, thereby combining assets that exhibit the least correlation with one another. In what follows, the RMD portfolio finds the vector of weights (ω) that maximizes these unexplained variances, subject to full investment and no-short-selling. Hence, the optimization problem is formulated as follows:

$$max\boldsymbol{\omega}^T\boldsymbol{R}\boldsymbol{\omega}$$

Subject to:

$$\textstyle\sum_{i=1}^{N}\omega_i,\ \text{and}\ \omega_i \geq 0, \forall i = 1,\ldots,N \tag{4}$$

The term $\boldsymbol{\omega}^T\boldsymbol{R}\boldsymbol{\omega}$ represents the weighted sum of unexplained variations. Maximizing this term means choosing portfolio weights that emphasize assets with minimal correlation to each other, thus maximizing diversification. The constraint $\sum\omega_i = 1$ ensures that the total investment across all assets equals the total capital available for investment. The constraint $\omega_i \geq 0$ ensures that no asset has a negative weight, meaning no short positions are taken in any asset.[2]

We can compute the R^2 value in Excel® using the formula: $= RSQ(\mu_{A1}, \mu_{A2})$, where μ_{A1} and μ_{A2} are the monthly returns of asset 1 and asset 2, respectively. Alternatively, we can obtain the respective R^2 by first computing the correlation between asset 1 and asset 2, $\rho_{1,2}$, and then taking the square of the correlation coefficient, i.e., $\rho^2_{1,2}$. In Excel®, we can implement this for each pair of assets using the formula: $(= POWER(CORREL(\mu_{A1}, \mu_2), 2))$.

3.2. Benchmark Portfolios

This section is devoted to explaining the benchmark portfolios used in our study. Benchmark portfolios serve as standards or points of reference against which the performance and characteristics of the RMD portfolio can be compared. The following benchmark portfolios are considered.

3.2.1. Equally Weighted (Naïve)

The equally weighted portfolio (1/N) (DeMiguel et al. 2009) allocates an equal proportion of the total investment to each asset, regardless of its individual characteristics. It is robust to estimation errors because it only relies on the number of assets in the portfolio. The primary assumption here is that the assets are uncorrelated, meaning each asset's performance is independent of the others. The equally weighted portfolio can be depicted as follows:

$$\omega_{EW} = \frac{1}{N} \tag{5}$$

where N is the number of assets in the portfolio.

3.2.2. Minimum Variance

The MVP (Clarke et al. 2006; Haugen and Baker 1991) aims to achieve the lowest possible portfolio volatility by minimizing the standard deviation of returns. This approach assumes accurate forecasts of the covariance matrix of asset returns. However, reliance on such forecasts can introduce estimation errors, particularly in large asset universes, leading to potential instability. Moreover, this method may result in overly concentrated portfolios, where a few assets dominate the risk exposure. The MVP can be formulated as follows:

$$\min\left\{\sigma_p = \sqrt{\omega_{MVP}^T \Sigma \omega_{MVP}} \middle| \mu_p = \sum_{i=1}^N \omega_i r_i;\ \sum_{i=1}^N \omega_i = 1\right\} \tag{6}$$

where Σ is the $N \times N$ covariance matrix of the asset returns, and r_i is the ith stock's market-adjusted return. Generally, the portfolio with the minimum variance can be computed by $\boldsymbol{\omega_{min}} = \frac{\boldsymbol{\Sigma^{-1}1}}{\boldsymbol{1^T\Sigma^{-1}1}}$, where Σ^{-1} is the inverse covariance of returns matrix. A portfolio's beta-adjusted portfolio mean is given by $\boldsymbol{\mu_p} = \boldsymbol{\omega}'\boldsymbol{\mu}$, and variance is given by $\boldsymbol{\sigma_p} = \boldsymbol{\omega}'\boldsymbol{\Sigma\omega}$, where $\boldsymbol{\Sigma}$ is the covariance matrix.

3.2.3. Mean Variance (Market)

The individual stock returns are computed as $r_t = ln\left(\frac{p_t}{p_{t-1}}\right)$, where p_t and p_{t-1} are the individual stock prices for months t and $t-1$, respectively. Individual asset beta, β_i, represents the trade-off between market return against the individual asset's return over the sample period. Based on the CAPM framework, the annualized return for each stock (μ_{β_i}) is computed as: $\mu_{\beta_i} = r_f + \beta_i\left(\mu_{mkt} - r_f\right)$, where r_f is the risk-free rate, assumed equal to 0.0544 (5.44%) in the paper, and μ_{mkt} = average return of the market weighted price index (MWPI) reported by SPX (SPX 2024a). The risk-free rate was selected based on the 1-year government bond yield rate of May 2019.[3]

With insights earlier studies from on CAPM and maximizing returns with a given risk (standard deviation) (Sharpe 1964; Merton 1972), we derive a market portfolio based on maximizing the Sharpe (1996) ratio, $SR = \left(\frac{\mu_p - r_f}{\sigma_p}\right)$, where μ_p, r_f and σ_p are the portfolio mean, risk-free rate and portfolio standard deviation, respectively. Hence, following Bai and Newsom (2011), we specify the programing problem as

$$\max\left\{SR = \frac{\mu_p - r_f}{\sigma_p} \middle| \mu_p = \sum_{i=1}^N \omega_i r_i; \sigma_p = \sqrt{\sum_{i,j}^N \sigma_{ij}\omega_i\omega_j};\ \sum_{i=1}^N \omega_i = 1\right\} \tag{7}$$

where N is the number of stocks, and σ_{ij} is the covariance between returns of stock i and stock j, and $\bar{r}_i$ is the annualized expected return of stock i, σ_{ij} is the maximum risk level specified by the investor, and ω_i is the weight of stock i in the portfolio. We use the changes in the market-weighted price index to proxy market return and derive beta-adjusted annualized stock returns based on the CAPM framework.

3.2.4. Semi-Variance

Next, we extend the analysis to examine downside risk only. First, we compute the monthly upside risk as $UR_t = \max\left(\bar{\mu} - \mu_{p_t}, 0\right)$, where μ_{p_t} is the monthly portfolio return with respect to weight, and $a\omega_i$, and $\bar{\mu}$ are the historical monthly returns. The monthly downside risk is given by $DR_t = \max\left(\mu_{p_t} - \bar{\mu}, 0\right)$. Then, we compute the annualized downside volatility, as $\bar{\sigma}_D = \sqrt{\frac{\sum_{i=1}^T \max\left(\mu_{p_t} - \bar{\mu}, 0\right)^2}{T-1}} \times 12$.

Hence,

$$\min\left\{\bar{\sigma}_D \middle| \bar{\mu}_p = \sum_{i=1}^N \omega_i r_i; \sigma_p = \sqrt{\sum_{i,j}^N \sigma_{ij}\omega_i\omega_j}; \sum_{i=1}^N \omega_i = 1\right\} \tag{8}$$

3.2.5. Max Target Ratio

With a target return of 6.75% ($T_g = 0.0675$), which was the rate of return offered by Fiji National Provident Fund in 2019,[4] we compute the Sortino ratio as $Sort = \frac{\overline{\mu}_p - T_g}{\overline{\sigma_D}}$, where $\overline{\sigma_D}$ is the annualized downside volatility. The program for optimization with the T-period sample is as follows:

$$\max\left\{ Sort = \frac{\overline{\mu} - T_g}{\overline{\sigma_D}} \middle| \mu_p = \sum_{i=1}^{N} \omega_i r_i;\ \overline{\sigma_D} = \sqrt{12\left[\frac{\sum_{i=1}^{T} \max(\mu_T - \mu_i, 0)^2}{T-1}\right]};\ \sum_{i=1}^{N} \omega_i = 1 \right\} \tag{9}$$

3.2.6. Maximum Skewness

Finally, the volatility skewness is defined as $\sigma_v = \frac{\sigma_U^2}{\sigma_D^2}$, where σ_U^2 and σ_D^2 are the variances of upside risk and downside risk, and $\sigma^2 = \sigma_U^2 + \sigma_D^2$. Hence, $0 < \sigma_v < 1$ implies higher downside risk in the portfolio, whereas $\sigma_v > 1$ implies greater upside risk in the portfolio. Hence, the program for maximum volatility skewness is specified as follows:

$$\max\left\{ \sigma_v = \frac{\sigma_U^2}{\sigma_D^2} \middle| \mu_p = \sum_{i=1}^{N} \omega_i r_i; \sigma_p = \sqrt{\sum_{i,j}^{N} \sigma_{ij}\omega_i\omega_j}; \sum_{i=1}^{N} \omega_i = 1 \right\} \tag{10}$$

3.2.7. Most Diversified Portfolio (MDP)

The MDP (Choueifaty and Coignard 2008) is calculated by comparing the weighted average volatility of the individual assets ($\omega_{MDP}\sigma$) to the total portfolio volatility ($\sqrt{\omega_{MDP}^T \Sigma \omega_{MDP}}$). This method aims to spread investments across assets with varying degrees of volatility and correlation to achieve maximum diversification benefits. The MDP can be depicted as follows:

$$\max\left\{ MDP = \frac{\omega_{MDP}\sigma}{\sqrt{\omega_{MDP}^T \Sigma \omega_{MDP}}} \middle| \mu_p = \sum_{i=1}^{N} \omega_i r_i;\ \sigma_p = \sqrt{\sum_{i,j}^{N} \sigma_{ij}\omega_i\omega_j};\ \sum_{i=1}^{N} \omega_i = 1 \right\} \tag{11}$$

where σ is an $N \times 1$ vector of the asset return volatilities.

4. Results

4.1. Descriptive Statistics and Asset Betas

This section presents the analysis and results based on historical return data covering 58 months from August 2019 to May 2024.[5] Table 1 presents a summary profile of the companies listed on the stock exchange. In terms of sector composition, there are two stocks in automotive (TTS and VBH); one each in banking (BCN), education (FBL), insurance (FIL), retail (RBG) and tourism (PDM); three in investment (FHL, KGF and VIL), five in manufacturing and wholesale (PBP, FMF, PGI, APP, RCF); and three in telecommunications and media (ATH, CFL, FTV). The annualized return is calculated using the compound method, considering the share price on the date the company was listed and the share price on 31 May 2024. Furthermore, 13/19 companies have market capitalization below 5%: APP, BCN, CFL, FBL, FIL, FTV, KFL, KGF, PBP, PDM, PGI, RCF and VBH (See Figure 1).

Table 1. Profile of listed stocks on SPX.

Symbol	Name	Date Listed	Sample End Date	Number of Trading Days	Sector	Date Listed Market Closing Price	Market Closing Price on 31 May-2024	Financial Year End	Annualized Return	Market Capitalization in Millions of FJD (% of Market Cap.)
APP	Atlantic & Pacific Packaging Company Ltd.	17-Aug-98	31-May-24	6502.98	Manufacturing and wholesale (Packaging)	$0.70	$3.09	30-Jun	5.75%	24.7 (0.72)
ATH	Amalgamated Telecom Holdings Ltd.	18-Apr-02	31-May-24	5577.83	Telecommunications and media (telecommunications and mobile services)	$1.14	$2.00	30-Jun	2.54%	1029.0 (30.01)
BCN	BSP Convertible Notes Ltd.	11-May-10	31-May-24	3544.57	Banking (Financial services)	$5.25	$31.00	31-Dec	12.63%	95.0 (2.77)
CFL	Communications (Fiji) Ltd.	20-Dec-01	31-May-24	5659.99	Telecommunications (Radio broadcasting)	$1.15	$6.17	31-Dec	7.48%	23.4 (0.68)
FBL	Free Bird Institute Ltd.	2-Feb-17	31-May-24	1846.85	Education	$2.00	$3.65	31-Dec	8.21%	8.7 (0.25)
FHL	Fijian Holdings Ltd.	20-Jan-97	31-May-24	6899.28	Investment (investment chain)	$0.18	$0.95	30-Jun	6.08%	304.6 (8.89)
FIL	FijiCare Insurance Ltd.	7-Dec-00	31-May-24	5920.96	Insurance	$0.60	$17.99	31-Dec	14.48%	159.2 (4.64)
FMF	FMF Foods Ltd.	27-Jul-79	31-May-24	11308.93	Manufacturing and wholesale (flour and related products)	$0.06	$1.78	30-Jun	7.56%	264.0 (7.70)

Table 1. *Cont.*

Symbol	Name	Date Listed	Sample End Date	Number of Trading Days	Sector	Date Listed Market Closing Price	Market Closing Price on 31 May-2024	Financial Year End	Annualized Return	Market Capitalization in Millions of FJD (% of Market Cap.)
FTV	Fiji Television Ltd.	24-Apr-97	31-May-24	6834.38	Telecommunications (Television broadcasting)	$1.02	$2.00	30-Jun	2.48%	20.6 (0.60)
KFL	Kontiki Finance Ltd.	4-Jul-18	31-May-24	1489.91	Investment (Financial services)	$1.14	$1.13	30-Jun	−0.15%	101.1 (2.95)
KGF	Kinetic Growth Fund Ltd.	16-Dec-04	31-May-24	4906.06	Investment (investment chain)	$1.05	$1.20	31-Dec	0.69%	4.6 (0.13)
PBP	Pleass Global Ltd.	4-Feb-09	31-May-24	3862.85	Manufacturing and wholesale (Bottled water)	$0.94	$7.95	31-Dec	13.93%	56.8 (1.66)
PDM	Port Denarau Marina	14-Aug-19	31-May-24	1209.60	Tourism (Tourism and hospitality)	$1.40	$2.30	31-Jul	10.34%	89.2 (2.60)
PGI	Pacific Green Industries	5-Jun-01	31-May-24	5796.69	Manufacturing and wholesale (Furniture)	$1.90	$1.08	31-Dec	−2.46%	8.2 (0.24)
RBG	RB Patel Group Ltd.	17-Jul-01	31-May-24	5767.69	Retail (Supermarket chain)	$0.21	$3.09	30-Jun	11.75%	463.5 (13.52)
RCF	The Rice Company of Fiji Ltd.	20-Jan-97	31-May-24	6899.28	Manufacturing and wholesale (Rice)	$0.50	$9.80	30-Jun	10.87%	58.8 (1.72)
TTS	Toyota Tsusho (South Sea) Ltd.	7-Jun-79	31-May-24	11343.45	Automotive (automotive trading)	$1.95	$20.00	31-Mar	5.17%	280.6 (8.19)

Table 1. *Cont.*

Symbol	Name	Date Listed	Sample End Date	Number of Trading Days	Sector	Date Listed Market Closing Price	Market Closing Price on 31 May-2024	Financial Year End	Annualized Return	Market Capitalization in Millions of FJD (% of Market Cap.)
VBH	V B Holdings Ltd.	1-Nov-01	31-May-24	5693.82	Automotive (property and fleet management)	$1.28	$6.00	31-Dec	6.84%	12.8 (0.37)
VIL	Vision Investments Ltd.	29-Feb-16	31-May-24	2080.90	Investment (investment chain)	$1.70	$4.09	31-Mar	10.63%	423.4 (12.35)

Notes: Number of trading days per year = 252, and total number of trading days from the day of listing to the sample end date is computed using Excel formula: $= \left(\left(\frac{DAYS(D3,C3)}{365} \right) * 252 \right)$, where D3 = Sample end date and C3 = date the respective company was listed. Annualized return is computed as: $= \left(\left(\frac{H3}{G3} \right)^{\left(\frac{1}{E3} \right)} - 1 \right) * 252$, where H3 is the market closing price on 31 May 2024, G3 = market closing price on the day of listing, and E3 = number of trading days per year. Sun Insurance Company (SUN) was listed on 15 August 2024, hence excluded from the sample. At the time of listing, SUN's market cap was FJD 144 million (4.03%). All the companies are based in and operating in Fiji. Source: authors' own computation and compilation based on data from SPX (2024a; 2024b, p. 3).

Table 2. Descriptive statistics (monthly return data from August 2019 to May 2024).

Statistics	APP	ATH	BCN	CFL	FBL	FHL	FIL	FMF	FTV	KFL	KGF	PBP	PDM	RBG	RCF	TTS	VBH	VIL	DMCWPI
Mean Return	0.013	−0.006	0.006	0.002	0.008	−0.003	0.038	0.002	−0.009	0.004	0.001	0.022	0.011	−0.008	0.004	0.010	−0.003	0.001	−0.001
Standard Error	0.007	0.008	0.006	0.002	0.009	0.017	0.008	0.015	0.012	0.008	0.001	0.010	0.008	0.013	0.006	0.008	0.004	0.010	0.004
Sample Variance	0.003	0.003	0.002	0.000	0.004	0.018	0.003	0.014	0.009	0.004	0.000	0.006	0.004	0.010	0.002	0.003	0.001	0.006	0.001
Kurtosis	15.004	2.511	4.218	26.266	12.000	1.942	2.142	35.988	14.730	2.524	41.424	17.674	4.544	39.718	30.123	10.773	6.453	10.173	3.140
Skewness	2.792	−0.030	0.708	4.005	1.427	0.730	1.333	4.791	1.561	0.897	6.234	3.702	1.905	−5.520	4.680	2.632	−0.365	1.584	0.324
Range	0.387	0.363	0.294	0.130	0.547	0.740	0.308	1.122	0.810	0.350	0.045	0.565	0.323	0.892	0.384	0.390	0.194	0.563	0.195
Minimum	−0.088	−0.178	−0.136	−0.034	−0.243	−0.322	−0.069	−0.340	−0.325	−0.168	0.000	−0.121	−0.089	−0.679	−0.095	−0.104	−0.111	−0.245	−0.096
Maximum	0.299	0.185	0.158	0.095	0.304	0.418	0.239	0.782	0.485	0.182	0.045	0.443	0.234	0.213	0.289	0.286	0.083	0.318	0.099
N (Months)	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58

Source: authors' own computation based on data from SPX (2024a).

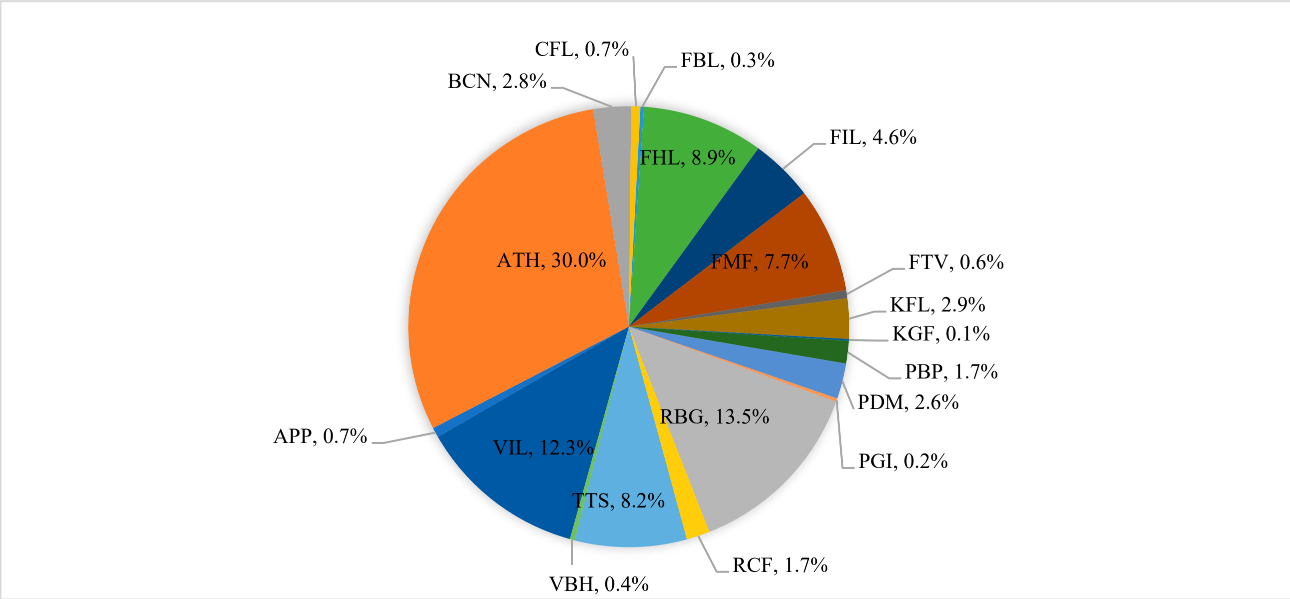


Figure 1. Market capitalization (%). Source: SPX (2024a, 2024b) and authors' own computation.

For analysis, we considered 18/19 companies listed on SPX at the time of study. We excluded one stock because it had no variability in its share price. Table 2 provides the descriptive statistics (mean returns, standard errors, variances, kurtosis, skewness, range, minimum and maximum returns) for the 18 stocks included in our analysis, and the market-weighted price index (MWPI) published by the South Pacific Stock Exchange (SPX). These data are sourced from SPX (2024a), and they form the basis for understanding the performance and risk characteristics of the assets in our portfolio.

Table 3 presents the beta coefficient and the respective *p*-values, the expected return, standard deviation, and market-adjusted return using the Capital Asset Pricing Model (CAPM). The beta values indicate the sensitivity of each asset's returns to market movements. The statistically significant values of stock beta (denoted by ***, **, *) highlight the strength of the asset's movement with respect to the overall market.

Table 3. Asset return–risk (annualized), adjusted return and trade profile.

Symbol	Beta	*p*-Value	Expected Return $E(r_i)$	Standard Deviation	Market (CAPM)—Adj. Return, $\bar{r}_{i,m}$	Number of Trades (Jul 2019–May 2024)	Consideration—FJD [Volume Traded]
APP	0.021	0.928	0.152	0.183	0.053	93	$201,269 [99,619]
ATH	1.258 ***	<0.001	−0.076	0.205	−0.022	817	$4,198,696 [2,079,256]
BCN	0.379 **	0.053	0.074	0.156	0.031	307	$3,381,869 [123,836]
CFL	0.118 *	0.072	0.022	0.052	0.047	88	$482,651 [75,774]
FBL	−0.043	0.881	0.095	0.228	0.057	93	$803,486 [266,528]
FHL	1.983 ***	<0.001	−0.031	0.459	−0.063	7016	$8,286,653 [8,244,432]

Table 3. *Cont.*

Symbol	Beta	p-Value	Expected Return $E(r_i)$	Standard Deviation	Market (CAPM)—Adj. Return, $\bar{r}_{i,m}$	Number of Trades (Jul 2019–May 2024)	Consideration—FJD [Volume Traded]
FIL	−0.012	0.963	0.452	0.202	0.055	439	$4,346,235 [598,217]
FMF	1.591 ***	0.001	0.024	0.407	−0.041	166	$4,019,393 [2,178,942]
FTV	0.715 *	0.084	−0.110	0.329	0.010	119	$1,485,997 [363,014]
KFL	0.387	0.160	0.044	0.218	0.030	820	$10,015,865 [10,190,076]
KGF	0.007	0.808	0.015	0.022	0.054	23	$90,111.80 [78,481]
PBP	−0.073	0.827	0.262	0.262	0.059	103	$440,551 [139,544]
PDM	−0.032	0.903	0.136	0.205	0.056	1273	$36,812,195 [23,808,791]
RBG	−1.131 ***	0.007	−0.095	0.341	0.127	862	$9,167,304 [2,723,851]
RCF	−0.068	0.731	0.052	0.154	0.059	257	$761,443 [76,842]
TTS	0.036	0.888	0.124	0.199	0.052	85	$8,616,426 [605,629]
VBH	−0.101	0.412	−0.036	0.097	0.061	41	$280,397 [42,590]
VIL	0.996 ***	0.002	0.012	0.267	−0.006	295	$30,855,896 [7,530,672]

Notes: Average per month market return (change in MWPI) over the sample period is $r_m = -0.055\%$; risk-free rate, $r_f = 5.44\%$ p.a; hence, the monthly rate based on $r_{f_{j12}} = \left\{(1+r_f)^{\frac{1}{12}} - 1\right\} = 0.004424081$; monthly market adjusted return is $r_{i,m} = \left\{r_{f_{j12}} + \beta_i\left(r_m - r_{f_{j12}}\right)\right\}$, and this is annualized as $\bar{r}_{i,m} = \left\{(1+r_{i,m})^{12} - 1\right\}$; ***, **, and * indicates significance at 1%, 5% and 10% level, based on p-values. Source: authors' own computation based on data from SPX (2024a).

For example, assets with $\beta > 1$ (ATH—telecommunications and media, FHL—investment, and FMF—manufacturing and wholesale) perform better (worse) than the market if the market moves positively (negatively). Similarly, assets with $0.5 < \beta < 1$ are FTV (telecommunications and media) and VIL (investment), which perform similarly to the market, although proportionately lower than the market. Other stocks with beta $0.1 < \beta < 0.5$ are BCN (banking), CFL (telecommunications and media), KFL (finance), and TTS (automotive), and others are positive but with a small beta $(0 < \beta \leq 0.1)$ (APP—manufacturing and wholesale, KGF—investment, and TTS—automotive). On the other hand, just one stock was noted to have $\beta < -1$, which was RBG (retail). Therefore, beta-adjusted annualized returns can provide deeper insight into a stock's performance against its risk profile for potential investors. Subsequently, using beta-adjusted returns can facilitate robust portfolio construction.

Considering the returns, we note that APP has a low beta (0.021), although its annualized average return is 15.2%, with a relatively high volatility (18.3%). Noting the stock's performance against the market, we find that the market-adjusted return is just 1.3%, which is significantly lower than the expected return based on monthly price changes. Examining a few other stocks, we note that some stocks are highly positively correlated with the market (examples: ATH, FHL, FMF), and their market-adjusted returns are higher (in absolute terms) relative to the expected returns. Another observation we make is that one of the stocks (i.e., PBP—manufacturing and wholesale sector) has a positive average return, but since its beta is negative ($\beta = -0.07$), the market-adjusted return is negative (-0.8%). Interestingly, some stocks with negative beta, such as RBG (retail business) with a $\beta = -1.13$ and VBH (automotive sector) with a $\beta = -0.10$, have negative average returns of -9.5% and -3.6%, respectively. However, the respective annualized market-adjusted returns are positive, with RBG's return of 12.9% and VBH's of 1.5%. Moreover, stocks like FBL (education) and FIL (insurance) have $\beta = -0.043$ and $\beta = -0.012$, respectively. Their respective annualized average returns are 9.5% and 45.2%. However, their market-adjusted returns are 0.6% and 0.5%, respectively.

Furthermore, we note that stocks with betas $+/-$ 0.1 range included APP ($\beta = 0.021$), CFL ($\beta = 0.12$), FBL ($\beta = -0.043$), FIL ($\beta = -0.012$), KGF ($\beta = -0.007$), PBP ($\beta = -0.073$), PDM ($\beta = -0.032$), RCF ($\beta = -0.068$), TTS ($\beta = 0.036$) and VBH ($\beta = -0.10$) (Table 3). We note that the p-values of these betas are above the 10% level, hence they are not statistically significant within the 1–10 percent level. Nevertheless, some observations are in order. We note that some of the stocks with low betas have comparatively low numbers of trading over the sample period, and most of them are low-cap stocks with less than 5% over the market capitalization (Table 1). For example (Table 3), the total numbers of trades over the sample period for stocks like APP, CFL, FBL, KGF, TTS and VBH were 93, 88, 93, 23, 85, and 41, respectively. Moreover, some of these stocks have relatively low percentages of market capitalization (below 1%) (APP, CFL, FBL and KGF) (Table 1). Moreover, stocks like FIL, PDM and RCF have small and negative betas, indicating that these stocks' prices tend to move opposite to the market. Moreover, although these stocks (FIL, PDM and RCF) have relatively high numbers of trades (439, 1273 and 257, respectively) and slightly higher percentages of market capitalization (4.64%, 2.60% and 1.72%), we note that for stocks like RCF, the volume traded (76,842) and considerations (FJD 761,443) remain low compared to high beta stocks; and for FIL and PDM, the stocks are traded in large amounts, but infrequently. In any case, a small beta is indicative that the stock prices are not highly sensitive to the overall market.

Next, we construct a portfolio using the RMD method. As mentioned earlier, the P-matrix contains the R-square values obtained from the pairwise regression results between the asset returns (Table 4). Each element P_{ij} in the matrix indicates the proportion of the variance of asset i that is explained by asset j. For example, if we take two stocks, say ATH and APP, then P_{21} represents how much of the variance of ATH is explained by APP. This matrix is essential in understanding the interdependencies and co-movements between different assets. The diagonal elements are 1, indicating that each asset explains its own variance completely. The off-diagonal elements close to 0 suggest low explanatory power between the assets, indicating the potential for diversification. For instance, the low values of $\mathbf{P}$ in most of the off-diagonal elements suggest that the assets have low correlations with each other, hence are favorable for diversification.

As discussed above, the R-matrix is derived from the P-matrix by computing $1 - {\rho_{ij}}^2$ for each pair of assets (Table 5). This matrix represents the unexplained variations between the asset pairs, and they are crucial for the RMD optimization process. Each element R_{ij} in the R-matrix derived from the pairwise regression denotes the unexplained variations. High values in the R-matrix indicate significant unexplained variation, suggesting that these assets move independently of each other, which is ideal for maximizing diversification. For example, in Table 5, we note that $R_{2,1}$ (where the subscript 2,1 denotes asset in row 2, column 1) is high (0.995). This implies that the relationship between the returns of ATH and APP has a substantial amount of unexplained variation (see Table 4). Because of the close link between correlation and variations, we further confirm this by noting the low correlation coefficient (0.007, Table 6) between the two asset returns. Thus, including these two assets in a portfolio could enhance diversification.

Table 4. P-Matrix.

	APP	ATH	BCN	CFL	FBL	FHL	FIL	FMF	FTV	KFL	KGF	PBP	PDM	RBG	RCF	TTS	VBH	VIL
APP	1.000																	
ATH	0.005	1.000																
BCN	0.028	0.000	1.000															
CFL	0.398	0.064	0.001	1.000														
FBL	0.133	0.034	0.003	0.013	1.000													
FHL	0.001	0.002	0.011	0.048	0.072	1.000												
FIL	0.006	0.006	0.003	0.002	0.003	0.001	1.000											
FMF	0.013	0.009	0.003	0.001	0.072	0.033	0.019	1.000										
FTV	0.010	0.016	0.005	0.025	0.004	0.000	0.028	0.000	1.000									
KFL	0.000	0.001	0.060	0.001	0.000	0.034	0.022	0.011	0.040	1.000								
KGF	0.001	0.003	0.001	0.009	0.000	0.000	0.000	0.000	0.002	0.001	1.000							
PBP	0.029	0.001	0.001	0.000	0.003	0.001	0.001	0.005	0.001	0.000	0.165	1.000						
PDM	0.000	0.016	0.003	0.004	0.000	0.000	0.015	0.000	0.006	0.016	0.004	0.003	1.000					
RBG	0.000	0.054	0.032	0.005	0.009	0.005	0.003	0.000	0.039	0.000	0.000	0.000	0.020	1.000				
RCF	0.004	0.010	0.000	0.001	0.002	0.000	0.004	0.000	0.007	0.002	0.001	0.001	0.000	0.006	1.000			
TTS	0.001	0.002	0.001	0.000	0.003	0.001	0.000	0.002	0.002	0.054	0.001	0.000	0.021	0.002	0.005	1.000		
VBH	0.006	0.000	0.001	0.002	0.000	0.046	0.006	0.000	0.008	0.015	0.000	0.000	0.001	0.003	0.219	0.009	1.000	
VIL	0.001	0.012	0.003	0.000	0.007	0.000	0.003	0.000	0.015	0.099	0.000	0.000	0.006	0.001	0.023	0.105	0.001	1.000

Source: Authors' own computation based on data from SPX (2024a).

Table 5. R-Matrix.

	APP	ATH	BCN	CFL	FBL	FHL	FIL	FMF	FTV	KFL	KGF	PBP	PDM	RBG	RCF	TTS	VBH	VIL
APP	0.000																	
ATH	0.995	0.000																
BCN	0.972	1.000	0.000															
CFL	0.602	0.936	0.999	0.000														
FBL	0.867	0.966	0.997	0.987	0.000													
FHL	0.999	0.998	0.989	0.952	0.928	0.000												
FIL	0.994	0.994	0.997	0.998	0.997	0.999	0.000											
FMF	0.987	0.991	0.997	0.999	0.928	0.967	0.981	0.000										
FTV	0.990	0.984	0.995	0.975	0.996	1.000	0.972	1.000	0.000									
KFL	1.000	0.999	0.940	0.999	1.000	0.966	0.978	0.989	0.960	0.000								
KGF	0.999	0.997	0.999	0.991	1.000	1.000	1.000	1.000	0.998	0.999	0.000							
PBP	0.971	0.999	0.999	1.000	0.997	0.999	0.999	0.995	0.999	1.000	0.835	0.000						
PDM	1.000	0.984	0.997	0.996	1.000	1.000	0.985	1.000	0.994	0.984	0.996	0.997	0.000					
RBG	1.000	0.946	0.968	0.995	0.991	0.995	0.997	1.000	0.961	1.000	1.000	1.000	0.980	0.000				
RCF	0.996	0.990	1.000	0.999	0.998	1.000	0.996	1.000	0.993	0.998	0.999	0.999	1.000	0.994	0.000			
TTS	0.999	0.998	0.999	1.000	0.997	0.999	1.000	0.998	0.998	0.946	0.999	1.000	0.979	0.998	0.995	0.000		
VBH	0.994	1.000	0.999	0.998	1.000	0.954	0.994	1.000	0.992	0.985	1.000	1.000	0.999	0.997	0.781	0.991	0.000	
VIL	0.999	0.988	0.997	1.000	0.993	1.000	0.997	1.000	0.985	0.901	1.000	1.000	0.994	0.999	0.977	0.895	0.999	0.000

Source: Authors' own computation based on data from SPX (2024a).

Table 6. Correlation (C-matrix).

	APP	ATH	BCN	CFL	FBL	FHL	FIL	FMF	FTV	KFL	KGF	PBP	PDM	RBG	RCF	TTS	VBH	VIL	MCWPI
APP	1.000																		
ATH	0.070	1.000																	
BCN	0.167	0.021	1.000																
CFL	0.631	0.253	−0.035	1.000															
FBL	0.365	0.185	−0.053	0.112	1.000														
FHL	−0.033	−0.042	0.106	0.218	−0.269	1.000													
FIL	−0.078	−0.075	−0.050	−0.041	−0.052	0.023	1.000												
FMF	−0.113	0.094	0.053	0.024	−0.268	0.181	0.139	1.000											
FTV	−0.099	0.127	0.074	0.159	0.060	−0.009	−0.167	−0.021	1.000										
KFL	0.003	0.024	0.245	−0.037	−0.003	0.183	−0.149	0.103	0.201	1.000									
KGF	−0.029	0.052	−0.027	−0.096	−0.018	0.006	−0.016	−0.006	−0.043	−0.025	1.000								
PBP	0.170	−0.034	−0.025	0.006	0.055	−0.031	−0.025	−0.070	−0.032	0.008	0.406	1.000							
PDM	−0.015	−0.127	0.056	0.067	0.018	0.021	−0.123	−0.020	0.075	0.128	−0.063	−0.052	1.000						
RBG	0.004	−0.232	−0.180	−0.068	0.094	−0.071	−0.052	−0.020	−0.198	0.015	0.007	0.017	0.141	1.000					
RCF	0.061	−0.099	0.014	−0.031	−0.047	0.005	0.062	0.018	0.083	0.039	−0.028	−0.030	−0.007	0.080	1.000				
TTS	0.039	−0.047	0.036	0.009	0.051	−0.022	−0.006	0.041	0.050	0.231	−0.035	−0.012	0.145	−0.049	0.071	1.000			
VBH	0.075	0.012	−0.035	0.042	0.010	−0.214	−0.076	−0.019	0.088	0.121	0.021	0.016	−0.027	−0.052	−0.468	−0.095	1.000		
VIL	0.034	0.110	0.057	0.018	0.081	0.018	−0.057	−0.011	0.123	−0.315	0.005	−0.008	−0.075	−0.038	−0.151	−0.325	−0.024	1.000	
MCWPI	0.012	0.648	0.256	0.238	−0.020	0.455	−0.006	0.411	0.229	0.187	0.033	−0.029	−0.016	−0.350	−0.046	0.019	−0.110	0.394	1.000

Source: Authors' own computation based on data from SPX (2024a).

In Tables 6 and 7, we also present the correlation (C-matrix) and the covariance matrices, respectively. The correlation matrix (Table 6) displays the correlation coefficients (ρ_{ij}), indicating how (the direction) the returns of two assets move relative to each other, with values ranging from −1 to 1. Low correlation values suggest that combining these assets could enhance diversification. In Table 7, we present the covariances (σ_{ij}) between asset returns, which measure the degree to which two assets' returns change together. These matrices form the foundation for constructing mean-variance type portfolios, including the RMD portfolio.

Table 7. Covariance Matrix.

	APP	ATH	BCN	CFL	FBL	FHL	FIL	FMF	FTV	KFL	KGF	PBP	PDM	RBG	RCF	TTS	VBH	VIL
APP	0.033																	
ATH	0.003	0.042																
BCN	0.005	0.001	0.024															
CFL	0.006	0.003	0.000	0.003														
FBL	0.015	0.009	−0.002	0.001	0.052													
FHL	−0.003	−0.004	0.008	0.005	−0.028	0.211												
FIL	−0.003	−0.003	−0.002	0.000	−0.002	0.002	0.041											
FMF	−0.008	0.008	0.003	0.001	−0.025	0.034	0.011	0.166										
FTV	−0.006	0.009	0.004	0.003	0.004	−0.001	−0.011	−0.003	0.108									
KFL	0.000	0.001	0.008	0.000	0.000	0.018	−0.007	0.009	0.014	0.047								
KGF	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000							
PBP	0.008	−0.002	−0.001	0.000	0.003	−0.004	−0.001	−0.007	−0.003	0.000	0.002	0.069						
PDM	−0.001	−0.005	0.002	0.001	0.001	0.002	−0.005	−0.002	0.005	0.006	0.000	−0.003	0.042					
RBG	0.000	−0.016	−0.010	−0.001	0.007	−0.011	−0.004	−0.003	−0.022	0.001	0.000	0.002	0.010	0.116				
RCF	0.002	−0.003	0.000	0.000	−0.002	0.000	0.002	0.001	0.004	0.001	0.000	−0.001	0.000	0.004	0.024			
TTS	0.001	−0.002	0.001	0.000	0.002	−0.002	0.000	0.003	0.003	0.010	0.000	−0.001	0.006	−0.003	0.002	0.040		
VBH	0.001	0.000	−0.001	0.000	0.000	−0.010	−0.001	−0.001	0.003	0.003	0.000	0.000	−0.001	−0.002	−0.007	−0.002	0.009	
VIL	0.002	0.006	0.002	0.000	0.005	0.002	−0.003	−0.001	0.011	−0.018	0.000	−0.001	−0.004	−0.003	−0.006	−0.017	−0.001	0.071

Source: Authors' own computation based on data from SPX (2024a).

In Table 8, we present 10 different portfolios. These include equally weighted (1/N) (column I), minimum variance (column II), market portfolio (column III), semi-variance (column IV), maximum Sortino (based on target return of 6.75%) (column V), and maximum volatility skewness (column VI). In column VII, we present the newly proposed maximum RMD portfolio (column VII) and its variations, i.e., RMD portfolio with positive Sharpe (column VIII), and RMD portfolio with Sharpe and Sortino ratios exceeding the risk-free rate and target rate, respectively (column IX); and in column IX, we present the most diversified portfolio.

Table 8. Portfolios under different methods and scenarios.

	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)	(X)
Statistics	**1/N (Equally Weighted)**	**Minimum Variance**	**Max Sharpe (CAPM)**	**Semi-Variance (min. Downside)**	**Max Sortino (CAPM)**	**Max. Vol Skewness (U/D)**	**Max. RMD**	**Max. RMD with Positive Sharpe**	**Max. RMD with Positive Sharpe (≥RF) and Sortino (≥TR)**	**Most Diversified Portfolio**
Unadjusted return (annual)	0.0619	0.0239	−0.0075	0.0153	−0.0950	0.0394	0.0637	0.0798	0.0704	0.0443
CAPM-adjusted return (annual) (μ_p)	0.0359	0.0523	0.0786	0.0539	0.1274	0.0030	0.0341	0.0544	0.0426	0.0464
Standard deviation (σ_p)	0.0589	0.0175	0.1032	0.0213	0.3406	0.2046	0.0600	0.0604	0.0551	0.0318
Sharpe ratio (CAPM)	−0.3384	−0.1011	0.2348	−0.0255	0.2144	−0.2511	−0.3393	0.0010	0.7741	1.4578
Sortino ratio (CAPM)	−0.8419	−1.8969	0.1224	−3.3462	0.1917	−0.8499	−0.8425	−0.2724	0.0675	0.2898
Downside volatility (σ_{p_D})	47.18%	33.39%	65.02%	16.34%	69.80%	28.54%	46.90%	56.51%	50.13%	48.82%
Volatility skewness (Up-variance/ Down variance)	1.2538	3.9796	0.2896	26.201	0.1872	6.2719	1.2823	0.5950	0.9894	1.099
RMD coefficient	0.9295	0.4976	0.6295	0.0479	0.000	0.6246	0.9303	0.9184	0.92812	0.8558
Diversification Ratio (*Div. ratio*)	3.7577	2.8710	1.7876	1.1508	1.0000	1.5505	3.7507	3.3480	3.8872	4.6486
Asset					**Weights**					
APP	0.056	0.000	0.000	0.000	0.000	0.000	0.039	0.044	0.041	0.000
ATH	0.056	0.000	0.000	0.000	0.000	0.079	0.057	0.020	0.042	0.047
BCN	0.056	0.016	0.000	0.001	0.000	0.000	0.062	0.058	0.061	0.045
CFL	0.056	0.141	0.000	0.008	0.000	0.000	0.044	0.055	0.049	0.021
FBL	0.056	0.003	0.000	0.000	0.000	0.277	0.052	0.072	0.060	0.034
FHL	0.056	0.000	0.000	0.000	0.000	0.000	0.056	0.000	0.031	0.027
FIL	0.056	0.015	0.043	0.001	0.000	0.000	0.063	0.076	0.069	0.055
FMF	0.056	0.000	0.000	0.000	0.000	0.512	0.061	0.016	0.042	0.011
FTV	0.056	0.000	0.000	0.000	0.000	0.000	0.058	0.042	0.051	0.010
KFL	0.056	0.005	0.000	0.000	0.000	0.000	0.049	0.051	0.050	0.003
KGF	0.056	0.688	0.000	0.976	0.000	0.000	0.059	0.068	0.064	0.239
PBP	0.056	0.000	0.024	0.000	0.000	0.004	0.058	0.071	0.063	0.025
PDM	0.056	0.009	0.000	0.001	0.000	0.072	0.063	0.075	0.068	0.036
RBG	0.056	0.005	0.279	0.001	1.000	0.033	0.059	0.117	0.082	0.033
RCF	0.056	0.034	0.166	0.003	0.000	0.000	0.055	0.066	0.060	0.109
TTS	0.056	0.013	0.012	0.001	0.000	0.000	0.058	0.071	0.063	0.055
VBH	0.056	0.057	0.475	0.005	0.000	0.022	0.052	0.068	0.060	0.204
VIL	0.056	0.012	0.000	0.001	0.000	0.000	0.055	0.029	0.045	0.047
Number of stocks in portfolio (%)	100%	72%	33%	50%	6%	39%	100%	94%	100%	94%

Source: Authors' own computation based on specific method and data from SPX (2024a).

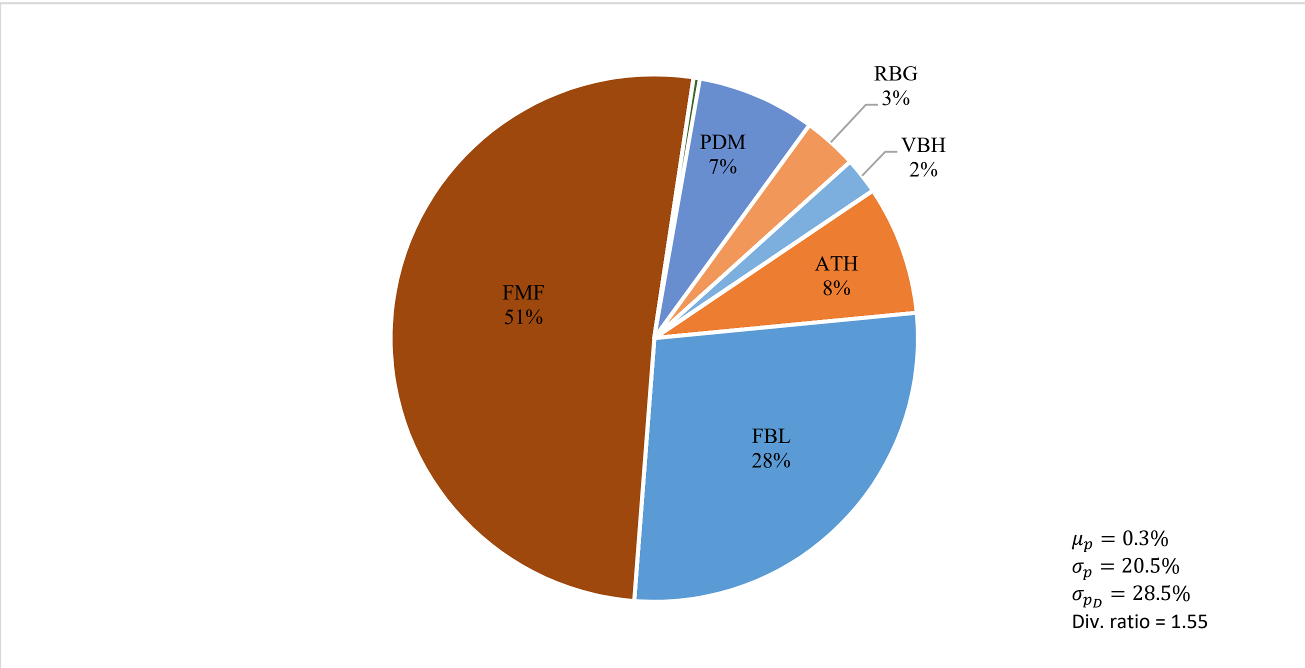
From these portfolios, we observe that three portfolios comprising 100% of the stock are the 1/N ($\mu_p = 3.6\%$, $\sigma_p = 5.9\%$) (column I), max. RMD ($mu_p = 3.4\%$, $\sigma_p = 6.0\%$) (column VII), and max. RMD with positive Sharpe and Sortino ($\mu_p = 4.26$, $\sigma_p = 5.5\%$) (column IX), with diversification ratios of 3.58, 3.75 and 3.89, respectively. However, these portfolios offer returns below the RF rate (of 5.44%), indicating that investing in all the stocks (100%) in either of the two portfolios (1/N and max. RMD) is not desirable. We noted that two portfolios—the max. Sortino (column V) and max. volatility skewness (column VI) have high standard deviations of 34.1% and 20.5%, respectively, and the respective returns are 12.7% and 0.3%. These two portfolios comprise 6% and 39% of the total stocks with diversification ratios of 1.0 and 1.6, respectively.

Moreover, a negative coefficient of Sharpe ratio could indicate possible underperformance of portfolios, and a negative Sortino ratio could indicate an 'unfair' risk–reward outcome. In our case, we note that when relaxing constraints on the Sharpe and Sortino ratios, there are five portfolios with negative Sharpe ratios (1/N, minimum variance, semi-variance, maximum volatility skewness, and max. RMD), and six portfolios with negative Sortino ratios (1/N, minimum variance, semi-variance, max. volatility skewness, max. RMD, and max. RMD with positive Sharpe).

Next, we analyze the portfolios against the RF rate (5.44%). We consider portfolios that offer returns similar to the RF rate, below the RF rate and above the RF rate.

4.2. Portfolios below the RF Rate and below 4%

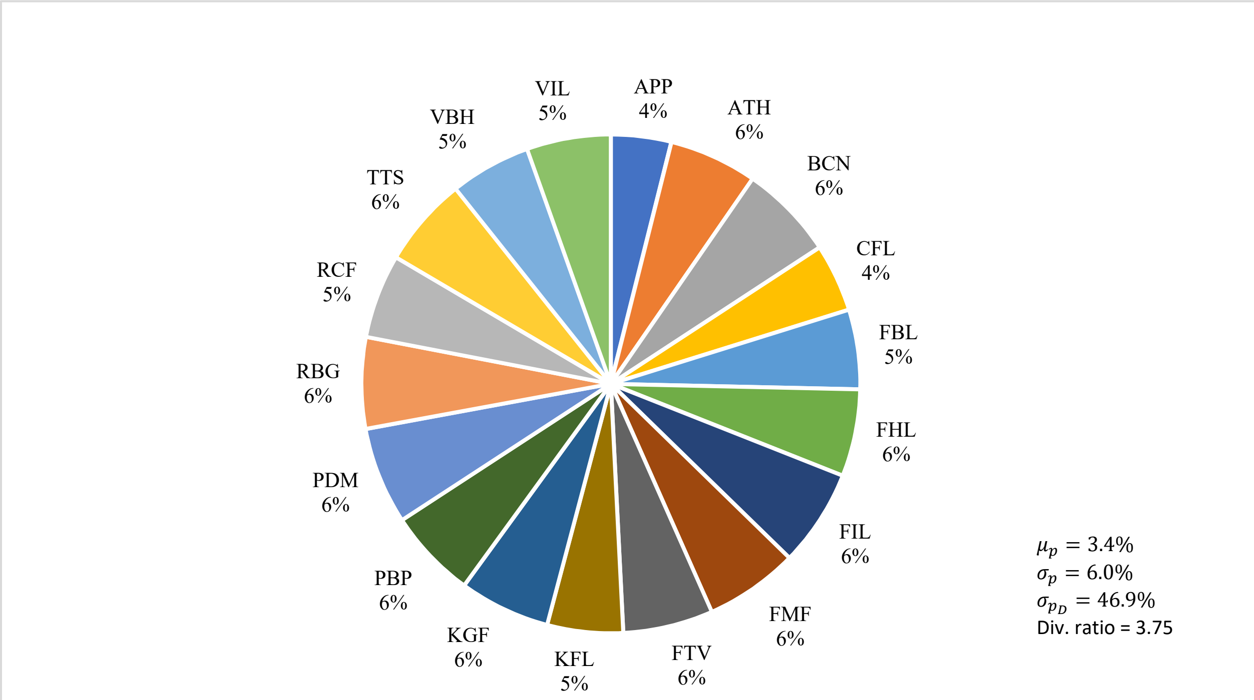
We find three portfolios have returns below the RF rate. These are, max. volatility skewness (column VI), max. RMD (column VII) and 1/N (column I). The portfolio that is farthest away from the RF rate is the max. volatility skewness (column VI), with a return and standard deviation of 0.3% and 20.5%, respectively. This portfolio comprises 39% of all the stocks, with a relatively large allocation in FMF and FBL, and a diversification ratio of 1.55 (see Figure 2a).



Source: Authors' own estimation

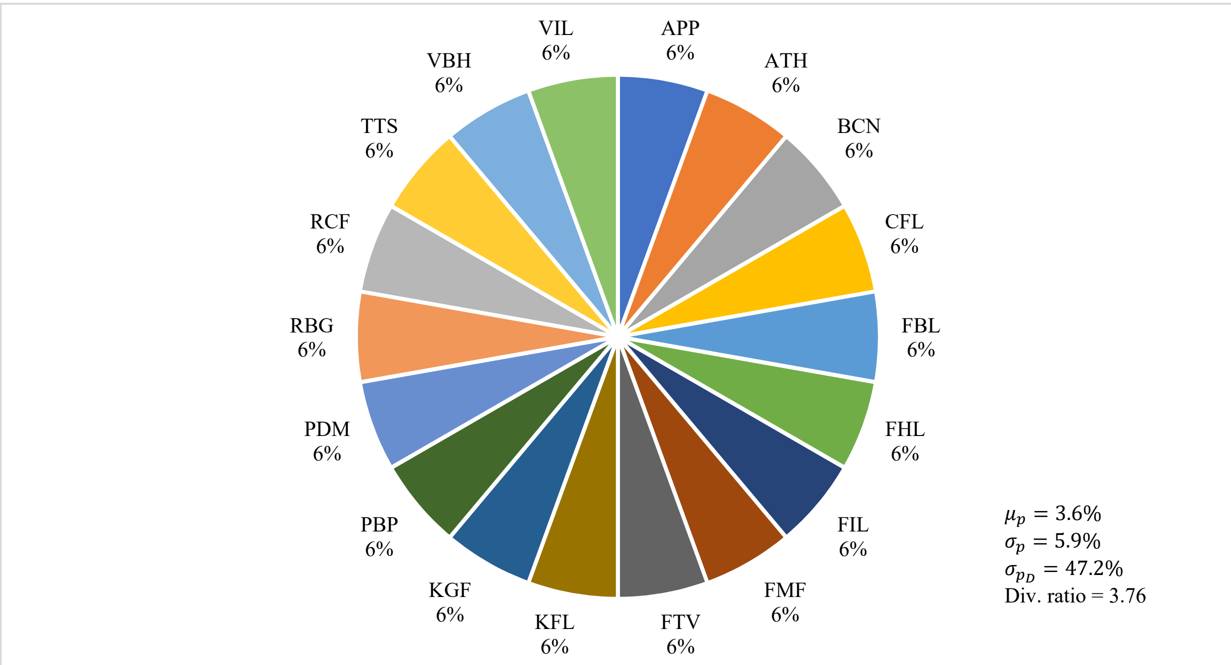
(**a**)

Figure 2. *Cont.*



Source: Authors' own estimation

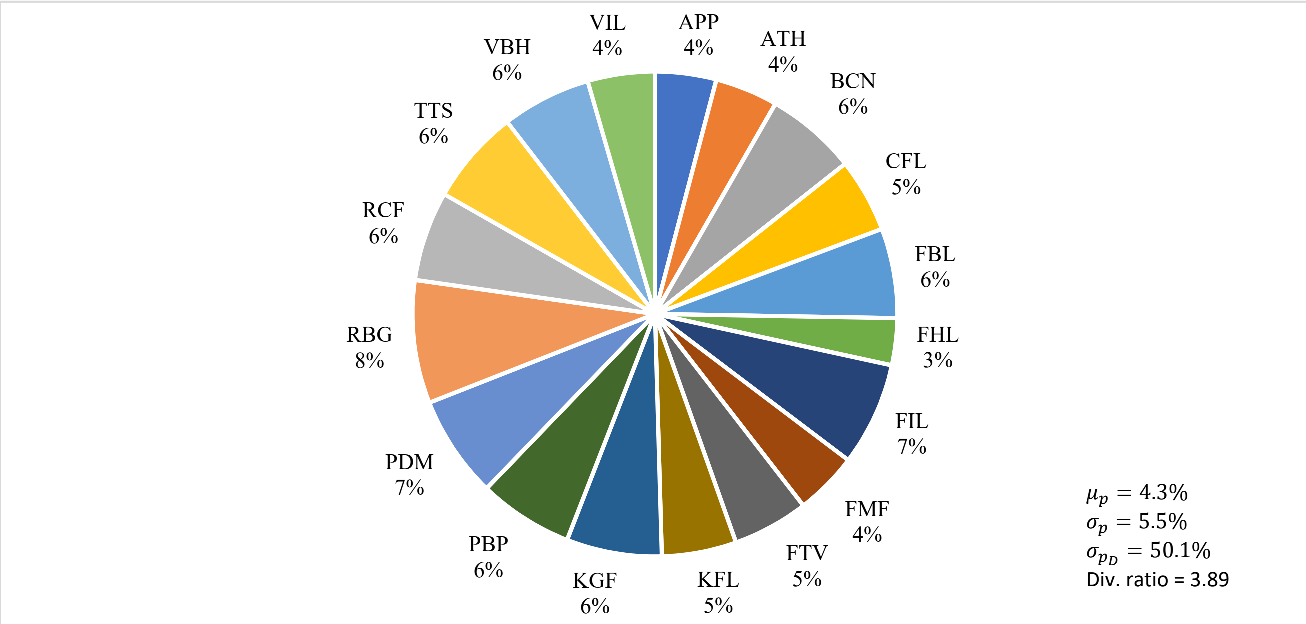
(**b**)



Source: Authors' own estimation

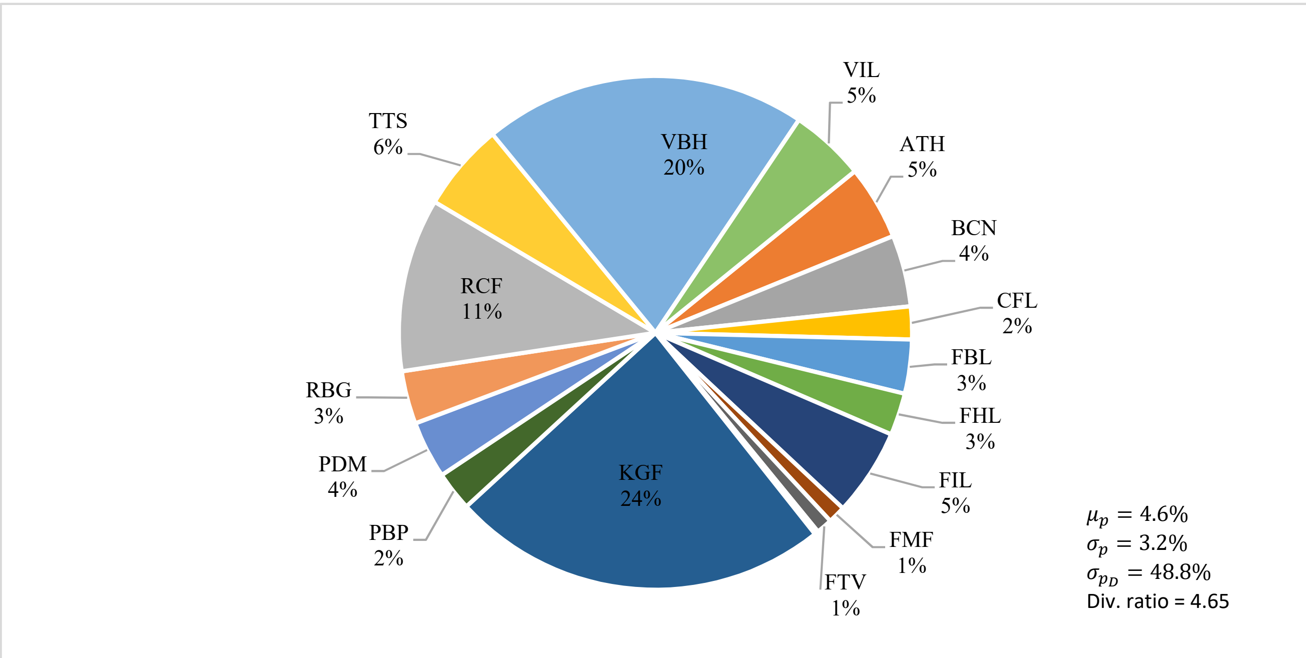
(**c**)

Figure 2. *Cont.*



Source: Authors' own estimation

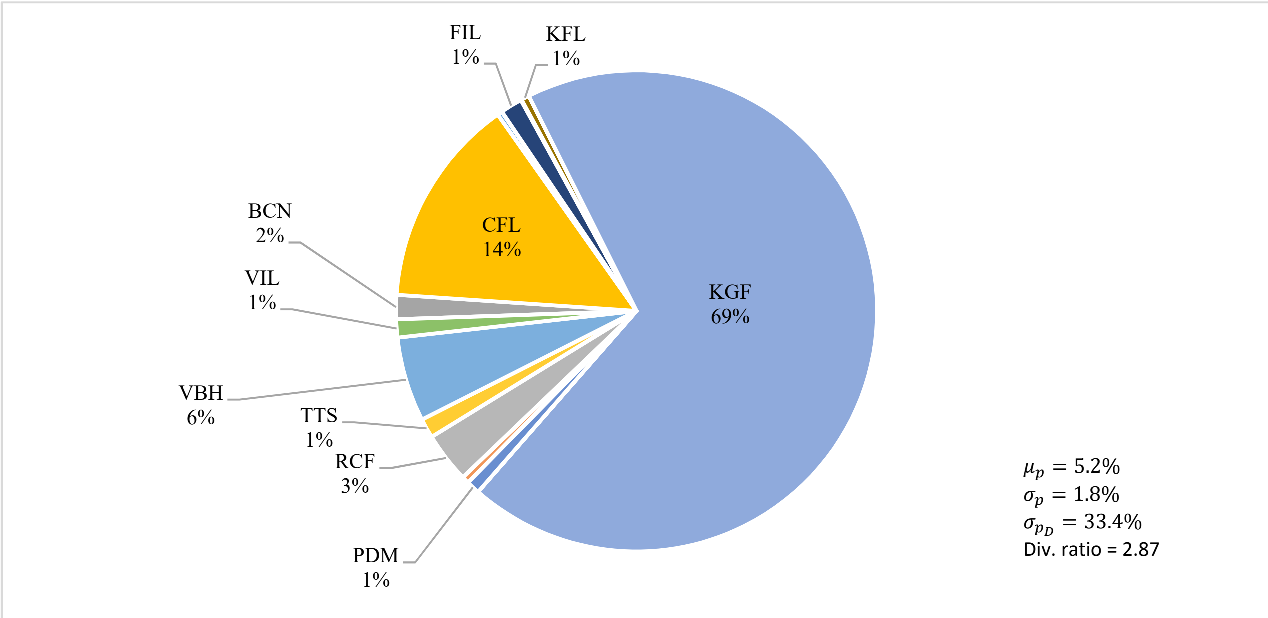
(**d**)



Source: Authors' own estimation

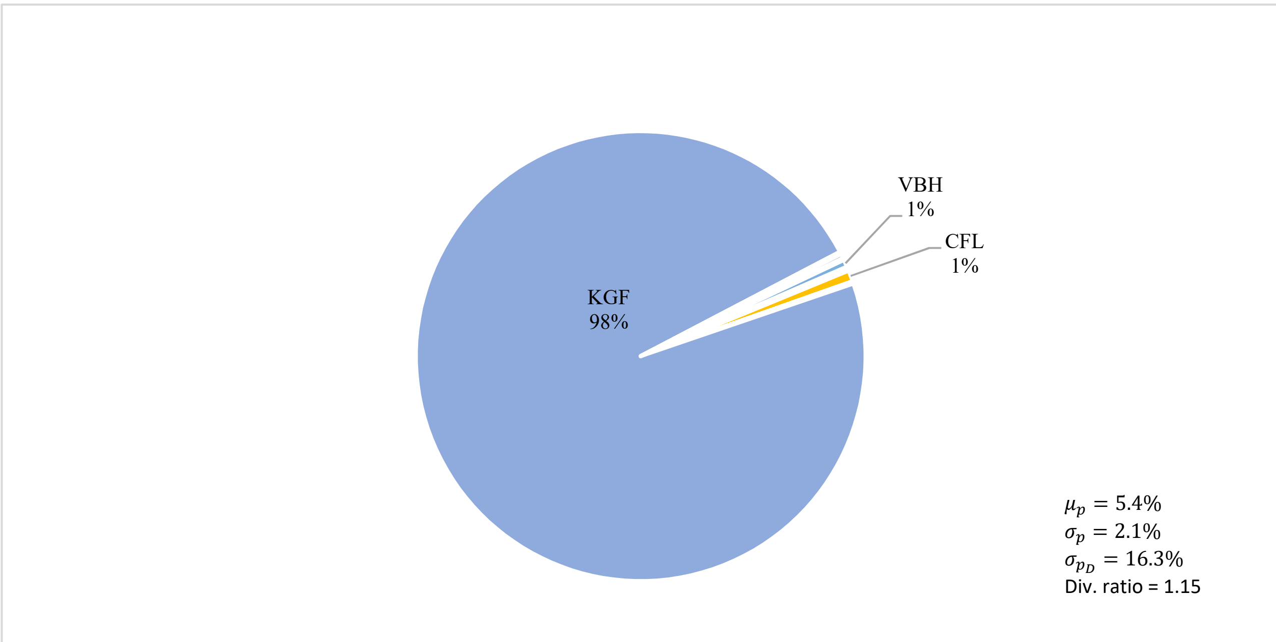
(**e**)

Figure 2. *Cont.*



Source: Authors' own estimation

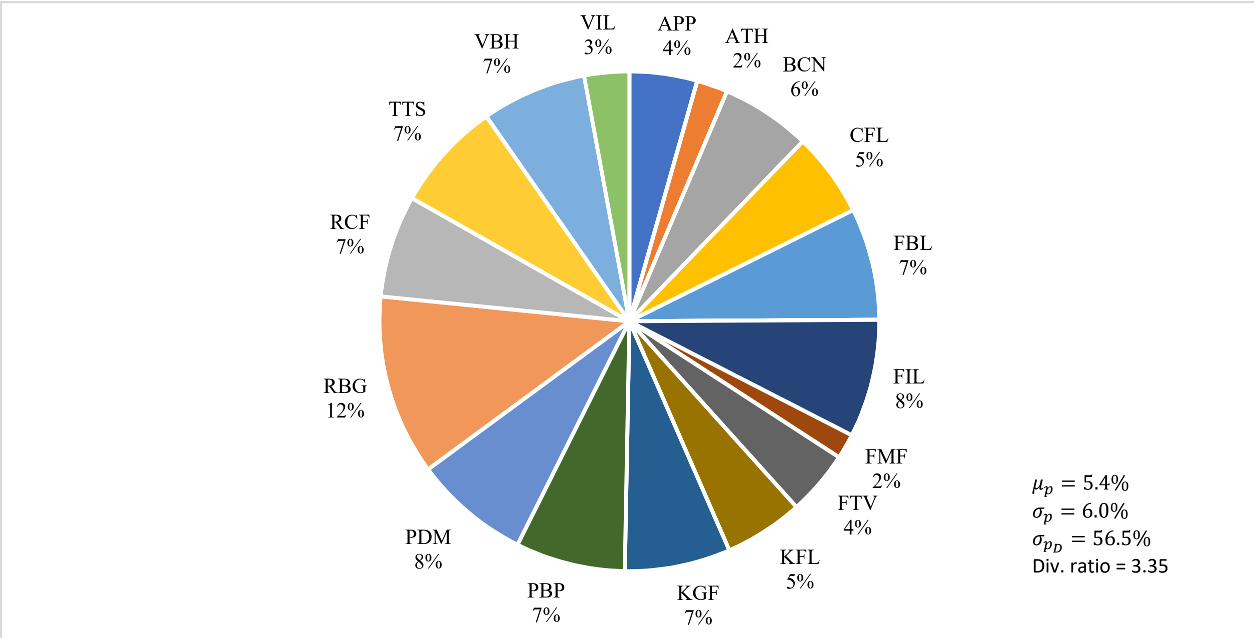
(**f**)



Source: Authors' own estimation

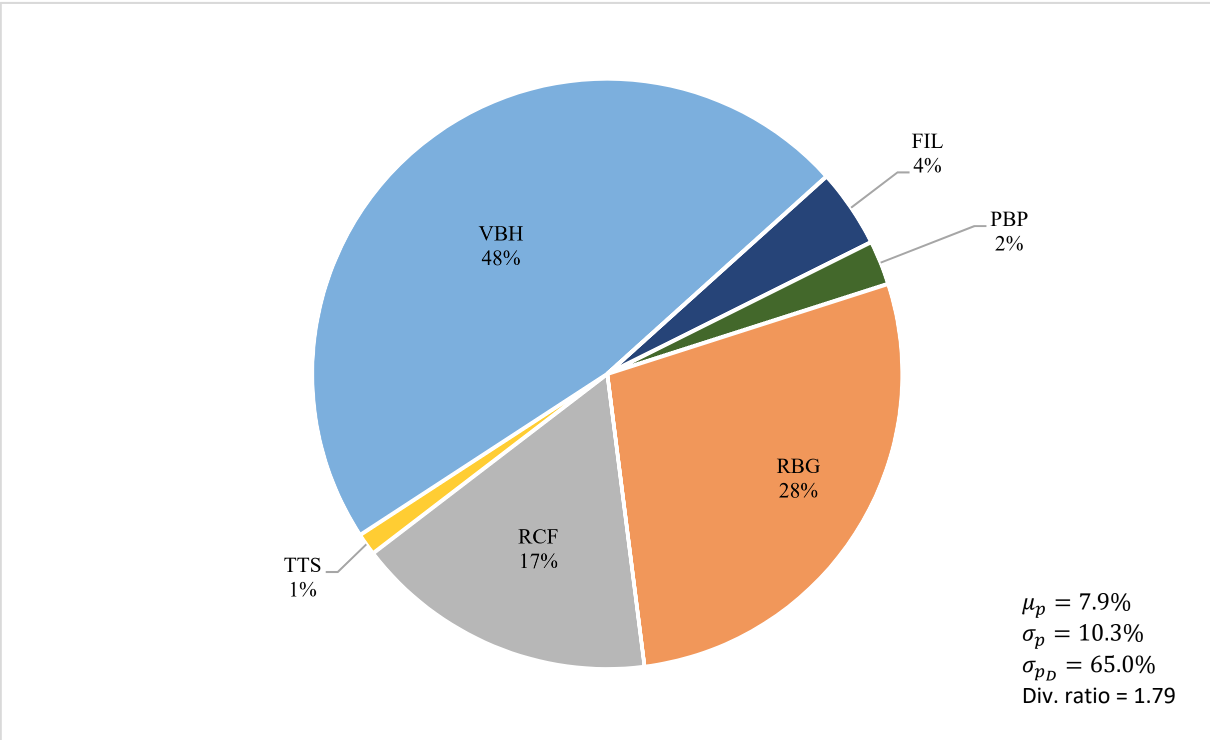
(**g**)

Figure 2. *Cont.*



Source: Authors' own estimation

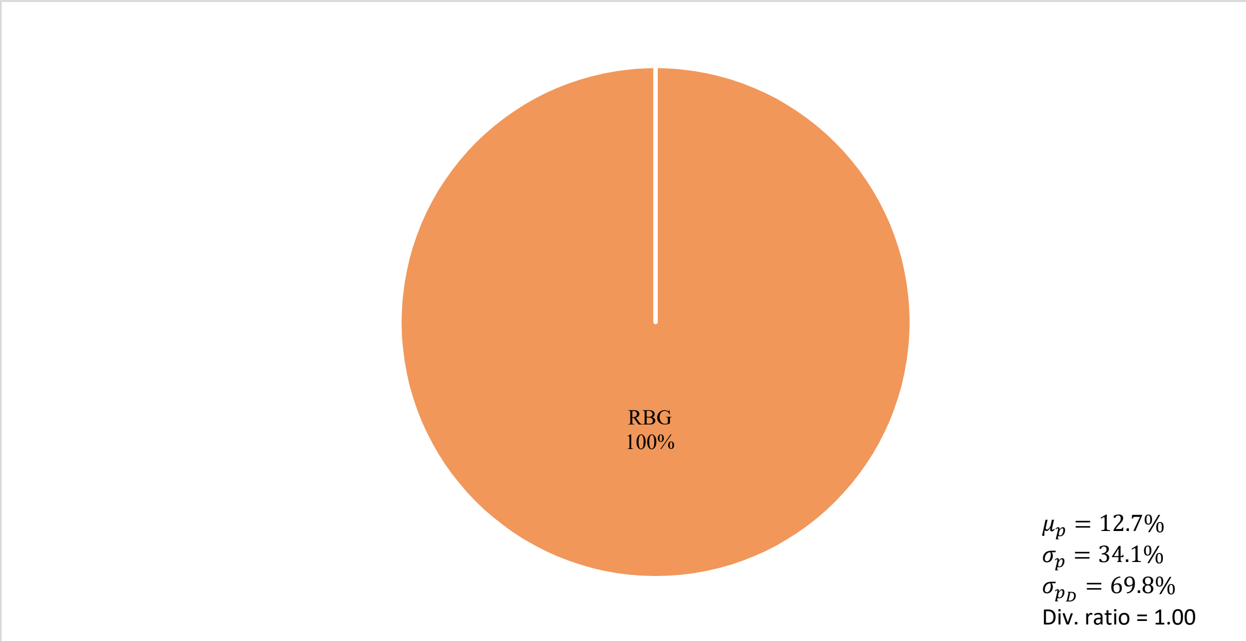
(**h**)



Source: Authors' own estimation

(**i**)

Figure 2. *Cont.*



Source: Authors' own estimation

(j)

Figure 2. (**a**) Max. volatility skewness. (**b**) Max. RMD portfolio. (**c**) 1/N portfolio. (**d**) Max. RMD with positive Sharpe and Sortino. (**e**) Most diversified portfolio. (**f**) Minimum variance portfolio. (**g**) Semi-variance portfolio. (**h**) Max. RMD with positive Sharpe. (**i**) Market portfolio (Max. Sharpe). (**j**) Maximum Sortino.

Next is the max. RMD portfolio (column VII), which comprises 100% (all) of the stocks and yields a return of 3.4% and standard deviation of 6.0% (Figure 2b). Compared to the equally weighted (1/N) portfolio, we note that the max. RMD portfolio yields a marginally lower return (column I, Figure 2c), although both portfolios' returns are below the RF rate.

4.3. Portfolios between 4 and 5% and below RF Rate (5.44%)

Next, we examine the portfolios that offer returns closer to the RF rate. We find that two portfolios could potentially achieve a return between 4 and 5%. These are the max. RMD with positive Sharpe and Sortino ($\mu_p = 4.3\%, \sigma_p = 5.5\%$) (column IX) and the most diversified portfolio ($\mu_p = 4.6\%$, $\sigma_p = 3.2\%$) (column X). As noted from the respective figures, the max. RMD with positive Sharpe and Sortino comprises all the stocks, although the distribution is slightly uneven (Figure 2d), whereas the most diversified portfolio comprises 94% of all the stock, with a relatively greater proportion allocated to VBH, KGF and RCF (Figure 2e). In both portfolios, we note positive Sharpe and Sortino ratios (although the former is based on setting the constraints explicitly), indicating relatively good portfolio performance and reasonable risk-reward tradeoffs.

4.4. Portfolios above 5% and below the Risk-Free Rate

We note that three portfolios yield returns above 5% p.a. These are minimum variance ($\mu_p = 5.2\%$, $\sigma_p = 1.8\%$, column II), semi-variance ($\mu_p = 5.4\%$, $\sigma_p = 2.1\%$, column IV) and max. RMD with positive Sharpe ($\mu_p = 5.4\%, \sigma_p = 6.0\%$, column VIII). These portfolios comprise 72%, 50% and 94% of the total stocks, respectively. Both the minimum variance (Figure 2f) and semi-variance (Figure 2g) indicate a relatively large allocation to KGF. Moreover, both the minimum variance and semi-variance portfolios have negative Sharpe ratios, with returns marginally lower than the RF rate, and the latter portfolio is relatively less diversified (Div. ratio = 1.15). The max. RMD portfolio was constrained by setting the Sharpe ratio to be positive (column VIII). In this case, we could obtain a return of

5.4%, which coincided with the RF rate, with a relatively high diversification ratio (Div. ratio = 3.35) (Figure 2h). Moreover, using a positive Sharpe ratio as a guide, this portfolio (max. RMD with positive Sharpe) performed better than seven other portfolios (1/N, minimum variance, semi-variance, max. volatility, max. RMD, max. RMD with positive Sharpe and Sortino, and most diversified portfolio).

4.5. Portfolios above the Risk-Free Rate (5.44%)

Next, we examine portfolios with returns higher than the risk-free rate. We find that two portfolios meet this criterion. We note that the market portfolio (column III) offers a return of 7.9%, with standard deviation of 10.3% (Sharpe ratio = 0.23, Sortino ratio = 0.12). This portfolio comprises 33% of all the stocks, has downside volatility of 65.0%, and a diversification ratio of 1.79 (see Figure 2i). The other portfolio is the max. Sortino (with target return of 6.45%), which offers a return of 12.7%, with standard deviation of 34.0% (column V). However, this portfolio is least diversified (Div. ratio = 1.0), comprising only one stock (RBG) (Figure 2j). While the return is higher than that of all the other portfolios, we note that this portfolio has the highest downside volatility (69.8%), lowest volatility skewness (0.19), and is the least robust (RMD coefficient = 0.00).

5. Conclusions

In this study, we implemented a newly proposed method of portfolio construction proposed by Bielstein et al. (2023), in a small country's stock market. While earlier studies have examined the small market using different methods, we differentiate this study in the following ways. First, earlier studies have used lower values for the risk-free and target rates (scenarios (see Kumar et al. 2022 and Kumar and Stauvermann 2022)). We considered a relatively high risk-free rate (5.44% p.a.) and target rate (6.75% p.a.), based on the 1-year government bond rate and superannuation rate of 2019 (starting period of our sample), respectively. Second, we have illustrated the applicability of Bielstein et al.'s (2023) RMD approach, in addition to including additional constraints like ensuring positive Sharpe and Sortino ratios. Additionally, using the recent data (from August 2019 to May 2024), we computed the CAPM-adjusted return for each stock based on the market-weighted price index (MWPI), and simulated different portfolios.

To keep the analysis manageable, we have separated the portfolios into four main categories. Considering the risk-free rate, we divided the portfolios into below 4%, between 4–5%, between 5% and the risk-free rate, and above the risk-free rate. We note that the max. RMD and 1/N portfolios provide comparable returns (3.4–3.6%) and contain all the stocks in the sample, although the latter is simpler to implement. To secure returns between 4 and 5%, we find that max. RMD with positive Sharpe and Sortino (as constraints) and the most diversified portfolio offer comparable returns (4.5–4.6%), although the latter has slightly lower standard deviation and downside volatility and contains 94% of all the stocks. Next, for portfolios with returns between 5% and the risk-free rate, we obtained three portfolios—the minimum variance, the semi-variance and the max. RMD with positive Sharpe; the latter coincides with the risk-free rate and contains the largest number (94%) of the stocks compared to the other two. Therefore, if diversification is the objective of an investor with at most a risk-free rate of return, then max. RMD with positive Sharpe can be considered. On the other hand, depending on the level of risk-averseness of an investor, the minimum-variance or the semi-variance portfolio can be recommended, with the latter having the lowest downside volatility.

We find two portfolios with returns above the risk-free rate—the market portfolio (max. Sharpe) and the maximum Sortino. Both portfolios are comparable in terms of the downside volatility (65–70%). Moreover, the latter portfolio (max. Sortino) yields the highest return (12.7%), although this portfolio is the least diversified (contains just a single stock), with the largest standard deviation and downside volatility among all the portfolios. On the other hand, the market portfolio (max. Sharpe) offers a return of 7.9%, with a reasonable level of diversification (contains 33% of the total stock). Therefore, investors with appetite for risk, diversification, and returns above the risk-free rate can consider the market portfolio as a suitable alternative.

While the results offer interesting insights, some caveats are in order. We have used the risk-free (1-year government bond) rate and the target return of the superannuation fund rate for the year 2019. However, we have noted that in subsequent years, there were significant declines in the 1-year government bond rates. The 1-year government bond rates for the years 2019, 2020, 2021, 2022, 2023 and 2024 were 5.44%, 3.22%, 1.01%, 0.14%, 0.13% and 0.93% (averaging around 1.74%). Furthermore, the savings rates on deposits offered by banks were 1.32% in 2018, 1.10% in 2019, 0.54% in 2020, 0.42% in 2021, 0.39% in 2022, 0.38 in 2023[6] and 0.31% in 2024 (averaging around 0.64%).[7] Based on the two average rates (savings and 1-year government bond), one can consider using the average of these rates as a relatively conservative risk-free rate. Similarly, the average of the interest rate offered by the superannuation fund from 2019 to 2024 can be applied to the target return.[8] We also acknowledge that our stock price data are from August 2019 to May 2024, which, although providing a substantial period for analysis, may not capture long-term market trends and behaviors. Market conditions and economic factors can vary significantly over different time periods, and a longer dataset might provide more robust conclusions. Although the study's focus on market-adjusted returns is a significant strength, it also introduces complexity in the analysis. In computing the market-adjusted return for each stock based on the CAPM framework, it must be noted that the returns are sensitive to the chosen market index (i.e., how the index was selected and/constructed) and its performance, and the choice of the risk-free rate, which could introduce biases or inconsistencies, especially in less liquid markets where index performance can be heavily influenced by a few large trades or specific sectors (and firms) and the government bond rates fluctuate significantly. We also note that at the time of completing this study, SPX listed a new company (SUN Insurance Limited—SUN) on 15 August 2024, which makes up 4.39% of the total market capitalization. This newly listed company is not included in the sample, although it is possible that SUN (in the insurance sector) may alter investor sentiments and behavior, hence impacting the overall market performance. Moreover, while the study emphasizes financial inclusion and literacy, the practical implementation of the proposed RMD portfolio in small markets requires careful consideration of domestic investor behavior, regulatory environments, and market infrastructure. The recommendations and findings might need adjustments to fit the specific regulatory and operational constraints of small markets. Noting these considerations, the portfolios presented in this study should be considered instructive only, supporting learning and exploration of the different methods of investment analysis. However, the results should not be treated as financial advice.

In conclusion, this study advances the understanding of portfolio optimization in small markets and provides the degrees of diversification and risk–return outcomes from different portfolios. Additionally, portfolios based on the RMD and RMD with additional constraints are compared with a set of benchmark portfolios. Nevertheless, noting the limitations of the study, further research to validate and extend the results in different market contexts and over longer time periods will provide additional insights. Moreover, future research could explore the feasibility and practical implications of implementing advanced portfolio strategies in small market environments, considering local market characteristics and investor needs.

Author Contributions: Conceptualization, R.R.K., H.G. and P.J.S.; methodology, R.R.K.; formal analysis, R.R.K.; writing—R.R.K., H.G. and P.J.S.; writing—review and editing, R.R.K., H.G. and P.J.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research receives no extra funding.

Data Availability Statement: Stock price data is available on the South Pacific Stock Exchange (SPX) website.

Acknowledgments: The authors thank the reviewers for their valuable comments and suggestions. All remaining errors are ours. Peter J. Stauvermann thankfully acknowledges the financial support of the Changwon National University 2023–2024 for ongoing research work.

Conflicts of Interest: The authors declare no conflict of interest.

Notes

1 https://www.un.org/sustainabledevelopment/economic-growth/#:~:text=Goal%208%20is%20about%20promoting,global%20economy%20under%20serious%20threat (accessed on 29 August 2024).

2 The no-short-selling assumption is highly relevant to Fiji's stock market.

3 https://www.rbf.gov.fj/bond-pricelist-and-yield-curve-2018-2019/#1608590112383-909852ff-3554 (accessed on 29 August 2024).

4 The choice of the target ratio is based on the interest earned on superannuation fund in 2019. https://www.parliament.gov.fj/wp-content/uploads/2019/11/Fiji-National-Provident-Fund-2019-Annual-Report.pdf (accessed on 29 August 2024).

5 Please note that SUN Insurance Company Ltd. (SUN) was listed on 15 August 2024, hence not included in the sample.

6 https://www.parliament.gov.fj/wp-content/uploads/2023/11/129-Reserve-Bank-of-Fiji-Annual-Report-August-2022%E2%80%93July-2023.pdf (accessed on 29 August 2024).

7 https://knoema.com/data/fiji+savings-rate#:~:text=Savings%20rate%20of%20Fiji%20fell,per%20annum%20in%20May%202024 (accessed on 29 August 2024).

8 https://www.rbf.gov.fj/category/bond-price-list-new/ (accessed on 29 August 2024).

References

Ahmed, Awais, Rizwan Ali, Abdullsh Ejaz, and Ishfaq Ahmad. 2018. Sectoral integration and investment diversification opportunities: Evidence from Colombo Stock Exchange. *Entrepreneurship and Sustainability Issues* 5: 514–27. [CrossRef] [PubMed]

Al Janabi, Mazin A. M. 2021. Multivariate portfolio optimization under illiquid market prospects: A review of theoretical algorithms and practical techniques for liquidity risk management. *Journal of Modelling in Management* 16: 288–309. [CrossRef]

Andreev, Nikolay. 2019. Robust Portfolio Optimization in an Illiquid Market in Discrete-Time. *Mathematics* 7: 1147. [CrossRef]

Bai, Lihui, and Paul Newsom. 2011. Teaching Utility Theory with an Application in Modern Portfolio Optimization. *Decision Sciences Journal of Innovative Education* 9: 107–12. [CrossRef]

Baumann, Philipp, and Norbert Trautmann. 2013. Portfolio-optimization models for small investors. *Mathematical Methods of Operations Research* 77: 345–56. [CrossRef]

Bielstein, Patrick, Jean-Paul Jaegers, and Paul Wesson. 2023. A Novel way to Diversify Portfolio Weights. *Wilmott Magazine*, October 11, Issue 128, November 2023 Issue. Available online: https://wilmott.com/wilmott-magazine-november-2023-issue/ (accessed on 29 August 2024).

Boasson, Vigdis, Emil Boasson, and Zhao Zhou. 2011. Portfolio optimization in a mean-semivariance framework. *Investment Management and Financial Innovations* 8: 58–68.

Booth, G. Geoffrey, and John Paul Broussard. 2016. The Sortino ratio and extreme value theory: An application to asset allocation. In *Extreme Events in Finance: A Handbook of Extreme Value Theory and Its Applications*. Wiley: pp. 443–64. Available online: https://onlinelibrary.wiley.com/doi/abs/10.1002/9781118650318.ch17 (accessed on 29 August 2024).

Choueifaty, Yves, and Yves Coignard. 2008. Toward Maximum Diversification. *The Journal of Portfolio Management* 35: 40–51. [CrossRef]

Clarke, Roger, Harindra De Silva, and Steven Thorley. 2006. Minimum-Variance Portfolios in the U.S. Equity Market. *The Journal of Portfolio Management* 33: 10–24. [CrossRef]

Clarke, Roger, Harindra De Silva, and Steven Thorley. 2011. Minimum-variance portfolio composition. *The Journal of Portfolio Management* 37: 31. [CrossRef]

Dash, Ashutosh, and Giridhari Mohanta. 2024. Fostering financial inclusion for attaining sustainable goals: What contributes more to the inclusive financial behaviour of rural households in India? *Journal of Cleaner Production* 449: 141731. [CrossRef]

De Athayde, Gustavo M., and Renato G. Flôres, Jr. 2004. Finding a maximum skewness portfolio—A general solution to three-moments portfolio choice. *Journal of Economic Dynamics and Control* 28: 1335–52. [CrossRef]

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal. 2009. Optimal Versus Naive Diversification: How Inefficient is the 1-N Portfolio Strategy? *The Review of Financial Studies* 22: 1915–53. [CrossRef]

Elton, Edwin J., and Martin J. Gruber. 1997. Modern portfolio theory, 1950 to date. *Journal of Banking & Finance* 21: 1743–59.

García, Fernando, Francisco Guijarro, Javier Oliver, and Rima Tamošiūnienė. 2018. Hybrid fuzzy neural network to predict price direction in the german DAX-30 index. *Technological and Economic Development of Economy* 24: 2161–78. [CrossRef]

Ghanbari, Hossein, Mojtaba Safari, Rouzbeh Ghousi, Emran Mohammadi, and Nawapon Nakharutai. 2023. Bibliometric analysis of risk measures for portfolio optimization. *Accounting* 9: 95–108. [CrossRef]

Haugen, Robert A., and Nardin L. Baker. 1991. The efficient market inefficiency of capitalization–weighted stock portfolios. *The Journal of Portfolio Management* 17: 35–40. [CrossRef]

Hilkevics, Sergejs, and Valentina Semakina. 2019. The classification and comparison of business ratios analysis methods. *Insights into Regional Development* 1: 48–57. [CrossRef]

Kalayci, Can B., Okkes Ertenlice, and Mehmet Anil Akbay. 2019. A comprehensive review of deterministic models and applications for mean-variance portfolio optimization. *Expert Systems with Applications* 125: 345–68. [CrossRef]

Kandpal, Vinay. 2020. Reaching sustainable development goals: Bringing financial inclusion to reality in India. *Journal of Public Affairs* 20: e2277. [CrossRef]

Kara, Alper, Haoyong Zhou, and Yifan Zhou. 2021. Achieving the United Nations' sustainable development goals through financial inclusion: A systematic literature review of access to finance across the globe. *International Review of Financial Analysis* 77: 101833. [CrossRef]

Koumou, Gilles Boevi. 2020. Diversification and portfolio theory: A review. *Financial Markets and Portfolio Management* 34: 267–312. [CrossRef]

Kumar, Ronald Ravinesh, and Peter Josef Stauvermann. 2022. Portfolios under Different Methods and Scenarios: A Case of Fiji's South Pacific Stock Exchange. *Journal of Risk and Financial Management* 15: 549. [CrossRef]

Kumar, Ronald Ravinesh, Peter Josef Stauvermann, and Aristeidis Samitas. 2022. An Application of Portfolio Mean-Variance and Semi-Variance Optimization Techniques: A Case of Fiji. *Journal of Risk and Financial Management* 15: 190. [CrossRef]

Maillard, Sébastien, Thierry Roncalli, and Jérôme Teïletche. 2010. The Properties of Equally Weighted Risk Contribution Portfolios. *The Journal of Portfolio Management* 36: 60–70. [CrossRef]

Markowitz, Harry Max. 1952a. Portfolio selection. *The Journal of Finance* 7: 77–91. [CrossRef]

Markowitz, Harry Max. 1952b. The Utility of Wealth. *Journal of Political Economy* 60: 43–50. [CrossRef]

Markowitz, Harry Max. 1991. Foundations of portfolio theory. *The Journal of Finance* 46: 469–77. [CrossRef]

Markowitz, Harry Max. 2010. Portfolio theory: As I still see it. *Annual Review of Financial Economics* 2: 1–23. [CrossRef]

Merton, Robert Cox. 1972. An analytic derivation of the efficient portfolio frontier. *Journal of Financial and Quantitative Analysis* 7: 1851–72. [CrossRef]

Milhomem, Danilo Alcantara, and Maria José Pereira Dantas. 2020. Analysis of new approaches used in portfolio optimization: A systematic literature review. *Production* 30: e20190144. [CrossRef]

Narkunienė, Judita, and Aurelija Ulbinaitė. 2018. Comparative analysis of company performance evaluation methods. *Entrepreneurship and Sustainability Issues* 6: 125–38. [CrossRef] [PubMed]

Ozili, Peterson K. 2022. Financial inclusion and sustainable development: An empirical association. *Journal of Money and Business* 2: 186–98. [CrossRef]

Reserve Bank of Fiji (RBF). 2017. *Fiji Financial Sector Development Plan 2016–2025*; Suva: The Reserve Bank of Fiji. Available online: https://www.rbf.gov.fj/wp-content/uploads/2020/03/Fiji-Financial-Sector-Development-Plan-2016-2025.pdf (accessed on 3 June 2024).

Righi, Marcelo Brutti, and Denis Borenstein. 2018. A simulation comparison of risk measures for portfolio optimization. *Finance Research Letters* 24: 105–12. [CrossRef]

Saliya, Candauda Arachchige. 2022. Stock market development and nexus of market liquidity: The case of Fiji. *International Journal of Finance & Economics* 27: 4364–82.

Salo, Ahti, Michalis Doumpos, Juuso Liesiö, and Constantin Zopounidis. 2024. Fifty years of portfolio optimization. *European Journal of Operational Research* 318: 1–18. [CrossRef]

Sharpe, William Forsyth. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance* 19: 425–42.

Sharpe, William Forsyth. 1994. The Sharpe ratio. *The Journal of Portfolio Management* 21: 49–58. [CrossRef]

Sharpe, William Forsyth. 1996. Mutual Fund Performance. *Journal of Business* 39: 119–38. [CrossRef]

Singh, Monika. 2019. Evolution of the Stock Market, Stocks, and Markets. *The Fiji Times*. July 16. Available online: https://www.pressreader.com/fiji/the-fiji-times/20190716/281852940146739 (accessed on 11 July 2024).

South Pacific Stock Exchange (SPX). 2024a. SPX Market Reports. Available online: https://marketreports.spx.com.fj/Company.aspx?code= (accessed on 3 June 2024).

South Pacific Stock Exchange (SPX). 2024b. Annual Report 2023. Available online: https://www.spx.com.fj/getattachment/Explore-SPX/Annual-Reports/SPX-AR-2023.pdf?lang=en-US (accessed on 11 June 2024).

Surtee, Taariq, and Imhotep Paul Alagidede. 2023. A novel approach to using modern portfolio theory. *Borsa Istanbul Review* 23: 527–40. [CrossRef]

Taljaard, Byran H., and Eben Maré. 2021. Why has the equal weight portfolio underperformed and what can we do about it? *Quantitative Finance* 21: 1855–68. [CrossRef]

Todoni, Marcela-Daniela. 2015. A post-modern portfolio management approach on CEE markets. *Procedia Economics and Finance* 32: 1362–76. [CrossRef]

Voica, Marian Cătălin. 2017. Financial inclusion as a tool for sustainable development. *Revista Română de Economie* 44: 121–29. Available online: https://revecon.ro/articles/2017-1/2017-1-10.pdf (accessed on 3 June 2024).

Whiteside, Barry. 2016. Recent Developments in the Fijian Stock Market. South Pacific Stock Exchange Listing Forum, Suva, September 22. Central Bankers' Speeches. Available online: https://www.bis.org/review/r160926j.htm (accessed on 11 July 2024).