Transmission Capacity of D2D communication under heterogeneous networks with Dual Bands

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***Abstract*—This paper analyzes the maximum achievable transmission capacity of the D2D communication system under heterogeneous networks. The heterogeneous networks contain two primary systems working on independent bands and D2D communication guarantees the target outage probabilities of both systems on each band. By utilizing stochastic geometry, the effects of the spatial densities and the transmission power allocation ratio on the achievable transmission capacity are presented. Moreover, the optimal transmission density of D2D pairs and the optimal power allocation ratio are derived. The maximum capacity of D2D communication is defined based on the former optimal value from theoretical results. It is shown that the optimal power allocation ratio is proportional to the product of bandwidth, node density and transmission power of two primary systems.**

***Index Terms*—Device-to-Device communication, network Capacity, (M)PPP model, power allocation**

I. Introduction

The fast development of wireless communication leads to the spectrum shortage [1] which impedes its further growth. While the cognitive radio [2] proposed by Mitola gives a light to solve the puzzle by improving spectrum efficiency greatly. As a type of such technology, the device-to-device (D2D) communication [3] is a kind of close range data transmission over a direct link and coexists with cellular networks in an underlay manner [4]. The D2D communication has advantages of enhancing network throughput, saving the power of user equipment and increasing an instantaneous data rate, which draws much attention in the recent years.

The transmission capacity is a fundamental issue of the heterogeneous system. Several results [5]-[7] in spectrum sharing environment were derived in the previous study. Huang analyzed the transmission capacity trade-off between cellular and mobile ad hoc networks [8] in uplink spectrum sharing. And in [9] Lee analyzed the achievable transmission capacity of secondary system in cognitive radio networks. The study based on a modified definition of network capacity as the spatial density of successful transmission per unit area while satisfying the outage probability constraints of both secondary and primary systems. And Vaze derived the transmission capacity of wireless ad hoc networks in [10], which considered the bidirectional data transmission.

However, in the former studies, second users only share spectrum with one primary system on a single band and transmit data with fixed powers. While the D2D communication is more flexible in coexistance with other systems and can increase transmission rate by aggregation of multi-bands [11]. Besides, the D2D terminals can further promote throughput and reduce interference by adjusting transmit power on each band. So the relationship between D2D users and primary system is more complicated and it is meaningful to extend D2D communication under the original scene to multi-bands spectrum sharing scenario.

In this paper, we consider the D2D communication coexisting with two primary systems, which work independently on two bands. We assume that the D2D communication can utilize both bands to transmit data and adjust the output power to achieve the maximum capacity. For protecting QoS of primary users in licenced bands, the D2D will not extend the outage constraints of the primary systems on each band. We analyze the influencing factor of the D2D transmission capacity and derive the maximum capacity in closed form expression. The simulation results show that the capacity of the D2D users is affected by interference between D2D nodes and primary systems, and the power allocation is determined by the node density, transmit power and spectrum bandwidth of the primary systems.

The rest of the paper is organized as follows. Section II describes the system model. In section III, the definition of D2D transmission capacity on the couple bands is given and the maximum capacity of D2D communication is provided with closed-form solutions of density and power allocation. Following this, the capacity is investigated under different network parameters with the analysis of influence by numerical results in section IV. Finally the conclusions are summarized in section V.

II. Scenario description and system model

A. Scenario description

The basic scenario contains two cellular systems and D2D pairs. The primary networks are deployed on two independent frequencies and D2D transmission will reuse the uplink bandwidths which are denoted as W_1 and W_2 respectively.

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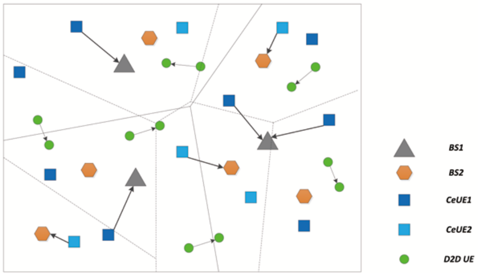

Fig. 1. The network model of two coexisting cellular systems with D2D transmission

The cellular uplink spectrum is divided in to *M* frequency-flat sub-channels by using OFDM. On each band the coexisting networks users (D2D users with cellular system 1 in W1 and D2D users with cellular system 2 in W2) uses full set of the sub-channels for the underlay sharing. In the cellular and D2D networks, a transmitter modulates signals by using frequency-hopping spread spectrum and the signals will hop randomly over all sub-channels assigned to the affiliated network.

We assume the D2D communication can use both bands to transmit data at the same time while the sum power on the couple bands is equal to a given value.

B. Network models

By using stochastic geometry [14], the coexisting network model is illustrated in Fig.1. There are three systems in the diagram, which are D2D, cellular system 1 and cellular system 2, denoted as S_0, S_1, S_2 respectively. We make the following assumptions:

Assumption 1. *The transmitters in the D2D pairs form a Poisson point process (PPP) on the two-dimensional plane, which is denoted as* S_0 *with the density* λ_0*. Each transmitter of D2D pair is associated with a receiver located at a distance* R_0 *and the transmission power of transmitters is* $P_i, i = 1, 2$ *on each band, where the sum power is equal to* P_0*.*

Assumption 2. *The base stations and uplink mobiles of cellular system 1 form two independent stationary PPPs. The density of mobiles is represented by* λ_1*, and the distance from a mobile to the BS is denoted as* R_1*.*

Assumption 3. *The base stations and uplink mobiles of cellular system 2 form two independent stationary PPPs. The density of mobiles is represented by* λ_2*, and the distance from a mobile to the BS is denoted as* R_2*.*

A typical point of a PPP is defined as a point selected using the procedure where every point of the PPP has the same probability of being selected. In order to evaluate outage probability, a typical receiver of system S_k is assumed to be located at the origin and it does not give any effect on statistics by the Palm probabilities of a Poisson process [12].

The propagation channel model contains path loss and Rayleigh fading δ_{ji}, hence, the received power at a typical receiver of system k from the ith node in system j can be defined as $P_j \delta_{ji} |X_{ji}|^{-\alpha}$, where P_j is the transmission power of system j, α is the path loss exponent, X_{ji} is the distance from the origin. For Rayleigh fading, it has an exponential distribution with unit mean.

In spectrum sharing on one of bands, the transmitting nodes in the same system and coexisting system will generate interference to a receiver at the same time, and the distribution of the interfering nodes in a single system j on band l can be modeled by marked Poisson Point process (MPPP), which is denoted as $\Pi_j^l = \{(X_{ji}, \delta_{ji})\}, \forall j \in \Phi_1, \Phi_1 = \{S_0, S_1\}$ on band 1 and $\Pi_j^l = \{(X_{ji}, \delta_{ji})\}, \forall j \in \Phi_2, \Phi_2 = \{S_0, S_2\}$ on band 2.

III. Achievable Transmission Capacity of D2D Transmission

A. Successful Transmission Probability on One Band

Since the interference received at the receiver of system S_k on band l is generated by transmitting nodes in other systems as well as in its own system, the SINR at the receiver becomes:

$$SINR_k = \frac{P_k \delta_{k0} R_k^{-\alpha}}{\sum\limits_{j \in \Phi_l} \sum\limits_{X_{ji} \in \Pi_j^l} P_j \delta_{ji} |X_{ji}|^{-\alpha} + N_0} \tag{1}$$

Where δ_{k0} is the fading factor on the transmitting power from the desired transmitter to the receiver, N_0 is the thermal noise power, and R_k is the distance between the transmitter and a typical receiver of system S_k. For we focus on the study of spectrum sharing which means the heterogeneous networks are interference limited and the thermal noise is negligible. So SIR can be used instead of SINR, and (1) is simplified to

$$SIR_k = \frac{\delta_{k0} R_k^{-\alpha}}{I_k} \tag{2}$$

Where $I_k = \sum\limits_{j \in \Phi_l} I_{kj}$ and $I_{kj} = \rho \sum\limits_{X_{ji} \in \Pi_j^l} \delta_{ji} |X_{ji}|^{-\alpha}$, $\rho = \frac{P_j}{P_k}$ stands for power ratio between system S_j and system S_k.

On each band, D2D transmission and the primary system should satisfy the smallest allowable value of SIR at the receiver to guarantee its desired decoding accuracy. Hence, the successful transmission of system S_k can be possible when $SIR_k \geq v_{th}^k$ where v_{th}^k is the target SIR of system S_k. The probability of successful transmission can be defined using formula as[9]:

$$P(SIR_k \geq v_{th}^k) = \exp\left\{-C_\alpha R_k^2 {v_{th}^k}^{2/\alpha} r_{kj} \lambda_j\right\} \tag{3}$$

where $r_{kj} = \rho^{2/\alpha}$, and $C_\alpha = \frac{2\pi}{\alpha} \Gamma\left(\frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right)$ with Gamma function $\Gamma(x) = \int_0^\infty y^{x-1} e^{-y} dy$.

The probability of successful transmission of system S_k is represented as follows:

$$P\left(SIR_k \geq v_{th}^k\right) = \exp\left\{-\varsigma_k \sum_{j \in \Phi} r_{kj} \lambda_j\right\} \tag{4}$$

Where $\varsigma_k = C_\alpha R_k^2 {v_{th}^k}^{2/\alpha}$. And the outage probability of the system S_k sharing the spectrum with system S_j in each band can be expressed as

$$P_k^o\left(\lambda_k,\lambda_j\right)=1-P\left(SIR_k\geq v_{th}^k\right) \tag{5}$$

B. Achievable Transmission Capacity on Dual Bands

The transmission capacity is defined as the maximum spatial density of successful transmissions while guaranteeing the target outage probability in [8]. The D2D transmission should guarantee the QoS of primary networks on both bands, which means the outage of cellular systems should be less than θ_1 on band1 and θ_2 on band2. Then the outage probability constraints are defined as $P_1^o\left(\lambda_1,\lambda_0\right)\leq\theta_1$ and $P_2^o\left(\lambda_2,\lambda_0\right)\leq\theta_2$.

Hence the number of successful transmissions in spectrum sharing environment is defined as the product of the spatial density and the actual successful transmission probability. We use the definition in [11] as D2D transmission capacity on one band and give the definition of the D2D achievable transmission capacity on the couple bands.

Definition 1. *The achievable transmission capacity of D2D transmission in spectrum sharing with two primary systems is defined as follows:*

$$\begin{aligned}f\left(\lambda_0^\theta,P_1^0,P_2^0\right)=&\tfrac{W_1\lambda_0^\theta}{W_1+W_2}\exp\left\{-\varsigma_0\sum_{j\in\Phi_1}r_{0j}\lambda_j^\theta\right\}\\&+\tfrac{W_2\lambda_0^\theta}{W_1+W_2}\exp\left\{-\varsigma_0\sum_{j\in\Phi_2}r_{0j}\lambda_j^\theta\right\}\end{aligned} \tag{6}$$

Where λ_0^θ is the spatial density of D2D pairs which satisfies the outage probability constraints of spectrum sharing systems as:

$$P_k^o\left(\lambda_k^\theta,\lambda_0^\theta\right)=1-\exp\left\{-\varsigma_k\sum_{i\in\{0,k\}}r_{ki}\lambda_i^\theta\right\}\leq\theta_k,k=1,2$$

C. Maximum Transmission Capacity of D2D Network over Dual Bands

The D2D is allowed to utilize the licensed spectrum as long as it does not disturb the reliable transmission of the primary system on a single band. That is it should guarantee the target outage probability of the cellular system on each band. Considering the difference of interference on couple bands, the D2D system can adjust the power on each band to gain the maximum sum-rate. The stand form of original maximum problem shows as follows:

$$\begin{aligned}\max\quad & f\left(\lambda_0^\theta,P_1^0,P_2^0\right)\\ s.t.\quad & \theta_1+e^{-\varsigma_1\left(\lambda_1+\left(\frac{P_1^0}{P_1}\right)^{2/\alpha}\cdot\lambda_0^\theta\right)}-1\geq0\\ & \theta_2+e^{-\varsigma_2\left(\lambda_2+\left(\frac{P_2^0}{P_1}\right)^{2/\alpha}\cdot\lambda_0^\theta\right)}-1\geq0\\ & P_1^0+P_2^0=P_0\\ & \lambda_0^\theta\geq0,\quad P_1^0\geq0,\quad P_2^0\geq0\end{aligned} \tag{7}$$

It is easy to prove that the feasible region **D** is compact and the object function is a continuous function on **D**. So according to the Weierstrass theorem [12], the object function attains a maximum on **D**. We can solve the above maximization problem by constructing Lagrangian function L under KKT condition[13].

By computing the value of each critical point over feasible region and comparing every value, we can obtain the maximum value of L. In practice, the value of $\lambda_0^\theta,P_1^0,P_2^0$ that maximizes L over is typically also the solution to the original maximization problem. Yet we can determine the definition region of the parameters and solve the maximum problem by derivative method in the definition domain. The further deduce shows as follows.

When the values of parameters have been set, the maximum of transmission capacity depends on λ_0^θ and power allocation ratio between two bands. If we define the parameter η as the factor of power allocation ratio, then P_1^0 and P_2^0 can be expressed as:

$$\begin{aligned}&P_1^0=\eta P_0\\&P_2^0=\left(1-\eta\right)P_0,\quad 0\leq\eta\leq1\end{aligned}$$

Make substitution of P_1^0 and P_2^0 in **Definition 1** and we can get the new object function as follows:

$$\begin{aligned}f\left(\lambda_0^\theta,\eta\right)=&\tfrac{W_1\lambda_0^\theta}{W_1+W_2}e^{-\varsigma_0\left(\lambda_0^\theta+\left(\frac{P_1}{\eta P_0}\right)^{2/\alpha}\cdot\lambda_1\right)}\\&+\tfrac{W_2\lambda_0^\theta}{W_1+W_2}e^{-\varsigma_0\left(\lambda_0^\theta+\left(\frac{P_2}{(1-\eta)P_0}\right)^{2/\alpha}\cdot\lambda_2\right)}\end{aligned} \tag{8}$$

So from the constraint conditions we can get the feasible region of the λ_0^θ. The two constraint inequalities are need to be guaranteed together. Reshape the constraint inequalities into following form:

$$\begin{aligned}\lambda_0^\theta\eta^{2/\alpha}&\leq\left(\tfrac{P_1}{P_0}\right)^{2/\alpha}\cdot\left(\tfrac{-\ln(1-\theta_1)}{\varsigma_1}-\lambda_1\right)\\ \lambda_0^\theta(1-\eta)^{2/\alpha}&\leq\left(\tfrac{P_2}{P_0}\right)^{2/\alpha}\cdot\left(\tfrac{-\ln(1-\theta_2)}{\varsigma_2}-\lambda_2\right)\end{aligned}$$

And then for $0\leq\eta\leq1$, the feasible region of λ_0^θ is $0<\lambda_0^\theta\leq\lambda_{0,\max}^\theta$, where

$$\begin{aligned}\lambda_{0,\max}^\theta=\min&\left\{\eta^{-2/\alpha}\left(\tfrac{P_1}{P_0}\right)^{2/\alpha}\cdot\left(\tfrac{-\ln(1-\theta_1)}{\varsigma_1}-\lambda_1\right),\right.\\&\left.(1-\eta)^{-2/\alpha}\left(\tfrac{P_2}{P_0}\right)^{2/\alpha}\cdot\left(\tfrac{-\ln(1-\theta_2)}{\varsigma_2}-\lambda_2\right)\right\}\end{aligned} \tag{9}$$

From the latter derivation, the opitimal value of η is typo with λ_0^θ and we can first determine the value of η^* and then obtain the feasible region of λ_0^θ.

If the node density of the primary system is too large and the link distance is long, there may be no possible value of λ_0^θ to guarantee the target outage probability because there is already too much interference between nodes within the same primary system.

After obtaining the definition domain of the objection function, we can determine the optimal solution of (7) by using partial derivative method as follows:

$$\begin{cases} \dfrac{\partial f\left(\lambda_0^\theta, \eta\right)}{\partial \lambda_0^\theta} = 0, \quad 0 \le \lambda_0^\theta \le \lambda_{0,\max}^\theta, & (a) \\ \dfrac{\partial f\left(\lambda_0^\theta, \eta\right)}{\partial \eta} = 0, \quad 0 \le \eta \le 1. & (b) \end{cases} \tag{10}$$

We can get η^* by make partial derivative of η and the equation (b) in (10) is expended as

$$\frac{\partial f\left(\lambda_0^\theta,\eta\right)}{\partial \eta} = \frac{2\lambda_0^\theta e^{-\varsigma_0 \lambda_0^\theta}}{\alpha(W_1+W_2)P_0}\left(\frac{W_1\lambda_1 P_1}{\eta^2}\left(\frac{P_1}{\eta P_0}\right)^{-1+2/\alpha} e^{-\varsigma_0\lambda_1\left(\frac{P_1}{\eta P_0}\right)^{2/\alpha}} - \frac{W_2\lambda_2 P_2}{(1-\eta)^2}\left(\frac{P_1}{(1-\eta)P_0}\right)^{-1+2/\alpha} e^{-\varsigma_0\lambda_2\left(\frac{P_2}{(1-\eta)P_0}\right)^{2/\alpha}}\right) = 0$$

Because $\frac{\partial f\left(\lambda_0^\theta,\eta\right)}{\partial \eta} = 0$ is a transcendental equation and we can only achieve the approximate solution by image method like Newton tangent method and so on. Yet by making transaction of above equation, we can get the approximate solution of η as follows:

$$\frac{W_1\lambda_1 P_1}{W_2\lambda_2 P_2}\left(\frac{\eta}{1-\eta}\right)^{-1-\frac{2}{\alpha}} = e^{-\varsigma_0\left(\lambda_2\left(\frac{P_2}{P_0}\right)^{\frac{2}{\alpha}}(1-\eta)^{\frac{\alpha}{2}} - \lambda_1\left(\frac{P_3}{P_0}\right)^{\frac{2}{\alpha}}\eta^{\frac{\alpha}{2}}\right)} \tag{11}$$

Due to the extraordinary small value of λ_1 and λ_2, the right hand-side of the equation is almost equal to one expect very close to the boundary of zero and one. And we can make the approximation as follows:

$$e^{-\varsigma_0\left(\lambda_2\left(\frac{P_2}{P_0}\right)^{\frac{2}{\alpha}}(1-\eta)^{\frac{\alpha}{2}} - \lambda_1\left(\frac{P_3}{P_0}\right)^{\frac{2}{\alpha}}\eta^{\frac{\alpha}{2}}\right)} \approx 1, 0 < \eta < 1 \tag{12}$$

So (12) can be expressed in the following approximate equation:

$$\frac{W_1\lambda_1 P_1}{W_2\lambda_2 P_2}\left(\frac{\eta}{1-\eta}\right)^{-1-2/\alpha} = 1 \tag{13}$$

From (13) we can get the optimal power allocation ratio and have a corollary as follows:

Corollary 1. *The optimal power allocation ratio can be defined as follows:*

$$\frac{P_1^0}{P_2^0} = \left(\frac{W_1\lambda_1 P_1}{W_2\lambda_2 P_2}\right)^{\frac{\alpha}{2+\alpha}} \tag{14}$$

From (14) we can see that the power allocation of D2D communication on two bands is in proportion to the product of bandwidth, node density and transmit power of the primary systems on each band. And we can get the optimal solution of η as:

$$\eta^* = \frac{1}{1+\left(\frac{\lambda_2 P_2 W_2}{\lambda_1 P_1 W_1}\right)^{\frac{\alpha}{2+\alpha}}} \tag{15}$$

After the value of η^* is determined, we can fix the definition domain of λ_0^θ. In the same manner, the equation (a) in (10) can be expanded as:

$$\frac{\partial f\left(\lambda_0^\theta,\eta\right)}{\partial \lambda_0^\theta} = \frac{\left(1-\varsigma_0\lambda_0^\theta\right)e^{-\varsigma_0\lambda_0^\theta}}{W_1+W_2}\left(W_1 e^{-\varsigma_0\lambda_1\left(\frac{P_1}{\eta P_0}\right)^{2/\alpha}} + W_2 e^{-\varsigma_0\lambda_2\left(\frac{P_2}{(1-\eta)P_0}\right)^{2/\alpha}}\right) = 0$$

And we get a unique critical point $\lambda_0^{\theta^*} = \frac{1}{\varsigma_0}$ over the $\mathbf{R}^+$. It's easy to prove the critical point is the global maximum value of the object function in spite of η. While if $\lambda_{0,\max}^\theta < \frac{1}{\varsigma_0}$, the object function will reach the maximum value on the right boundary of feasible region. So we have the following conclusion:

Corollary 2. *The optimal density of D2D pairs can be defined as follows:*

$$\lambda_0^{\theta*} = \begin{cases} \lambda_{0,\max}^\theta & , \lambda_{0,\max}^\theta < \dfrac{1}{\varsigma_0}, \\ \dfrac{1}{\varsigma_0} & , \lambda_{0,\max}^\theta \ge \dfrac{1}{\varsigma_0} \end{cases}. \tag{16}$$

The global optimal density of D2D pairs over the $\mathbf{R}^+$ is only determined by the transmission distance of D2D link and the threshold of successful transmission, but whether it can reach the global point is decided by the feasible region. If the main interference with D2D receiver is from the primary system, the optimal density of the D2D pairs can be only reached at the $\lambda_0^{\theta*}$ which is less than the global optimal value. While, if the interference from the primary system transmitter is weak, the D2D transmissions construct an interference limited system due to its own density. And the optimal density of D2D pairs is also the global optimal point.

Based on the optimal value of D2D pair density and power allocation ratio between two bands, we can get the maximum transmission capacity of D2D communication.

Corollary 3. *The maximum transmission capacity of D2D communication can be derived as follows:*

$$C_{\max} = \frac{\lambda_0^{\theta*} e^{-\varsigma_0\lambda_0^{\theta*}}}{(W_1+W_2)}\left(W_1 e^{\varsigma_0\lambda_1\left(\left(\frac{P_1}{P_0}\right)\cdot\left(1+\left(\frac{\lambda_2 P_2 W_2}{\lambda_1 P_1 W_1}\right)^{\frac{\alpha}{2+\alpha}}\right)\right)^{\frac{2}{\alpha}}} + W_2 e^{\varsigma_0\lambda_2\left(\left(\frac{P_2}{P_0}\right)\cdot\left(1+\left(\frac{\lambda_2 P_2 W_2}{\lambda_1 P_1 W_1}\right)^{\frac{\alpha+2}{\alpha}}\right)\right)^{\frac{2}{\alpha}}}\right) \tag{17}$$

IV. NUMERICAL RESULTS

In this section, the theoretical results in Section III are verified by simulations. Fig.2 presents the achievable transmission capacity of D2D transmission on the feasible region of the power allocation ratio and the density of D2D pairs. The curved face of the achievable capacity constructs an upward protruding surface and has a unique maximal point.

Fig.3 and Fig.4 show the achievable capacity of the D2D communication varies with a single parameter (the shadow part in the figure stands for the feasible region of λ_0^θ), which means one of the parameters(λ_0^θ or η) is fixed and the other parameter is available.

Comparing Fig.3 with Fig.4, we can see that if the interference of the primary system is temperate (which means the the feasible reigon of λ_0^θ is large and vice versa), the global optimal density of D2D pairs can be achieved. Besides the optimal power allocation ratio is in the middle of definition interval of

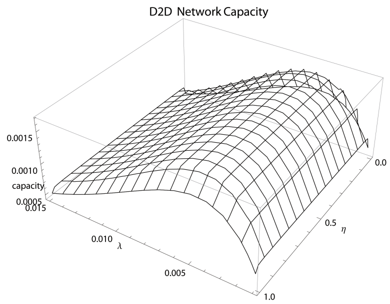

Fig. 2. Achievable Capacity of D2D transmission over λ_0^θ and η(the system parameters are set as $W_1 = 10MHz, W_2 = 5MHz$,$\lambda_1 = 1 \times 10^{-5}m^{-2}, \lambda_2 = 3 \times 10^{-5}m^{-2}$, $\theta_1 = \theta_2 = 0.05$),$R_1 = 5m$,$v_1 = v_2 = v_3 = 3$ and $\alpha = 4$

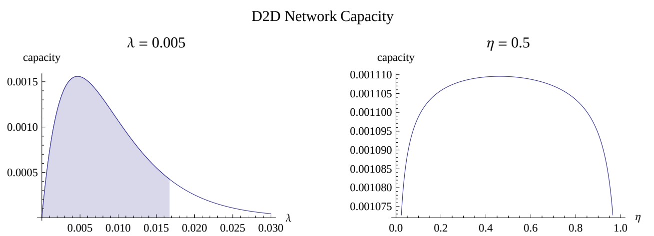

Fig. 3. Achievable Capacity of D2D transmission over λ_0^θ or η (the system parameters are set as $W_1 = 10MHz, W_2 = 5MHz$,$\lambda_1 = 1 \times 10^{-5}m^{-2}, \lambda_2 = 2 \times 10^{-5}m^{-2}$, $\theta_1 = \theta_2 = 0.05$,$R_1 = 5m$, $v_1 = v_2 = v_3 = 3$ and $\alpha = 4$)

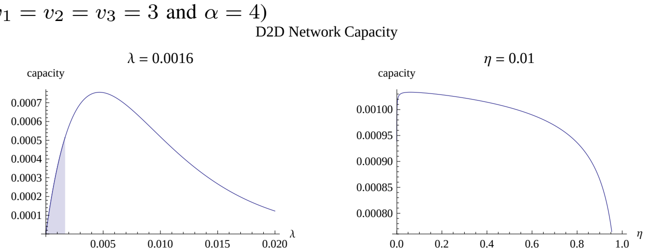

Fig. 4. Achievable Capacity of D2D transmission over λ_0^θ or η (the system parameters are set as $W_1 = 5MHz, W_2 = 20MHz$,$\lambda_1 = 1 \times 10^{-5}m^{-2}, \lambda_2 = 1 \times 10^{-4}m^{-2}$, $\theta_1 = \theta_2 = 0.05$,$R_1 = 5m$, $v_1 = v_2 = v_3 = 3$ and $\alpha = 4$)

η. When the interference from the primary system is strong, the best density of D2D pair is achieved on the boundary of the constraints and the strategy of power allocation is pouring out a majority of power on one of the two bands almost with the other one almost unused.

By adjusting the power on each band, we can further improve the capacity of the D2D transmission, which is determined by the bandwidth, power and density of the primary system nodes. The basic trend shows that the wider the bandwidth is, the more sensitive the D2D communication capacity is to the product of density and transmit power of nodes in primary system and vice versa.

The denser the nodes in a primary system are, the lower the output power of a transmitter is, which will benefits avoiding the interference between different nodes. Therefore the multiplication between the value of node density and the transmit power is limited to the successful receiving threshold.

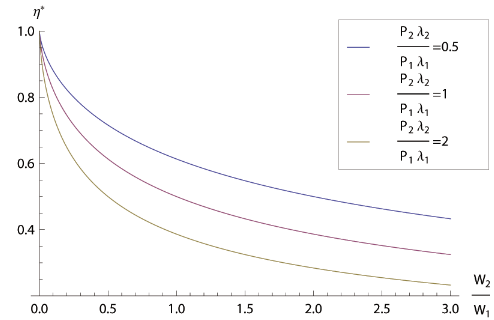

Fig. 5. The optimal η^* according to bandwidth coefficient between two bands

If the ratio of the above product is fixed between the two primary systems and the bandwidth ratio between them is adjustable, we can get the tend of optimal power allocation ratio η^* according to $\frac{W_2}{W_1}$. From Fig.5 we can conclude that the power allocated to band 1 declines with the increase of $\frac{W_2}{W_1}$ because the average interference on the band2 gets smaller due to the expanding of band2. And the larger $\frac{P_2\lambda_2}{P_1\lambda_1}$ is, the faster the curve declines, which is in according with the results of Fig.3 and Fig.4.

V. Conclusion

In this paper, the maximum achievable transmission capacity of the D2D communication under the outage probability constraints is presented, which is based on the optimal density of D2D pairs and the power allocation ratio between two bands. Based on the theoretical results, we verify that the optimal density of D2D pairs is determined by the interference from the primary system on both bands. If the interference from primary system is weak, the density of D2D pairs can reach global optimal point. Otherwise the optimal density of D2D pairs is equal to the maximum density in the feasible region. The optimal power allocation ratio is in proportion to the product of band-width, node density and transmission power of the two primary systems.

These conclusions give quantitative insights into the effect of the interference and the parameter of primary systems to the performance of D2D communication. Although the spectrum sharing is based on couple bands, the results can be extended to multi-band as well.

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