

# Adaptive MBER Space-Time DFE Assisted Multiuser Detection for SDMA Systems

S. Chen, A. Livingstone and L. Hanzo

School of Electronics and Computer Science  
University of Southampton, Southampton SO17 1BJ, U.K.  
E-mails: {sqc,al902,lh}@ecs.soton.ac.uk

ABSTRACT

**In this contribution we propose a space-time decision feedback equalization (ST-DFE) assisted multiuser detection (MUD) scheme for multiple antenna aided space division multiple access systems. A minimum bit error rate (MBER) design is invoked for the MUD, which is shown to be capable of improving the achievable bit error rate performance over that of the minimum mean square error (MMSE) design. An adaptive MBER ST-DFE-MUD is proposed using the least bit error rate algorithm, which is demonstrated to consistently outperform the least mean square (LMS) algorithm, while achieving a lower computational complexity than the LMS algorithm for the binary signalling scheme. Simulation results demonstrate that the MBER ST-DFE-MUD is more robust to channel estimation errors as well as to error propagation imposed by decision feedback errors, compared to the MMSE ST-DFE-MUD.**

## I. INTRODUCTION

In an effort to further increase the achievable system capacity, antenna arrays can be employed for supporting multiple users in a space division multiple access (SDMA) communications scenario [1]-[10]. We investigate a space-time (ST) decision feedback equalization (DFE) assisted multiuser detection (MUD) scheme for multiple receiver antenna aided SDMA systems. To interpret the multiuser-supporting capability of such a novel SDMA system [11], it is useful to relate it to classic code division multiple access (CDMA) multiuser systems [9]. In a CDMA system, each user is separated by a unique user-specific spreading code. By contrast, an SDMA system differentiates each user by the associated unique user-specific channel impulse response (CIR) encountered at the receiver antennas. In this analogy, the unique user-specific CIR plays the role of a user-specific CDMA signature. However, owing to the non-orthogonal nature of the CIRs, an effective MUD is required for separating the users in an SDMA system.

The most popular SDMA-receiver design is constituted by the minimum mean square error (MMSE) MUD [5],[8]-[12]. However, as recognized by [13] in a CDMA context and by [14] in an adaptive beamforming-based MUD scenario, a better strategy is to choose the detector's coefficients by directly minimizing the system's bit error ratio (BER). For the single-user single-antenna system, the minimum BER (MBER) equalization design has been proposed [15]-[18]. This paper studies the MBER ST-DFE-MUD in the context of SDMA and derives an adaptive MBER ST-DFE-MUD based on the least bit error rate (LBER) algorithm. It is shown that the MBER ST-DFE-MUD design results in an enhanced BER

performance in comparison to the MMSE design. Moreover, unlike the MMSE design whose performance degrades significantly owing to decision feedback errors in the presence of multi-user feedback loops, the MBER ST-DFE-MUD is very robust to the error propagation. The MBER ST-DFE-MUD is also shown to be more robust to channel estimation errors than the MMSE design. It is demonstrated that the LBER ST-DFE-MUD consistently outperforms the least mean square (LMS) based ST-DFE-MUD and yet it has a lower computational complexity than the latter in the case of the binary phase shift keying (BPSK) modulation scheme.

## II. SYSTEM MODEL

Consider the multiple antenna aided SDMA system supporting  $M$  users, where each of the  $M$  users is equipped with a single transmit antenna and the receiver is assisted by an  $L$ -element antenna array. The symbol-rate received signal samples  $x_l(k)$  for  $1 \leq l \leq L$  are given by

$$x_l(k) = \sum_{m=1}^M \sum_{i=0}^{n_C-1} c_{i,l,m} s_m(k-i) + n_l(k) = \bar{x}_l(k) + n_l(k), \quad (1)$$

where  $n_l(k)$  is a complex-valued Gaussian white noise process with  $E[|n_l(k)|^2] = 2\sigma_n^2$ ,  $\bar{x}_l(k)$  denotes the noise-free part of the  $l$ th receive antenna's output,  $s_m(k)$  is the  $k$ th transmitted symbol of user  $m$ , and  $\mathbf{c}_{l,m} = [c_{0,l,m} \ c_{1,l,m} \ \dots \ c_{n_C-1,l,m}]^T$  denotes the tap vector of the CIR connecting the user  $m$  and the  $l$ th receive antenna. For notational simplicity, we have assumed that each of the  $(M \times L)$  CIRs has the same length of  $n_C$ . We assume furthermore that BPSK modulation is employed and hence we have  $s_m(k) \in \{\pm 1\}$ .

A bank of the  $M$  ST-DFEs constitutes the MUD, and the soft outputs of the  $M$  ST-DFEs are given by

$$y_m(k) = \sum_{l=1}^L \sum_{i=0}^{n_F-1} w_{i,l,m}^* x_l(k-i) + \sum_{q=1}^M \sum_{i=1}^{n_B} b_{i,q,m}^* \hat{s}_q(k-d-i), \quad (2)$$

for  $1 \leq m \leq M$ , where  $\hat{s}_m(k)$  denotes the estimate of  $s_m(k)$ ,  $\mathbf{w}_{l,m} = [w_{0,l,m} \ w_{1,l,m} \ \dots \ w_{n_F-1,l,m}]^T$  denotes the feedforward filter weight vector of the  $m$ th user's detector associated with the  $l$ th receive antenna, while  $\mathbf{b}_{q,m} = [b_{1,q,m} \ b_{2,q,m} \ \dots \ b_{n_B,q,m}]^T$  denotes the  $m$ th user's detector feedback filter weight vector associated with the  $q$ th user detector's feedback. Again, for notational simplicity, we have assumed that each of the  $M$  ST-DFEs has the same decision delay  $d$ , all the feedforward filters have the same order  $n_F$ , and all the feedback filters have the same order  $n_B$ . The  $M$  detectors' decisions are defined by

$$\hat{s}_m(k-d) = \text{sgn}(y_{R_m}(k)), \quad 1 \leq m \leq M, \quad (3)$$

where  $y_{R_m}(k) = \Re[y_m(k)]$ . Define  $\mathbf{x}_l(k) = [x_l(k) \ x_l(k-1) \ \dots \ x_l(k-n_F+1)]^T$ ,  $\hat{\mathbf{s}}_{B_q}(k) = [\hat{s}_q(k-d-1) \ \dots \ \hat{s}_q(k-$

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$d - n_B)]^T$ ,  $\mathbf{w}_m = [\mathbf{w}_{1,m}^T \mathbf{w}_{2,m}^T \cdots \mathbf{w}_{L,m}^T]^T$ ,  $\mathbf{x}(k) = [\mathbf{x}_1^T(k) \mathbf{x}_2^T(k) \cdots \mathbf{x}_L^T(k)]^T$ ,  $\mathbf{b}_m = [\mathbf{b}_{1,m}^T \mathbf{b}_{2,m}^T \cdots \mathbf{b}_{M,m}^T]^T$ , and  $\hat{\mathbf{s}}_B(k) = [\hat{\mathbf{s}}_{B_1}^T(k) \hat{\mathbf{s}}_{B_2}^T(k) \cdots \hat{\mathbf{s}}_{B_M}^T(k)]^T$ . Then the output of the  $m$ th ST-DFE can be written as

$$\begin{aligned} y_m(k) &= \sum_{l=1}^L \mathbf{w}_{l,m}^H \mathbf{x}_l(k) + \sum_{q=1}^M \mathbf{b}_{q,m}^H \hat{\mathbf{s}}_{B_q}(k) \\ &= \mathbf{w}_m^H \mathbf{x}(k) + \mathbf{b}_m^H \hat{\mathbf{s}}_B(k). \end{aligned} \quad (4)$$

We will choose the ST-DFE structure's parameters as follows:  $d = n_C - 1$ ,  $n_F = n_C$  and  $n_B = n_C - 1$ . This choice of the DFE structure's parameters is sufficient for guaranteeing that the two classes of noise-free signal states are always linearly separable at the detector's output and therefore they guarantee an adequate performance [15]. With  $n_F = n_C$  and  $d = n_B = n_C - 1$ , let us introduce the two overall CIR matrices as

$$\mathbf{C}_F = \begin{bmatrix} \mathbf{C}_{F_1} \\ \mathbf{C}_{F_2} \\ \vdots \\ \mathbf{C}_{F_L} \end{bmatrix} \quad \text{and} \quad \mathbf{C}_B = \begin{bmatrix} \mathbf{C}_{B_1} \\ \mathbf{C}_{B_2} \\ \vdots \\ \mathbf{C}_{B_L} \end{bmatrix}, \quad (5)$$

where  $\mathbf{C}_{F_l}$  and  $\mathbf{C}_{B_l}$  are given by

$$\mathbf{C}_{F_l} = [\mathbf{C}_{F_{l,1}} \quad \mathbf{C}_{F_{l,2}} \quad \cdots \quad \mathbf{C}_{F_{l,M}}] \quad (6)$$

and

$$\mathbf{C}_{B_l} = [\mathbf{C}_{B_{l,1}} \quad \mathbf{C}_{B_{l,2}} \quad \cdots \quad \mathbf{C}_{B_{l,M}}], \quad (7)$$

respectively, with the  $n_F \times (d+1)$  and  $n_F \times n_B$  dimensional CIR matrices  $\mathbf{C}_{F_{l,m}}$  and  $\mathbf{C}_{B_{l,m}}$  defined by

$$\mathbf{C}_{F_{l,m}} = \begin{bmatrix} c_{0,l,m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} \\ 0 & c_{0,l,m} & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_{1,l,m} \\ 0 & \cdots & 0 & c_{0,l,m} \end{bmatrix} \quad (8)$$

and

$$\mathbf{C}_{B_{l,m}} = \begin{bmatrix} 0 & \cdots & 0 \\ c_{n_C-1,l,m} & \ddots & \vdots \\ \vdots & \ddots & 0 \\ c_{1,l,m} & \cdots & c_{n_C-1,l,m} \end{bmatrix}, \quad (9)$$

respectively. Let us define furthermore  $\mathbf{s}_F(k) = [\mathbf{s}_{F_1}^T(k) \mathbf{s}_{F_2}^T(k) \cdots \mathbf{s}_{F_M}^T(k)]^T$ ,  $\mathbf{s}_B(k) = [\mathbf{s}_{B_1}^T(k) \mathbf{s}_{B_2}^T(k) \cdots \mathbf{s}_{B_M}^T(k)]^T$  and  $\mathbf{n}(k) = [\mathbf{n}_1(k) \mathbf{n}_2(k) \cdots \mathbf{n}_L(k)]^T$ , where  $\mathbf{s}_{F_m}(k) = [s_m(k) s_m(k-1) \cdots s_m(k-d)]^T$ ,  $\mathbf{s}_{B_m}(k) = [s_m(k-d-1) s_m(k-d-2) \cdots s_m(k-d-n_B)]^T$  and  $\mathbf{n}_l(k) = [n_l(k) n_l(k-1) \cdots n_l(k-n_F+1)]^T$ . Then the received signal vector  $\mathbf{x}(k)$  is modeled as

$$\mathbf{x}(k) = \mathbf{C}_F \mathbf{s}_F(k) + \mathbf{C}_B \mathbf{s}_B(k) + \mathbf{n}(k). \quad (10)$$

Under the assumption that the past decisions are correct, we have  $\hat{\mathbf{s}}_B(k) = \mathbf{s}_B(k)$  and the received signal vector can be expressed as  $\mathbf{x}(k) = \mathbf{C}_F \mathbf{s}_F(k) + \mathbf{C}_B \hat{\mathbf{s}}_B(k) + \mathbf{n}(k)$ . Thus, the decision feedback

can be viewed as a translation of the original observation space  $\mathbf{x}(k)$  into a new space  $\mathbf{r}(k)$  [15]

$$\begin{aligned} \mathbf{r}(k) &\triangleq \mathbf{x}(k) - \mathbf{C}_B \hat{\mathbf{s}}_B(k) = \mathbf{C}_F \mathbf{s}_F(k) + \mathbf{n}(k) \\ &= \bar{\mathbf{r}}(k) + \mathbf{n}(k). \end{aligned} \quad (11)$$

In the translated space  $\mathbf{r}(k)$ , the original ST-DFE described by (4) is "translated" into a ST "linear equalizer" described as

$$y_m(k) = \mathbf{w}_m^H \mathbf{r}(k) = \mathbf{w}_m^H (\bar{\mathbf{r}}(k) + \mathbf{n}(k)) = \bar{y}_m(k) + e_m(k), \quad (12)$$

where  $e_m(k)$  is Gaussian distributed, having a zero mean and  $E[|e_m(k)|^2] = 2\mathbf{w}_m^H \mathbf{w}_m \sigma_n^2$ . Note that we have  $\mathbf{r}(k) = [\mathbf{r}_1^T(k) \mathbf{r}_2^T(k) \cdots \mathbf{r}_L^T(k)]^T$  with  $\mathbf{r}_l(k) = [r_l(k) r_l(k-1) \cdots r_l(k-n_F+1)]^T$ . The elements of  $\mathbf{r}_l(k)$  can be computed recursively according to [15]

$$\begin{aligned} r_l(k-i) &= z^{-1} r_l(k-i+1) - \sum_{m=1}^M c_{n_C-i,l,m} \hat{s}_m(k-d-1), \\ &\quad \text{for } i = n_F - 1, n_F - 2, \dots, 1, \\ r_l(k) &= x_l(k), \end{aligned} \quad (13)$$

where  $z^{-1}$  defines the unit delay operator. The detector structure of (12) with the space translation (13) is exactly the same as the original DFE structure (4). The feedback coefficient vector  $\mathbf{b}_m$  does not simply "disappear". It has in fact been set to its "optimal value", which is  $\mathbf{b}_m = -\mathbf{C}_B^H \mathbf{w}_m$ .

### III. MINIMUM BIT ERROR RATE MULTIUSER DETECTION

Let us denote the  $N_s = 2^{M(d+1)}$  number of possible sequences of  $\mathbf{s}_F(k)$  as  $\mathbf{s}^{(q)}$ ,  $1 \leq q \leq N_s$ . Denote furthermore the  $m(d+1)$ th element of  $\mathbf{s}^{(q)}$ , corresponding to the symbol  $s_m(k-d)$ , as  $s_{m,d}^{(q)}$ . The noise-free part of the  $m$ th detector input signal  $\bar{\mathbf{r}}(k)$  assumes values from the signal set defined as  $\mathcal{R}_m \triangleq \{\bar{\mathbf{r}}^{(q)} = \mathbf{C}_F \mathbf{s}^{(q)}, 1 \leq q \leq N_s\}$ . Similarly, the noise-free part of the  $m$ th detector's output  $\bar{y}_{R_m}(k) = \Re[\bar{y}_m(k)]$  assumes values from the scalar set

$$\mathcal{Y}_{R_m} \triangleq \{\bar{y}_{R_m}^{(q)} = \Re[\mathbf{w}_m^H \bar{\mathbf{r}}^{(q)}], 1 \leq q \leq N_s\}. \quad (14)$$

The probability density function (PDF) of  $y_{R_m}(k)$  is a Gaussian mixture given by [13],[14]

$$p_m(y_R) = \frac{1}{N_s \sqrt{2\pi} \sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}} \sum_{q=1}^{N_s} e^{-\frac{(y_R - \bar{y}_{R_m}^{(q)})^2}{2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}}, \quad (15)$$

where  $\bar{y}_{R_m}^{(q)} \in \mathcal{Y}_{R_m}$ . Thus the BER of the  $m$ th ST-DFE associated with weight vector  $\mathbf{w}_m$  is given by

$$P_E(\mathbf{w}_m) = \frac{1}{N_s} \sum_{q=1}^{N_s} Q\left(g_R^{(q)}(\mathbf{w}_m)\right), \quad (16)$$

where  $Q(\bullet)$  is the usual error  $Q$ -function and

$$g_R^{(q)}(\mathbf{w}_m) = \frac{\text{sgn}(s_{m,d}^{(q)}) \bar{y}_{R_m}^{(q)}}{\sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}} = \frac{\text{sgn}(s_{m,d}^{(q)}) \Re[\mathbf{w}_m^H \bar{\mathbf{r}}^{(q)}]}{\sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}}. \quad (17)$$

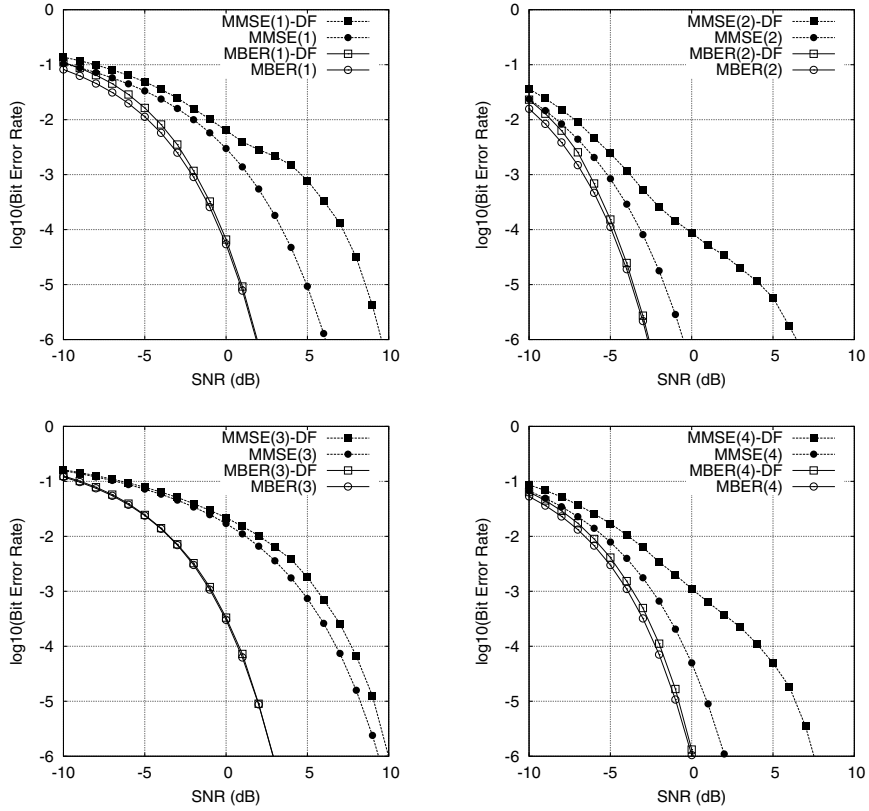


Fig. 1. Theoretical and simulated bit error rate comparison of the MMSE and MBER ST-DFE-MUDs for users 1 to 4 of the 4-user 4-antenna time-invariant system, where DF indicates simulated BER with detected symbols being fed back.

TABLE I  
SYSTEM'S CIRs FOR A 4-ANTENNA 4-USER TIME-INVARIANT SDMA SYSTEM.

$C_{l,m}(z)$	$m = 1$	$m = 2$	$m = 3$	$m = 4$
$l = 1$	$(0.6 + j0.7)$ $+(0.8 + j0.5)z^{-1}$ $+(0.3 + j0.4)z^{-2}$	$(-0.1 - j0.2)$ $+(0.4 + j0.5)z^{-1}$ $+(0.3 - j0.2)z^{-2}$	$(0.7 + j0.5)$ $+(0.6 + j0.4)z^{-1}$ $+(0.5 + j0.5)z^{-2}$	$(0.8 - j0.4)$ $+(-0.6 + j0.5)z^{-1}$ $+(0.3 + j0.3)z^{-2}$
$l = 2$	$(0.1 + j0.2)$ $+(0.4 + j0.3)z^{-1}$ $+(0.5 + j0.4)z^{-2}$	$(0.9 + j0.2)$ $+(0.3 + j0.7)z^{-1}$ $+(0.2 + j0.2)z^{-2}$	$(-0.3 - j0.3)$ $+(0.4 + j0.2)z^{-1}$ $+(-0.2 + j0.4)z^{-2}$	$(0.3 + j0.3)$ $+(0.4 + j0.4)z^{-1}$ $+(0.5 + j0.5)z^{-2}$
$l = 3$	$(-0.1 + j0.3)$ $+(0.6 - j0.5)z^{-1}$ $+(0.2 + j0.4)z^{-2}$	$(0.5 + j0.6)$ $+(-0.3 - j0.4)z^{-1}$ $+(0.2 + j0.4)z^{-2}$	$(0.2 - j0.3)$ $+(0.4 - j0.5)z^{-1}$ $+(0.6 + j0.3)z^{-2}$	$(0.1 + j0.8)$ $+(0.7 + j0.6)z^{-1}$ $+(0.8 + j0.5)z^{-2}$
$l = 4$	$(0.8 + j0.9)$ $+(0.6 + j0.5)z^{-1}$ $+(0.5 + j0.3)z^{-2}$	$(0.4 + j0.4)$ $+(0.4 + j0.4)z^{-1}$ $+(0.4 + j0.4)z^{-2}$	$(0.1 + j0.2)$ $+(-0.3 - j0.4)z^{-1}$ $+(0.3 + j0.2)z^{-2}$	$(0.4 + j0.6)$ $+(0.5 + j0.3)z^{-1}$ $+(0.2 + j0.3)z^{-2}$

Note that the BER is invariant to a positive scaling of  $\mathbf{w}_m$ .

The MBER solution for the  $m$ th detector is then defined as the weight vector that minimizes the error probability (16)

$$\mathbf{w}_{(\text{MBER})_m} = \arg \min_{\mathbf{w}_m} P_E(\mathbf{w}_m). \quad (18)$$

The gradient of  $P_E(\mathbf{w}_m)$  with respect to  $\mathbf{w}_m$  is given by

$$\begin{aligned} \nabla P_E(\mathbf{w}_m) &= \frac{1}{2N_s \sqrt{2\pi} \sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}} \sum_{q=1}^{N_s} e^{-\frac{(\hat{y}_{R_m}^{(q)})^2}{2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}} \\ &\times \text{sgn} \left( s_{m,d}^{(q)} \right) \left( \frac{\hat{y}_{R_m}^{(q)} \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{w}_m} - \hat{\mathbf{r}}^{(q)} \right). \quad (19) \end{aligned}$$

Given the gradient (19), the optimization problem (18) can be solved iteratively by commencing from an appropriate initialization point using a gradient optimization algorithm. The simplified conjugate gradient algorithm of [19],[13] provides an efficient means of finding an MBER solution for the optimization problem (18).

#### IV. ADAPTIVE MINIMUM BIT ERROR RATE IMPLEMENTATION

The Parzen window method [20]-[22] provides an efficient means of estimating a PDF. Given a block of  $K$  training samples  $\{\mathbf{r}(k), s_m(k-d)\}_{k=1}^K$ , a Parzen window density estimate of the

PDF in (15) takes the form

$$\tilde{p}_m(y_R) = \frac{1}{K\sqrt{2\pi}\rho_n} \sum_{k=1}^K e^{-\frac{(y_R - y_{R_m}(k))^2}{2\rho_n^2}}, \quad (20)$$

where  $\rho_n^2$  is the chosen kernel variance. Based on the estimated PDF (20), an approximate BER is given by

$$\tilde{P}_E(\mathbf{w}_m) = \frac{1}{K} \sum_{k=1}^K Q\left(\tilde{g}_R^{(k)}(\mathbf{w}_m)\right) \quad (21)$$

with

$$\tilde{g}_R^{(k)}(\mathbf{w}_m) = \frac{\text{sgn}(s_m(k-d))y_{R_m}(k)}{\rho_n}. \quad (22)$$

This approximation is an adequate one, provided that the width  $\rho_n$  is chosen appropriately.

To derive a sample-by-sample adaptive algorithm for updating the detector's weight vector  $\mathbf{w}_m$ , consider a single-sample estimate of  $p_m(y_R)$

$$\tilde{p}_m(y_R, k) = \frac{1}{\sqrt{2\pi}\rho_n} e^{-\frac{(y_R - y_{R_m}(k))^2}{2\rho_n^2}}. \quad (23)$$

Conceptually, from this single-sample PDF "estimate", we have a single-sample or instantaneous BER "estimate"  $\tilde{P}_E(\mathbf{w}_m, k)$ . Using the instantaneous stochastic gradient  $\nabla \tilde{P}_E(\mathbf{w}_m, k)$  gives rise to a stochastic gradient adaptive algorithm, which we referred to as the LBER algorithm

$$\mathbf{w}_m(k+1) = \mathbf{w}_m(k) + \mu \frac{\text{sgn}(s_m(k-d))}{2\sqrt{2\pi}\rho_n} e^{-\frac{y_{R_m}^2(k)}{2\rho_n^2}} \mathbf{r}(k). \quad (24)$$

The adaptive gain  $\mu$  as well as the kernel width  $\rho_n$  are the two algorithmic parameters that have to be set appropriately. Specifically, they are chosen to ensure adequate performance in terms of both the achievable convergence rate and steady-state BER misadjustment. Note that there is no need to normalize the weight vector to a unit-length after each update. It can readily be shown that for the BPSK case, the LBER ST-DFE is computationally simpler than the LMS ST-DFE, imposing about half the computational complexity required by the LMS algorithm [14].

## V. SIMULATION STUDY

**Time-invariant system.** The system supported  $M = 4$  users with  $L = 4$  receiver antennas. The 16 CIRs are listed in Table I, each having  $n_C = 3$  taps. In the simulations all the 16 CIRs were normalized using  $C_{l,m}(z)/|C_{l,m}(z)|$  to provide a channel gain of unity. As the length of the CIRs was  $n_C = 3$ , the ST-DFE structure was defined by  $n_F = 3$ ,  $d = 2$  and  $n_B = 2$ . The theoretical BER curves of the MMSE and MBER ST-DFE-MUDs, computed using the BER expression of (16), are plotted in Fig. 1 over a range of signal to noise ratio (SNR) conditions. It can be seen that the MBER ST-DFE-MUD provided better BER performance than the MMSE ST-DFE-MUD. The BER calculated using the expression (16) represents the theoretical best-case performance, since it was obtained assuming that the correct symbols were fed back

in the ST-DFE-MUD's feedback loop. For the sake of investigating the effects of decision feedback induced error propagation, the BERs of the MMSE and MBER ST-DFE-MUDs were also calculated using simulations with the error-prone detected symbols being fed back, and the results are also depicted in Fig. 1, in comparison to the corresponding theoretical best-case performance. It can be seen that the MBER ST-DFE-MUD is significantly more robust to error propagation than the MMSE ST-DFE-MUD. We also added the Gaussian white noise with standard deviation 0.1 to each tap of the CIRs to represent channel estimation errors. The theoretical BERs of the MMSE and MBER ST-DFE-MUDs obtained based on the "estimated" CIRs and averaged over 10 "estimations" are illustrated in Fig. 2, in comparison to the performance derived using perfect channel knowledge. It can be seen that the performance degradation due to imperfect channel estimates is less serious for the MBER ST-DFE-MUD than for the MMSE one.

**Slow fading system.** The system again supported 4 users with 4 receive antennas. However, fading channels were simulated and each of the 16 CIRs had  $n_C = 3$  taps. Magnitudes of the CIR taps were uncorrelated Rayleigh processes, each having the root mean power of  $\sqrt{0.5} + j\sqrt{0.5}$ . The normalized Doppler frequency for the simulated system was  $10^{-6}$ , which for a carrier of 900 MHz and a symbol rate of 3 Msymbols/s corresponded to a user velocity of 1 m/s (3.6 km/h). Continuously fluctuating fading was used, which provided a different fading magnitude and phase for each transmitted symbol. The ST-DFE structure parameters were set to  $d = 2$ ,  $n_F = 3$  and  $n_B = 2$ . The step size for the LMS algorithm was chosen as  $\mu = 0.005$ , while for the LBER algorithm the step size  $\mu = 0.1$  and kernel variance  $\rho_n^2 = 9\sigma_n^2$ . The transmission frame structure consisted of 50 training symbols followed by 450 data symbols. The BER of an adaptive ST-DFE-MUD was calculated using Monte Carlo simulation with the detected symbols being fed back. Fig. 3 compares the BER of the LBER ST-DFE-MUD for user 2 with that of the LMS based one. The BERs for the other three users, not shown here due to space limitation, are similar to the BER for user 2 shown in Fig. 3. It can be seen that the LBER ST-DFE-MUD consistently outperformed the LMS ST-DFE-MUD.

## VI. CONCLUSIONS

A novel minimum bit error rate design has been proposed for the ST-DFE-MUD employed in multiple antenna aided SDMA systems. It has been demonstrated that this MBER design is capable of achieving better performance and hence of improving the attainable system capacity, compared to the MMSE design. An adaptive implementation of the MBER ST-DFE-MUD has also been derived based on the LBER algorithm, which has been shown to consistently outperform the LMS algorithm and yet maintaining a lower computational complexity than the latter for BPSK modulation. Another interesting result observed in this study is that the MBER ST-DFE-MUD is significantly more robust against the error propagation caused by error-prone detected symbols used in the MUD's feedback loop, in comparison to the MMSE ST-DFE-MUD.

## REFERENCES

- [1] J.H. Winters, J. Salz and R.D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Communications*, Vol.42, No.2, pp.1740-1751, 1994.

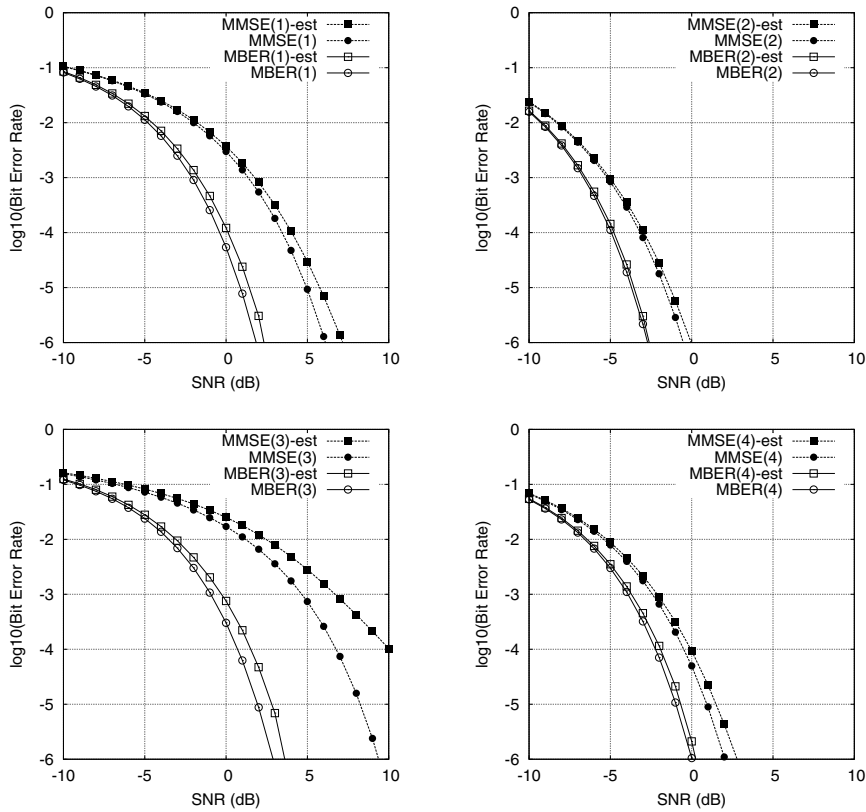


Fig. 2. Theoretical bit error rate comparison of the MMSE and MBER ST-DFE-MUDs for users 1 to 4 of the 4-user 4-antenna time-invariant system, where est indicates imperfect channel estimates were used.

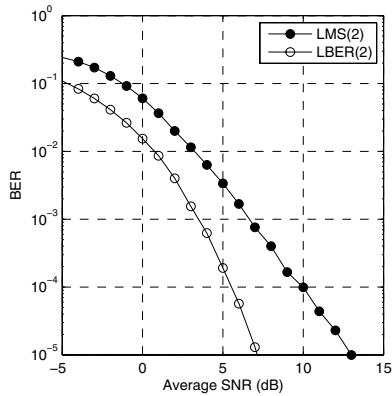


Fig. 3. Simulated bit error rate comparison of the LMS and LBER ST-DFE-MUDs for user 2 of the 4-user 4-antenna slow fading system.

[2] A.J. Paulraj and C.B. Papadias, "Space-time processing for wireless communications," *IEEE Signal Processing Magazine*, Vol.14, No.6, pp.49–83, 1997.

[3] G. Tsoulos, M. Beach and J. McGeehan, "Wireless personal communications for the 21st century: European technological advances in adaptive antennas," *IEEE Communications Magazine*, Vol.35, No.9, pp.102–109, 1997.

[4] J.H. Winters, "Smart antennas for wireless systems," *IEEE Personal Communications*, Vol.5, No.1, pp.23–27, 1998.

[5] A.J. Paulraj and B.C. Ng, "Space-time modems for wireless personal communications," *IEEE Personal Communications*, Vol.5, No.1, pp.36–48, 1998.

[6] P. Vandenameele, L. van Der Perre and M. Engels, *Space Division Multiple Access for Wireless Local Area Networks*. Boston: Kluwer Academic Publishers, 2001.

[7] J.S. Blough and L. Hanzo, *Third Generation Systems and Intelligent Wireless Networking – Smart Antenna and Adaptive Modulation*. Chichester: John Wiley, 2002.

[8] A. Paulraj, R. Nabar and D. Gore, *Introduction to Space-Time Wireless Communications*. Cam-

bridge: Cambridge University Press, 2003.

[9] L. Hanzo, L.-L. Yang, E.-L. Kuan and K. Yen, *Single- and Multi-Carrier DS-SS: Multi-User Detection, Space-Time Spreading, Synchronisation, Standards and Networking*. IEEE Press - John Wiley, 2003.

[10] A.J. Paulraj, D.A. Gore, R.U. Nabar and H. Bölcskei, "An overview of MIMO communications – A key to gigabit wireless," *Proc. IEEE*, Vol.92, No.2, pp.198–218, 2004.

[11] L. Hanzo, M. Münster, B.J. Choi and T. Keller, *OFDM and MC-CDMA*. West Sussex, England: John Wiley and IEEE Press, 2003.

[12] D.N.C. Tse and S.V. Hanly, "Linear multiuser receivers: effective interference, effective bandwidth and user capacity," *IEEE Trans. Information Theory*, Vol.45, No.2, pp.641–657, 1999.

[13] S. Chen, A.K. Samangan, B. Mulgrew and L. Hanzo, "Adaptive minimum-BER linear multiuser detection for DS-SS signals in multipath channels," *IEEE Trans. Signal Processing*, Vol.49, No.6, pp.1240–1247, 2001.

[14] S. Chen, N.N. Ahmad and L. Hanzo, "Adaptive minimum bit error rate beamforming," *IEEE Trans. Wireless Communications*, Vol.4, No.2, pp.341–348, 2005.

[15] S. Chen, B. Mulgrew, E.S. Chng and G. Gibson, "Space translation properties and the minimum-BER linear-combiner DFE," *IEE Proc. Communications*, Vol.145, No.5, pp.316–322, 1998.

[16] C.C. Yeh and J.R. Barry, "Adaptive minimum bit-error rate equalization for binary signaling," *IEEE Trans. Communications*, Vol.48, No.7, pp.1226–1235, 2000.

[17] B. Mulgrew and S. Chen, "Adaptive minimum-BER decision feedback equalisers for binary signalling," *Signal Processing*, Vol.81, No.7, pp.1479–1489, 2001.

[18] S. Chen, L. Hanzo and B. Mulgrew, "Adaptive minimum symbol-error-rate decision feedback equalization for multi-level pulse-amplitude modulation," *IEEE Trans. Signal Processing*, Vol.52, No.7, pp.2092–2101, 2004.

[19] M.S. Bazaraa, H.D. Sherali and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms*. New York: John Wiley, 1993.

[20] E. Parzen, "On estimation of a probability density function and mode," *The Annals of Mathematical Statistics*, Vol.33, pp.1066–1076, 1962.

[21] B.W. Silverman, *Density Estimation*. London: Chapman Hall, 1996.

[22] A.W. Bowman and A. Azzalini, *Applied Smoothing Techniques for Data Analysis*. Oxford: Oxford University Press, 1997.