Capacity Bounds for Relay-Aided Wireless Multiple Multicast with Backhaul

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***Abstract*—We investigate the capacity bounds for relay-aided two-source two-destination wireless networks with backhaul support between source nodes. Each source multicasts its own message to all destinations with the help of an intermediate relay node, which is full-duplex and shared by both sources. We are aiming to characterize the capacity region of this model given discrete memoryless Gaussian channels. We establish three capacity upper bounds by relaxing the cut-set bound, and by extending two capacity bounds originally derived for MIMO relay channels. We also present one lower bound by using decoding-and-forward relaying combined with network beam-forming.**

I. Introduction

Wireless communications have recently seen rapid progress both in academy and industry, and the use of relay as well as advanced cooperative communication techniques has the potential to further boost both the communication range and data rate. The full understanding of such systems, even for the original three-node relay network, is still not ready yet. In the last 30 years, numerous research efforts have been casted on the relay networks. In [1], [2], capacity bounds and various cooperative strategies for three-node relaying networks (source-relay-sink, or two cooperative sources and one sink) have been studied, with successive decoding, sliding-window forward decoding, or backward decoding techniques used at the sink. The relay (or the other source) uses decode-and-forward (DF) or compress-and-forward (CF) to aid the transmission. In [3] cooperative strategies and coding schemes are investigated for multiple-access relay channels (MARC) involving multiple sources and a single destination, and for broadcast relay channels (BRC) where a single source transmits messages to multiple destinations. Recent results on capacity bounds for multiple-source multiple-destination relay networks, [4]–[6] and references therein, have provided valuable insights into the benefits of relaying, either half-duplex or full-duplex. Apart from introducing dedicated relay nodes to help the transmission, one can also utilize cooperative strategies among sources and/or among destinations [7], [8] with the help of orthogonal conferencing channels.

In this paper, we aim to characterize the capacity regions when source cooperation and relaying are combined together. More specifically, we focus on a relay-aided two-source two-destination multicast network with backhaul support, as shown in Figure. 1. Source $\mathcal{S}_1$ intends to multicast[1] its message W_1 at rate R_1 to two geographically separated destinations $\mathcal{D}_1$ and $\mathcal{D}_2$, with the help of a relay $\mathcal{R}$. At the same time, source $\mathcal{S}_2$ also multicasts its message W_2 at rate R_2 to both destinations. The relay will forward the information it receives in previous time slot to both destinations. The transmissions from two sources and from the relay use the same channel resource (i.e. co-channel transmission) and will mix up at all the receiving terminals ($\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{R}$). This model arises from downlink wireless cellular networks where two base stations multicast to two mobile terminals, one in each cell, with the help of a dedicated relay deployed at the common cell boundary. This model is interesting since it is a combination of relaying, MARC, BRC, and sources cooperation. It can be extended to more general networks by tuning the channel gains within the range $[0, \infty)$. In this paper, we are interested in the scenario without cross channels between $\mathcal{S}_1$ and $\mathcal{D}_2$, or $\mathcal{S}_2$ and $\mathcal{D}_1$. In wireless cellular networks, such cross channels are normally too weak to be used or technically suppressed by the system.

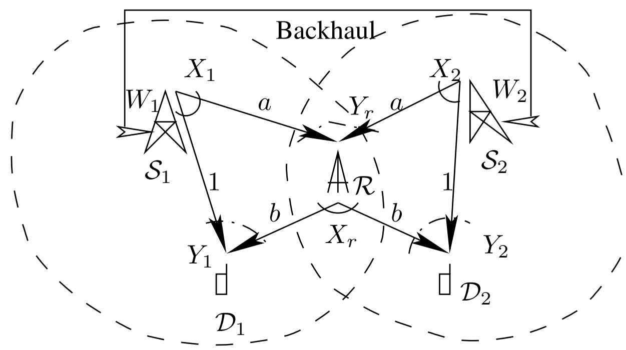

Figure 1. Two source nodes $\mathcal{S}_1$ and $\mathcal{S}_2$, connected with backhaul, multicast information W_1 and W_2 respectively to both destinations $\mathcal{D}_1$ and $\mathcal{D}_2$, with aid from a full-duplex relay node $\mathcal{R}$.

The rest of this paper is organized as follows. The system model is introduced in Section II. A lower bound given by networked beam-forming is presented in Section III, and three capacity upper bounds are established in Section IV. Numerical results are presented in Section V and concluding remarks are shown in Section VI.

Notations: Capital letter X indicates a real valued random variable and $p(X)$ indicates its probability density/mass function. $X^{(n)}$ denotes a vector of random variables of length n and $I(X;Y)$ denotes the mutual information between X and Y. $C(x) = \frac{1}{2}\log_2(1+x)$ is the Gaussian capacity function.

[1] In some other papers and books, "multicast" is also referred as "broadcast" but with only common messages.

II. SYSTEM MODEL

To simplify our analysis, we consider a symmetric channel gain scenario in Figure 1 (extension to non-symmetric channel gains is straightforward),

$$Y_1^{(n)} = X_1^{(n)} + bX_r^{(n)} + Z_1^{(n)}, \quad (1a)$$

$$Y_2^{(n)} = X_2^{(n)} + bX_r^{(n)} + Z_2^{(n)}, \quad (1b)$$

$$Y_r^{(n)} = aX_1^{(n)} + aX_2^{(n)} + Z_r^{(n)}, \quad (1c)$$

where $a \geq 0$ is the normalized channel gain for the source-relay links and $b \geq 0$ for the relay-destination links. $X_i^{(n)}$, $Y_i^{(n)}$, $Z_i^{(n)}$, $i = 1, 2, r$ are n-dimensional transmitted signals, received signals, and noise, respectively. The noise components $Z_i(k)$, $i = 1, 2, r$ and $k = 1, ..., n$ are i.i.d. zero-mean unit-variance Gaussian random variables. Assuming perfect synchronization, $\mathcal{S}_1$ and $\mathcal{S}_2$ can cooperate with $\mathcal{R}$ and get coherent combining gains (i.e., beamforming) at the sinks, as stated in [1]–[3]. An average power constraint

$$\frac{1}{n}\sum_{k=1}^{n} X_i^2(k) \leq P_i, \quad i = 1, 2, r, \quad (2)$$

is assumed throughout this paper. We note that in practice, the backhaul has much higher capacity and lower error rates than the forward wireless channels. Therefore, in our model the backhaul is assumed to be error-free and of sufficiently high capacity, which makes our system closely related to the MIMO relay channel scenario, as studied in [9], [10]. However, we emphasize three main differences between the system investigated in this paper and the MIMO relay scenario with a two-antenna source node. Firstly, in our system each source/antenna is subject to an individual power constraint as stated in 2, while in the MIMO relay channel model a sum-power constraint is applied at the source node, which essentially means a larger achievable rate region. Secondly, in our system the relay combines messages from each source by performing NC rather than forwarding them separately through orthogonal channels. Since network coding is preferred in symmetric rate scenarios (otherwise we have to append zeros at the shorter message), its efficiency is limited by the source-relay channels, especially when the two source-relay channels are not symmetry. Last but no the least, our system model can be easily extended to the finite-rate backhaul scenario where only partial cooperation between source nodes is possible. Therefore, capacity lower bounds derived for the MIMO relay channel may not be directly relevant to our scenario. However, the corresponding upper bounds, e.g., Theorem 3.1 in [9] and the upper bound (9) in [10], are still valid. We will introduce them to our system and modify them accordingly to establish new upper bounds.

III. CAPACITY LOWER BOUND

A cooperative transmission strategy, namely networked beam-forming (NBF) with a full duplex decode-and-forward relay, has been proposed in [13] for non-perfectly synchronized signal at the relay. We will introduce the achievable rate presented in [13] but for perfectly synchronization scenario to serve as the capacity lower bound. The main results of NBF are listed here with a brief outline of the constructive proof.

Similar to [1]–[4], source $\mathcal{S}_i$, $i = 1, 2$, divides its messages W_i into B blocks $W_{i,1}, \ldots, W_{i,B}$ with nR_i bits each. The signals transmitted at $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{R}$ are formulated in a beam-forming fashion to take advantage of the coherent combining gain. The transmission process requires $B + 2$ blocks in total, and each transmission is over n channel uses, and assuming the backhual is used for free, the overall rate is $\frac{Bk_i}{(B+2)n}$ bits per channel use, which converges to $R_i = \frac{k_i}{n}$ when B goes to infinity.

During block $b - 1$, $(W_{1,b-1}, W_{2,b-1})$ are exchanged via the backhaul and formulated into a network coded message $W_b = f(W_{1,b-1}, W_{2,b-1})$ by some function $f(\cdot)$; at block b the coded message W_b is transmitted by both sources and the relay receives and decodes it afterwards; at block $b + 1$, W_b is transmitted by $\mathcal{R}$, i.e.,

$$X_{r,b+1}^{(n)} = \sqrt{P_r} U^{(n)}(W_b).$$

$U^{(n)}(W)$ is a codeword of the message W, and we relate their dependence in the way of an encoding function. Each codeword is generated in the usual memoryless fashion. Since the message W_b (hence the signal $X_{r,b+1}^{(n)}$) at the relay is known by both sources before its transmission, $\mathcal{S}_1$ and $\mathcal{S}_2$ can cooperative their transmission of new message W_{b+1} with the relaying message W_b and transmit at block $b + 1$

$$X_{1,b+1}^{(n)} = \sqrt{\alpha_1 P_1} V^{(n)}(W_{b+1}, W_b) + \sqrt{(1-\alpha_1)P_1} U^{(n)}(W_b),$$

$$X_{2,b+1}^{(n)} = \sqrt{\alpha_2 P_2} V^{(n)}(W_{b+1}, W_b) + \sqrt{(1-\alpha_2)P_2} U^{(n)}(W_b),$$

where $0 \leq \alpha_1, \alpha_2 \leq 1$ are power allocation parameters.

The decoding process is as follows: the relay performs *successive decoding* [1] to decode W_b, $b = 1, 2, ..., B$, and the destinations utilize *backward decoding* [11] to decode W_b, $b = B, B - 1, ..., 1$. Since $\mathcal{S}_1$ and $\mathcal{S}_2$ transmit the same NC message W_b, the achievable sum-rate can be split arbitrarily between them. Therefore in NBF strategy only the constraints over the sum-rate matter. The following rate region is achievable by NBF,

$$\begin{aligned} R_1 + R_2 < \min \Big\{ & C\left(P_1 + b^2 P_r + 2b\sqrt{(1-\alpha_1)P_1 P_r}\right), \\ & C\left(a^2\left(\sqrt{\alpha_1 P_1} + \sqrt{\alpha_2 P_2}\right)^2\right), \\ & C\left(P_2 + b^2 P_r + 2b\sqrt{(1-\alpha_2)P_2 P_r}\right)\Big\}, \end{aligned} \quad (3)$$

with the union taken over the power allocation parameters $0 \leq \alpha_1, \alpha_2 \leq 1$. The terms in (3) indicate the constraints at $\mathcal{D}_1$, $\mathcal{R}$, and $\mathcal{D}_2$, respectively.

For the symmetric scenario where $P_1 = P_2 = P_r = P$ (therefore $R_1 = R_2 = R$), by setting $\alpha_1 = \alpha_2 = \alpha$ in (3), the achievable symmetric rate R is given by

$$R < \max_{0 \leq \alpha \leq 1} \min \left\{ \frac{1}{2} C(4a^2 P\alpha), \frac{1}{2} C\left(\left(1 + b^2 + 2b\sqrt{1-\alpha}\right) P\right) \right\}.$$

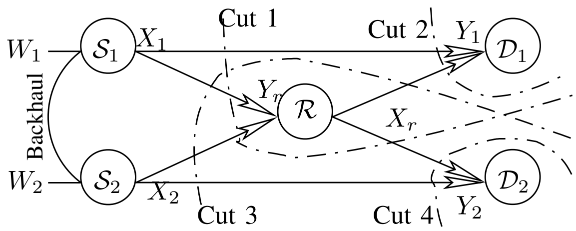

Figure 2. The sum multicast capacity is bounded by the cut-set bound based on the four cuts shown in the figure.

IV. Capacity Upper Bounds

The cut-set bound [12] on the sum rate $R_1 + R_2$ will be derived for our model, and two upper bounds for the MIMO relay channels given by [9] and [10] will also be discussed as references.

A. Upper Bound from the Cut-Set Bound

As in [12], the cut-set bound for the sum-rate R_1+R_2 over the cooperative relay network shown in Fig. 1 can be derived based on the four cuts shown in Fig. 2, i.e.,

$$\begin{aligned} C_{cut-set} = &\sup_{p(X_1,X_2,X_r)} \min \{I(X_1,X_2;Y_1,Y_r|X_r),\\ &I(X_1,X_r;Y_1), I(X_1,X_2;Y_2,Y_r|X_r), I(X_2,X_r;Y_2)\}\,. \end{aligned} \tag{4}$$

To model the potential correlation among X_1, X_2 and X_r due to cooperation, we partition the transmitting signals as follows

$$\begin{aligned} X_r^{(n)} &= \sqrt{P_r}U^{(n)},\\ X_1^{(n)} &= \sqrt{\alpha_1' P_1}S_1^{(n)} + \sqrt{\alpha_1'' P_1}V^{(n)} + \sqrt{(1-\alpha_1'-\alpha_1'')P_1}U^{(n)},\\ X_2^{(n)} &= \sqrt{\alpha_2' P_2}S_2^{(n)} + \sqrt{\alpha_2'' P_2}V^{(n)} + \sqrt{(1-\alpha_2'-\alpha_2'')P_2}U^{(n)}, \end{aligned}$$

where S_1, S_2, V, and U are independent random variables with zero-mean and unit-variance to represent respectively, the individual signals for user 1 and user 2, the cooperative signal between two sources, and the cooperative signal with the relay. The received signals in (1) therefore becomes

$$\begin{aligned} Y_1^{(n)} &= \left(b\sqrt{P_r} + \sqrt{(1-\alpha_1'-\alpha_1'')P_1}\right)U^{(n)} + \sqrt{\alpha_1' P_1}S_1^{(n)}\\ &\qquad + \sqrt{\alpha_1'' P_1}V^{(n)} + Z_1^{(n)},\\ Y_2^{(n)} &= \left(b\sqrt{P_r} + \sqrt{(1-\alpha_2'-\alpha_2'')P_2}\right)U^{(n)} + \sqrt{\alpha_2' P_2}S_2^{(n)}\\ &\qquad + \sqrt{\alpha_2'' P_2}V^{(n)} + Z_2^{(n)},\\ Y_r^{(n)} &= a\left(\sqrt{\alpha_1'' P_1} + \sqrt{\alpha_2'' P_2}\right)V^{(n)} + a\sqrt{\alpha_1' P_1}S_1^{(n)} + Z_r^{(n)}\\ &+ a\sqrt{\alpha_2' P_2}S_2^{(n)} + a\left(\sqrt{(1-\alpha_1'-\alpha_1'')P_1} + \sqrt{(1-\alpha_2'-\alpha_2'')P_2}\right)U^{(n)}. \end{aligned}$$

For Cut 2 and Cut 4, it can be verified that [12]

$$\begin{aligned} I(X_1,X_r;Y_1) &\le C\left(P_1 + b^2 P_r + 2b\sqrt{(1-\alpha_1'-\alpha_1'')P_1P_r}\right),\\ I(X_2,X_r;Y_2) &\le C\left(P_2 + b^2 P_r + 2b\sqrt{(1-\alpha_2'-\alpha_2'')P_2P_r}\right), \end{aligned} \tag{5}$$

where the equalities are achieved by joint Gaussian distributed signals (X_1,X_2,X_r). Note that for Cut 1, we have

$$\begin{aligned} &I(X_1,X_2;Y_1,Y_r|X_r) = h(Y_1,Y_r|X_r) - h(Y_1,Y_r|X_1,X_2,X_r)\\ &= h(Y_r|X_r,Y_1) + h(Y_1|X_r) - h(Y_1|X_1,X_2,X_r)\\ &\qquad - h(Y_r|X_1,X_2,X_r)\\ &= h(Y_r|X_r,Y_1) + I(X_1,X_2;Y_1|X_r) - h(Y_r|X_1,X_2,X_r)\\ &= h(Y_r|X_r,Y_1) - h(Y_r|X_r) + I(X_1,X_2;Y_1|X_r)\\ &\qquad + h(Y_r|X_r) - h(Y_r|X_1,X_2,X_r)\\ &= I(X_1,X_2;Y_1|X_r) + I(X_1,X_2;Y_r|X_r) - I(Y_1;Y_r|X_r). \end{aligned} \tag{6}$$

Similarly for Cut 3 we have

$$\begin{aligned} &I(X_1,X_2;Y_2,Y_r|X_r)\\ &= I(X_1,X_2;Y_2|X_r) + I(X_1,X_2;Y_r|X_r) - I(Y_2;Y_r|X_r). \end{aligned} \tag{7}$$

It is hard to find a suitable distribution $p(X_1,X_2,X_r)$ that can maximize (6) and (7). Therefore the direct calculation of $C_{cut-set}$ turns out to be a hard problem to solve. Alternatively, given the fact that $I(Y_1;Y_r|X_r) \ge 0$ and $I(Y_2;Y_r|X_r) \ge 0$, we can find an upper bound $C_{upper1} \ge C_{cut-set}$ by removing the $I(Y_1;Y_r|X_r)$ and $I(Y_2;Y_r|X_r)$ from (6) and (7), respectively,

$$\begin{aligned} I(X_1,X_2;Y_1,Y_r|X_r) &\le I(X_1,X_2;Y_1|X_r) + I(X_1,X_2;Y_r|X_r),\\ I(X_1,X_2;Y_2,Y_r|X_r) &\le I(X_1,X_2;Y_2|X_r) + I(X_1,X_2;Y_r|X_r). \end{aligned} \tag{8}$$

Since all the items in RHS of (8) are simultaneously maximized by joint Gaussian distribution, i.e.,

$$\begin{aligned} &I(X_1,X_2;Y_1|X_r) \le C\left((\alpha_1'+\alpha_1'')P_1\right),\\ &I(X_1,X_2;Y_2|X_r) \le C\left((\alpha_2'+\alpha_2'')P_2\right),\\ &I(X_1,X_2;Y_r|X_r) \le\\ &C\left(a^2\left[(\alpha_1'+\alpha_1'')P_1 + (\alpha_2'+\alpha_2'')P_2 + 2\sqrt{\alpha_1''\alpha_2''P_1P_2}\right]\right), \end{aligned} \tag{9}$$

by combining it with (5), we can find the upper bound as

$$\begin{aligned} R_1 + R_2 < C_{upper1} = &\sup_{\substack{\alpha_1',\alpha_1'',\alpha_2',\alpha_2''\ge 0\\ 0\le\alpha_1'+\alpha_1''\le 1,\ 0\le\alpha_2'+\alpha_2''\le 1}} \min\\ &\left\{C\left(P_1 + b^2P_r + 2b\sqrt{(1-\alpha_1'-\alpha_1'')P_1P_r}\right),\right.\\ &C\left(P_2 + b^2P_r + 2b\sqrt{(1-\alpha_2'-\alpha_2'')P_2P_r}\right),\\ &C\left((\alpha_1'+\alpha_1'')P_1\right) +\\ &C\left(a^2\left[(\alpha_1'+\alpha_1'')P_1 + (\alpha_2'+\alpha_2'')P_2 + 2\sqrt{\alpha_1''\alpha_2''P_1P_2}\right]\right),\\ &C\left((\alpha_2'+\alpha_2'')P_2\right) +\\ &\left.C\left(a^2\left[(\alpha_1'+\alpha_1'')P_1 + (\alpha_2'+\alpha_2'')P_2 + 2\sqrt{\alpha_1''\alpha_2''P_1P_2}\right]\right)\right\}. \end{aligned} \tag{10}$$

For the symmetric rate scenario where $P_1 = P_2 = P_r = P$, by setting $\alpha_1' = \alpha_2' = \alpha'$ and $\alpha_1'' = \alpha_2'' = \alpha''$ (10) becomes

$$\begin{aligned} C_{upper1}^R = &\sup_{\substack{\alpha',\alpha''\ge 0\\ 0\le\alpha'+\alpha''\le 1}} \min\left\{\frac{1}{2}C\left(P(1+b^2+2b\sqrt{1-\alpha'-\alpha''}\right),\right.\\ &\left. 1/2 * \left[C\left((\alpha'+\alpha'')P\right) + C\left(2a^2P(\alpha'+2\alpha'')\right)\right]\right\}. \end{aligned} \tag{11}$$

B. Upper Bounds from MIMO Relay Channels

As stated in Sec. II, by modifying the channel and power allocation parameters accordingly (real signal and noise, single receiver antenna), the capacity upper bounds given by [9] and [10] are still valid and therefore can serve as baselines to bound the capacity regions.

As in Theorem 3.1 of [9], we can group the two single-antenna source nodes together for a new source node with $M_1 = 2$ transmit antennas. The relay has $N_1 = 1$ receive antenna and $M_2 = 1$ transmit antenna. Each of the destination node has $N = 1$ receive antenna. Since the transmitting power constraint has been incorporated into the signals X_i as described in (2), we simply set the power parameters $\eta_1 = \eta_2 = \eta_3 = 1$. According to the system model described in (1a) and (1c), the virtual MIMO relay channel defined by $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{R}$ and $\mathcal{D}_1$ has the following channel matrices: The source-relay channel $H_1 = [a, a]$, the source-destination channel $H_2 = [1, 0]$ and the relay-destination channel $H_3 = b$. We can set the covariance of X_r as $\Sigma_{22} = E[X_r^2] = P_r$ and the covariance matrix of $[X_1\ X_2]$ as

$$\Sigma_{11} = E\{[X_1\ X_2]' * [X_1\ X_2]\} = \begin{bmatrix} P_1 & \lambda\sqrt{P_1P_2} \\ \lambda\sqrt{P_1P_2} & P_2 \end{bmatrix},$$

where $0 \leq \lambda \leq 1$ is introduced to model the potential correlation between X_1 and X_2. The capacity upper bound defined by Theorem 3.1 of [9] therefore can be written as

$$R_1 + R_2 < C_{upper2} = \max_{0\leq\rho,\lambda\leq1} \min\left\{C_1^G, C_2^G, C_3^G, C_4^G\right\}, \tag{12}$$

where C_1^G and C_2^G are obtained from (8) and (9) in [9] and C_3^G and C_4^G by replacing H_2 by $\hat{H}_2 = [0, 1]$.

For symmetric scenarios, we can translate (12) to bound the symmetric rate $R_1 = R_2 = R$ as follows

$$R < C_{upper2}^R = \max_{0\leq\rho,\lambda\leq1} \min\left\{\frac{1}{2}C\left(P(1 + b^2 + 2b\rho)\right), \frac{1}{2}C\left(P(1-\rho^2)[1 + 2a^2(1+\lambda) + a^2P(1-\rho^2)(1-\lambda^2)]\right)\right\}.$$

Note that the bound C_{upper2} is not tight in general for two reasons: (i) the equality in (4) of [9] is achieved only if $M_1 \leq M_2$, which is not the case here; (ii) the optimization (12) is non-convex on ρ and therefore cannot guarantee global optimum. The overcome these limitations, a joint covariance matrix has been introduced in [10] as follows

$$\boldsymbol{R}_{SR} = \begin{bmatrix} P_1 & \lambda\sqrt{P_1P_2} & \rho\sqrt{P_1P_r} \\ \lambda\sqrt{P_1P_2} & P_2 & \mu\sqrt{P_2P_r} \\ \rho\sqrt{P_1P_r} & \mu\sqrt{P_2P_r} & P_r \end{bmatrix}, \tag{13}$$

where $0 \leq \rho, \lambda, \mu \leq 1$ are correlation coefficients. By setting the number of antennas $N_s = 2$ at the source, $N_R = 1$ at Relay and $N_D = 1$ at destinations, with channel matrices

$$H_0 = [1, 0],\ H_1 = [a, a],\ H_2 = b,\ \hat{H}_0 = [0, 1]$$

and the auxiliary construction matrices

$$\boldsymbol{D}_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ \boldsymbol{D}_R = [0\ 0\ 1],$$

the upper bound given by (9) of [10] can be written as

$$\begin{aligned} R_1 + R_2 < C_{upper3} = \max_{0\leq\rho,\lambda,\mu\leq1} \min \\ \Big\{C\left(P_1 + b^2P_r + 2b\rho\sqrt{P_1P_r}\right), \\ C\left(P_2 + b^2P_r + 2b\mu\sqrt{P_2P_r}\right), \\ \frac{1}{2}\log_2\Big((1+P_1)(1 + a^2(P_1 + P_2 + 2\lambda\sqrt{P_1P_2})) \\ -a^2(P_1 + \lambda\sqrt{P_1P_2})^2\Big), \\ \frac{1}{2}\log_2\Big((1+P_2)(1 + a^2(P_1 + P_2 + 2\lambda\sqrt{P_1P_2})) \\ -a^2(P_2 + \lambda\sqrt{P_1P_2})^2\Big)\Big\}. \end{aligned} \tag{14}$$

For symmetric scenarios, we can get from (14) that

$$R < C_{upper3}^R = \max_{0\leq\rho\leq\lambda\leq1} \min\left\{\frac{1}{2}C\left(P(1 + b^2 + 2b\rho)\right), \frac{1}{2}C\left(P(1 + 2a^2(1+\lambda) + a^2P(1-\lambda^2))\right)\right\}. \tag{15}$$

Note that the upper bound given by (9) of [10] is derived based on the sum-power constraint, which means it is in general loose for our case where only per-antenna/user power constraint is applied.

C. A Tighter Upper Bound C_{upp}

Based on the upper bounds C_{upper1}, C_{upper2} and C_{upper3}, we can obtain a tighter upper bound by taking their minimum,

$$R_1 + R_2 < C_{upp} = \min\left\{C_{upper1}, C_{upper2}, C_{upper3}\right\}. \tag{16}$$

It is similar for the symmetric rate upper bound C_{upp}^R.

V. Numerical Results

In this section, we present the numerical results on the capacity bounds on the symmetric rate scenarios with different power and channel gain parameters to compare these bound for different link quality. For power constraint $P_1 = P_2 = P_r = P$ and channel gains a^2 and b^2, the Signal-to-noise ratios are P/σ^2 for the source-destination link, a^2P/σ^2 for the source-relay link, and b^2P/σ^2 for the relay-destination link, respectively. The results for non-symmetric cases are similar and therefore omitted.

In Fig. 3, we compare these upper bounds discussed in Section IV with fixed transmitting power $P/\sigma^2 = 5$dB and relay-destination channel gain $b^2 = 0$dB but varying the source-relay channel gain a^2. When the source-relay channel is weak, the scenario where the DF relay strategy performs bad, the gap between upper and lower bounds is large, upp to 0.5 bits per channel use. When a^2 is large, however, the NBF scheme together with a DF relay turns to be optimal.

In Fig. 4, we keep transmitting power $P/\sigma^2 = 5$dB and the relay-destination channel gain $a^2 = 0$dB and but varying relay-destination channel gain b^2. For a weak relay-destination channel, NBF with DF relay performs well and the capacity gap is small, within 0.03 bits per channel use. For strong relay-destination channel, the gap is 0.16 bits per channel use.

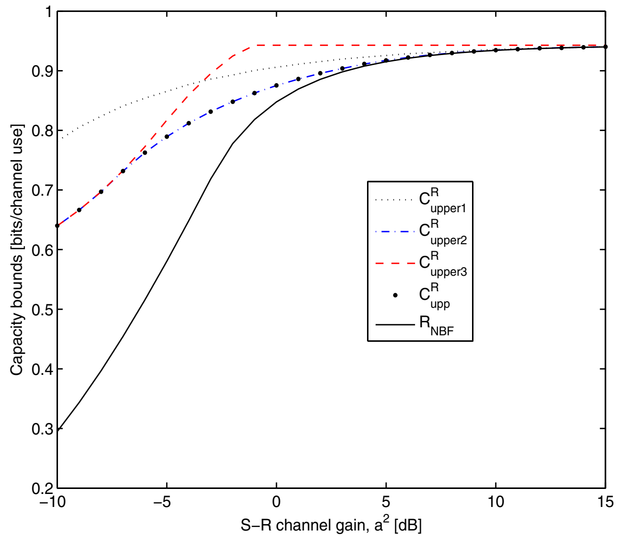

Figure 3. Capacity bounds for varying source-relay channel gain a^2 with fixed transmitting power $P/\sigma^2 = 5$dB and relay-destination channel gain $b^2 = 0dB$.

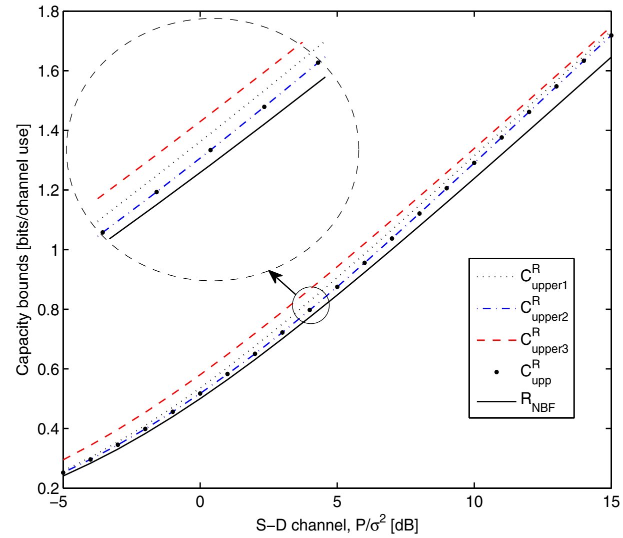

Figure 5. Capacity bounds for varying transmitting power P/σ^2 but fixed source-relay channel gain $a^2 = 0$dB and relay-destination channel gain $b^2 = 0dB$.

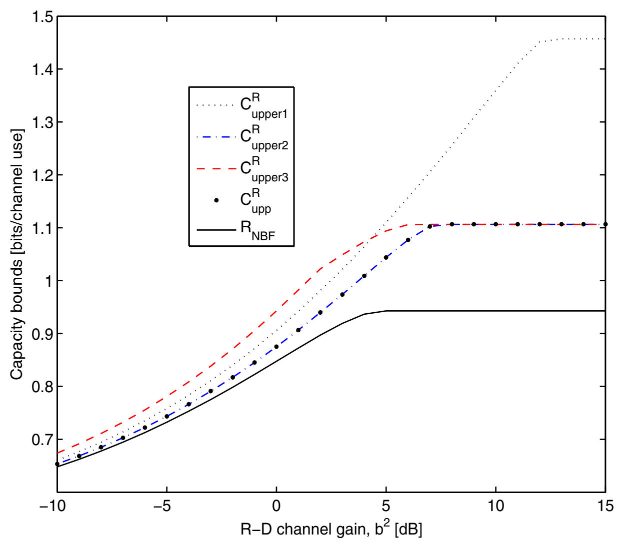

Figure 4. Capacity bounds for varying relay-destination channel gain b^2 with fixed transmitting power $P/\sigma^2 = 5$dB and source-relay channel gain $a^2 = 0dB$.

In Fig. 5, we investigate the asymptotic performance of different bounds with varying transmitting power P/σ^2 but fixed source-relay channel gain $a^2 = 0$dB and relay-destination channel gain $b^2 = 0$dB. A capacity gap of 0.07 bits per channel use can be observed at high SNR.

VI. Conclusions

We have studied a relay-aided two-source two-sink wireless multicast network with a backhual link between the source nodes. We provided three upper bounds on the capacity region and one lower bound given by an cooperative strategies using a full-duplex DF relay. The gap between the upper bounds and the lower bounds are still large in most of the regions but converge in some specific cases, as illustrated in the our numerical results. Further research on the capacity bounds are needed to make deeper understanding of the capacity regions of such building blocks in wireless networks.

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