New methodology for improving the inspection policies for degradation model selection according to prognostic measures

Thi Phuong Khanh Nguyen, Mitra Fouladirad, Antoine Grall

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Khanh T.P Nguyen
Production Engineering Laboratory, Toulouse INP, ENIT, Tarbes, France
Emails: thi-phuong-khanh.nguyen@enit.fr
Mitra Fouladirad, Antoine Grall
ICD/LM2S, UMR CNRS 6281, Université de Technologie de Troyes, Troyes, France
Email: {mitra.fouladirad, antoine.grall}@utt.fr

***Abstract*—Health monitoring data are vital for failure prognostic and maintenance planning. Continuous monitoring data or frequent inspections can provide a large amount of information on degradation evolution and therefore ensure the quality of deterioration modeling and the lifetime prognostic accuracy. However, they are usually very costly, and sometimes inpractible in real engineering applications. Therefore, it is essential to address the issue of the appropriate amount of monitoring data. This paper proposes a new methodology to help the companies improving their actual inspection/monitoring policy to reduce operation and maintenance costs but also ensure the information quality. We investigate different types of inspection policies including periodic or non-periodic ones by considering multiples functions of the system degradation state that are linear, concave or convex. The best policies are chosen based on a multi-objective optimization problem dealing with the inspection cost and the information level. The advantages and disadvantages of the proposed methodology are discussed through numerous numerical examples for different types of degradation process, particularly Wiener and Gamma processes that have been largely addressed in the framework of degradation modeling.**

Keywords–Prognostic prediction, RUL prediction, Non-periodic inspection, Model selection, Degradation process

I. Introduction

Nowadays, the implementation of Prognostic and Health Management (PHM) solutions leads to improve the competitiveness of companies by increasing production availability, avoiding contract penalties and reducing maintenance costs. Indeed, PHM is concerned with estimating component health and predicting Remaining Useful Life (RUL) of a given system. Prognostic methods can be classified into three main approaches: model-based prognostics, data-driven prognostics and hybrid. Data-driven approaches uses very often machine learning algorithms to transform monitoring data before failure into compatible models [16], [27]. However, it is difficult to acquire such large amount of training data to built suitable machine learning algorithms. Model-based approaches allow us to overcome this drawback by taking into account the physical degradation phenomenon of the system and deriving appropriate mathematical models [9], [32], [37]. Furthermore, prior knowledge on the degradation phenomenon helps us to limit the list of candidate models. It can improve PHM procedure. A hybrid approach allows benefiting from the strengths of both categories [5], [6]. It can be based on a study of physical degradation phenomena to propose a list of models and an estimation of their parameters based on the acquired data. In this framework, one important issues is to select in the competing candidate list the model which is the best to describe the underlying degradation phenomenon in the presence of deteriorating data [36], [38].

For model selection, numerous papers in literature propose decision criteria that depend mainly on the specific purpose for which the model can be used, see for instance [3], [12], [38]. Authors in [22] introduce a set of metrics to assess the prognostic ability of a given model when degradation data and true failure times are known. [2] proposes a method to compare degradation models based on the quality of reliability assessment method. [14] develops an approach for online assessing the performance of a prognostic method in situations of very poor knowledge on the degradation process. To our knowledge, no existing research address the issue of the relevant number of monitoring data for model selection. Continuous monitoring data provide a large amount of information and therefore can help improving the model selection accuracy. However, continuous monitoring can be very costly and is sometimes impossible to implement in practice. Limits due to the monitoring technology or to human resources can lead to some constraints on the time between two successive inspections. Among different inspection policies, the policy with more frequent inspections can capture more information on the degradation evolution. Nevertheless, it is also very costly. This paper aims to address the gap in the literature by investigating how to find a balance between the monitoring cost and the necessary information level for degradation modeling. The competing models are assumed to follow the Wiener and Gamma processes that have been largely addressed in the framework of degradation modeling [13], [26], [29]. In fact, the Wiener process is very popular in engineering and deterioration modeling when the health indicator evolves non-monotonically. The statistical properties of the failure time of a Wiener process are studied in [7], [8]. Since the 1970s, this process has been widely studied in reliability and lifetime analysis. Authors in [11] used the Wiener process with drift to model accelerated life testing data. In [33], [34] the impact of measurement errors on the degradation of self-regulating heating cables is analyzed. Authors in [19], [20], [35] also focused on the stopping time (failure time) of Wiener process and expanded the existing theoretical results in this domain. Recently, the Brownian motion with non-linear drift has attracted more attention in engineering problems and residual lifetime estimation, see for example [24], [30], [31]. When the degradation path is monotonically increasing, the Gamma process which a is a positive stochastic process with independent increments is a suitable candidate to model the degradation. Hence very often the Gamma process is used to describe the model a deterioration caused by the accumulation of wear [1]. This process is a jump process which can be roughly considered as a succession of the frequent arrival of tiny increments. This rough description makes it relevant to model gradual deterioration such as corrosion, erosion, wear of structural components, concrete creep, crack growth [4]. Moreover, the Gamma process has a probability distribution function which permits feasible mathematical developments. This process has been widely used in deterioration modeling for condition-based maintenance (see [29]).

Considering a list of model candidates including the Wiener and Gamma processes, the paper proposes a new methodology to investigate optimal inspection policies that are less expensive but ensure information quality for model selection. Therefore, it could help companies to improve their actual monitoring policy for reducing operation and maintenance costs.

The remainder of the paper is as follows. Section II describes the problem statement. Section III presents the proposed methodology to optimize inspection policies for model selection. In Section IV, a set of models under consideration is exposed. Section V is dedicated to illustration of the performance of our methodology through numerical examples. Finally, Section VI presents the conclusion and further research work.

II. Problem statement

A regular inspection policy provides a large amount of information for degradation modeling and ensures the prognostic precision of system failure. However, it can be very costly and is sometimes impossible to implement in practice. Therefore, the question is how to find a balance between the inspection policy cost and the required information level for degradation modeling. This paper aims to address this issue. A methodology is proposed to improve the inspection policy, i.e to reduce the inspection cost while ensuring the information quality.

The system state is assumed to be observed through periodic or non-periodic inspections. Each inspection instantaneously reveals the system state. For periodic inspections, the length between two successive inspections Δ_{T_i} is a constant. For non-periodic inspections, Δ_{T_i} is a function of the degradation state observed at the previous inspection.

In practice, due to technological and logistic constraints, the inspection interval is always greater than $\Delta_{\min}$, which is the minimum achievable time between two successive inspections. Therefore the periodic inspection policy with inspection interval equal to $\Delta_{\min}$ is the policy which can collect the maximum information about the system degradation evolution. This latter may also be the most costly. Hereafter, this inspection policy is called the Benchmark policy and is considered as a reference to compare the efficiency of the other policies.

It is necessary to optimize inspection policies to find a balance between the monitoring cost and the minimum information required for relevant degradation modeling. A "relevant" degradation model is obtained when the model selection results are as good as for the Benchmark policy. In detail, data is firstly acquired following different inspection policies, including the periodic or non-periodic inspections. Corresponding to every set of data, a degradation model selection procedure is then performed. The results of the model selection procedure are compared to the results obtained by the Benchmark policy. If the number of inspections of a given policy is small while the selection result based on this policy is consistent with the one based on the Benchmark policy, this policy is considered as better than the Benchmark policy. The optimal inspection policy is the one having a minimal number of observations and ensuring the best consistency with the Benchmark policy.

III. Methodology to improve the inspection policy for degradation model selection

We firstly define an investigation set of different types of inspection policies, see subsection III-A. Afterward, corresponding to each inspection policy, the model selection procedure is performed, see subsection III-B. Finally, in subsection III-C we propose a criteria set to optimize the inspection policies such that the selection model results based on these policies are acceptable compared to the Benchmark policy.

A. Inspection policies for acquiring degradation information

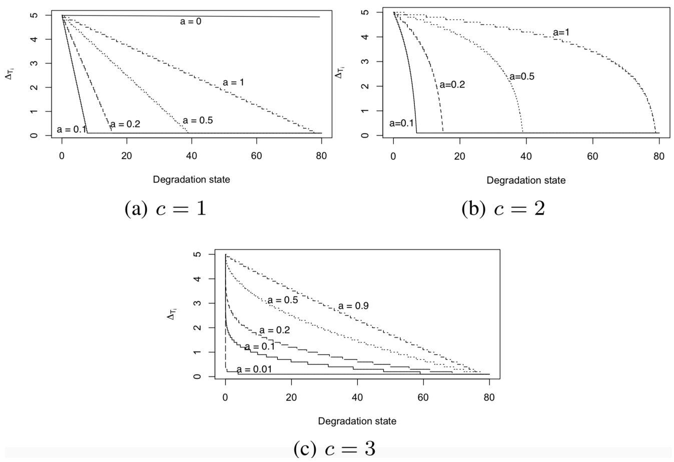
An inspection policy is considered as relevant when the number of inspections is small, but the collected data is informative enough. For instance, for a new system, it is not necessary to often inspect the degradation state. On the opposite it is preferable to observe frequently the degradation state for an old system. Therefore, the function characterizing the length between two successive inspections ($\Delta_{T_i} = T_i - T_{i-1}$) is generally a decreasing function of the observed degradation state at time T_{i-1} denoted by X_{i-1}. Consider a fixed failure threshold L. The duration Δ_{T_i} before the next inspection time T_i is assumed to be one of the functions described hereafter.

1) Case 1: a linear function,

$$\begin{aligned} \Delta_{T_i} &= \max\left(b - b \cdot \frac{X_{i-1}}{a \cdot L}, \Delta_{\min}\right); \text{ if } a > 0 \\ &= b; \qquad\qquad\qquad\qquad\qquad \text{if } a = 0 \end{aligned} \tag{1}$$

2) Case 2: a concave function,

$$\Delta_{T_i} = \max\left(\frac{b}{\ln(aL)} \ln(aL - X_{i-1}), \Delta_{\min}\right) \tag{2}$$



(a) $c = 1$ (b) $c = 2$

(c) $c = 3$

Fig. 1. Illustration of different forms of inspection policy based on the degradation state when $b = 5$

3) Case 3 : a convex function,

$$\Delta_{T_i} = \max\left(b - b\left(\frac{X_{i-1}}{L}\right)^a, \Delta_{\min}\right); \qquad (3)$$

Considering $\Delta_{\min}$ is given, an inspection policy is characterized by 3 parameters (a, b, c) where:

- The parameter c characterizes the case number i.e. the shape of the decreasing function which can be linear ($c = 1$), concave ($c = 2$) or convex ($c = 3$).
- The parameter b represents the first inspection time, $T_1 = \Delta_{T_1} = b$, $\Delta_{\min} \leq b \leq L$.
- The parameter a adjusts the decreasing rate of Δ_{T_i}, $a \in [0, 1]$. When the degradation state exceeds the threshold $a \cdot L$, $X_{i-1} \geq a \cdot L$, the periodic inspection with $\Delta_{T_i} = \Delta_{\min}$ will be applied.

Figure 1 illustrates different inspection functions. When $c = 1$, see Figure 1(a), Δ_{T_i} constantly decreases with system degradation state. When $a = 0$, a periodic inspection policy is considered and the period length is defined by b i.e. $\Delta_{T_i} = b$. When $c = 2$, see Figure 1(b), Δ_{T_i} is slowly decreasing in early stage (when the degradation state is still insignificant) while it rapidly declines later. Contrarily, when $c = 3$, see Figure 1(c), Δ_{T_i} rapidly decreases in early stage while it slowly declines when system is more degraded.

B. Procedure of model selection

The model selection procedure has the following steps:

1) **Data acquisition.** The degradation data are recorded at the inspection times T_i, $i = 1, 2, \cdots n$ until it exceeds a given threshold $\tau \cdot L$, where $\tau \in [0, 1]$. For the Benchmark inspection policy, the length between two inspections are constant an equal to $\Delta_{T_i} = \Delta_{\min}$. For other periodic or non-periodic policies, Δ_{T_i} is a function of the degradation state observed at the previous inspection time. The considered function Δ_{T_i} have been introduced in subsection III-A.
2) **Parameter estimation.** In reliability engineering, parameter estimation refers to the process of using sample data (degradation records at inspection times) to find the model parameters allowing the best data fitting. Among numerous parameter estimation methods, the Maximum Likelihood Estimation (MLE) is one of the most widely used in statistics [18]. It proposes a general approach to determine the values of the model parameters through the maximization of the likelihood function.
Suppose that n is the number of collected degradation records, x_j is the degradation records at the jth observation time ($1 \leq j \leq n$). Let $p(x_j, t_j | x_{(j-1)}, t_{(j-1)}; \theta)$ denote the transition probability density function of the degradation process from the state $x_{(j-1)}$ at time $t_{(j-1)}$ to the state x_j at time t_j knowing that θ is the vector of model parameters. The log-likelihood function for an uncensored sample dataset is given by:

$$L(\theta) = \sum_{j=1}^{n} \log(p(x_j, t_j | x_{(j-1)}, t_{(j-1)}; \theta)) \qquad (4)$$

The optimal parameter vector is given by $\theta^* = \arg\max_\theta L(\theta)$. In this paper, we propose to use Nelder-Mead (NM) method as a numerical method to maximize the likelihood or the log-likelihood function. The NM algorithm was firstly proposed by Nelder-Mead in 1965 to solve an unconstrained optimization problem. As it is only based on the evaluation of the objective function values, but no derivatives, this algorithm could be used for objective functions whose closed form derivatives are difficult to obtain. Although it generally lacks rigorous convergence properties it can reach satisfactory results after the first few iterations in practice [25]. The choice of this generic algorithm is to emphasize that the optimization problem can be solved numerically easily.
3) **Evaluation of selection criteria.** The candidate models are compared according to different criteria focusing on the prognostic measures. They are described hereafter:
   - Prognostic accuracy criterion (PAC) is a measure that evaluates the precision of Residual Useful Lifetime (RUL) estimation corresponding to a given amount of observation data within a fixed observation period. Let L be the failure threshold of the system and t_F the failure time i.e. the first time the degradation level crosses L. Let t_τ be the first time the degradation level exceeds the threshold $L_\tau = \tau \cdot L$ with $\tau \in [0, 1]$. The prognostic accuracy criterion (PAC) is defined in this paper as a prognostic measure for small values of ϵ. It evaluates the probability that the RUL of component calculated at time t_τ and denoted by RUL_{t_τ} remains between the ϵ-bounds: $\epsilon^+ = (1+\epsilon)t_F - t_\tau$ and $\epsilon^- = (1-\epsilon)t_F - t_\tau$. The PAC is defined as follows:

$$PAC = \mathbb{P}[\epsilon^- \leq RUL_{t_\tau} \leq \epsilon^+] \qquad (5)$$

The choice of ϵ, ζ, and τ depend on the operation requirements. The approach to evaluate the predicted RUL distribution is presented later in section

IV. Among candidate models, the one having the maximal PAC will be selected.

- Hybrid Criterion (HyC) is a measure that allows to take into account the goodness-of-fit information of observed data in the evaluation of the RUL estimation accuracy. The HyC is calculated as the sum between the logarithm of PAC indicator based on RUL_{t_τ} and the mean log-likelihood of the observed degradation increments without exceeding the threshold L_τ:

$$HyC = -\log(\mathbb{P}[\epsilon^- \le RUL_{t_\tau} \le \epsilon^+) - \frac{1}{n}\begin{pmatrix} \log p(x_1, t_1|X_0, 0; \theta) \\ + \sum_{i=2}^{n|(x_n \le \tau \cdot L)} \log p(x_i, t_i|x_{i-1}, t_{i-1}; \theta) \end{pmatrix} \tag{6}$$

where n is the number of observation points. For the HyC criterion, the model with the minimal value of HyC will be chosen

4) **Normalization of selection criteria.** According to each selection criterion, the best model will be chosen. For model selection, we are interested in the ranking of candidate models according to the identified selection criterion more than its measured values. Moreover, in order to be able to compare the consistency of ranking results of different selection criteria, normalized values need to be considered. The latter allows us to adjust values measured on different scales to a common scale, for example between 0 and 1.
Let CR_i be the measured values corresponding to the Hybrid Criterion of the ith model in a set of M considered models, $CR_i \in R^+$. The normalized values is given by:

$$\overline{CR}_i = \frac{\min_{i \in M}(CR_i)}{CR_i} \tag{7}$$

For the Prognostic Accuracy criteria, let CR_i be the value of the PAC indicator for the ith model in a set of M candidates. The normalized value is defined as:

$$\overline{CR}_i = \frac{CR_i}{\max_{i \in M}(CR_i)} \tag{8}$$

After normalization, according to all criteria, the value of $\overline{CR}_i$ falls into $[0, 1]$. The model i with $\overline{CR}_i = 1$ is the best one.

C. Optimization criteria of inspection policies for model selection

For model selection, the optimal inspection policy is the one having a small number of observations but still offers sufficient information for the model selection procedure. In detail, considering Benchmark monitoring as a reference policy, an optimal inspection policy satisfies the following conditions:

1) It has the minimal ratio between its inspection numbers and the inspection numbers of the Benchmark policy. Let n_j and n_s be respectively the observation numbers of the jth inspection policy and the Benchmark monitoring policy, the first optimality criterion is defined by:

$$OC1_j = \frac{n_j}{n_s} \tag{9}$$

For an identified degradation process, the number of observations for each inspection policy is always smaller that the one of the Benchmark policy. Therefore, the ratio between them falls into $[0, 1]$. Its value is the proportion of the sample size obtained from the jth inspection policy by comparison with the one of the Benchmark policy.

2) It has the minimal inconsistency score obtained from its resulting model selection when compared with the standard result based on information acquired from the Benchmark policy. The model selection result are characterized by the vector $\overline{CR}$ in space $\mathbb{R}^M$ where M is the number of considered models. The M components of $\overline{CR}$ are evaluated by Eq.(7) or (8). Let $\overline{CR}^s$ and $\overline{CR}^j$ be respectively the standard model-selection-result obtained from Benchmark policy and the model-selection-result based on jth inspection policy. The inconsistency score between them is given by:

$$OC2_j = \frac{1}{M}\|\overline{CR}^s - \overline{CR}^j\|_p \tag{10}$$

where $\|.\|_p$ is a given norm. Note that we have to divide the "distance" between $\overline{CR}^s$ and $\overline{CR}^j$ by M in order to avoid the impact of the size of model candidate set, M.

IV. Presentation of Candidate Models

In the framework of this paper, the set of candidate models is based on Brownian and Gamma family, which has widely been considered for degradation modeling, [26], [29].

A. Wiener process and its extensions

To consider a general degradation modeling framework and take into account the possible existing physical models, it is possible to introduce stochastic differential equations (SDE) based on a standard Brownian motions B_t:

$$dX_t = m(X_t, t)dt + \sigma(X_t, t)dB_t,$$

$(m, \sigma) : \mathbb{R} \times \mathbb{R}^+ \mapsto \mathbb{R}$ are respectively the drift and the diffusion coefficient. These equations appear at the beginning of 20th century in statistical mechanics and have been thoroughly formulated by Itô [15], [28]. Such equations can be considered as derived from existing physical models by adding Gaussian "white noises" on measurements. They permit a wide range of degradation modeling due to the flexibility of the structure and functional parameters.

In this section, some specific Generalized Wiener processes and extensions derived from SDE are presented. For each case, the differential equation, the related distribution functions, and some stochastic properties are exposed.

1) Diffusion processes: A Diffusion process has the following properties. X_t is solution of the SDE $dX_t = \mu(t)dt + \sigma dB_t$, where $\mu(t)$ is a function of t and B_t is a standard Brownian motion. The transition probability to $X_t = x$ knowing that $X_s = y$ is given by:

$$p(x,t|y,s) = \frac{1}{\sqrt{4\pi\sigma(t-s)}} exp\left(-\frac{(x+M(t,s)-y)^2}{4\sigma^2(t-s)}\right), \tag{11}$$

where $M(t,s)$, $\gamma(t,s)$ are given by:

$$M(t,s) = -\int_s^t \mu(u)du, \quad \gamma(t,s) = \int_s^t \frac{\sigma^2(u)}{2}du \tag{12}$$

The mean and variance values of X_t are given by:

$$\mathbb{E}[X_t] \quad = -M(t,0), \text{Var}[X_t] = \sigma^2 t \tag{13}$$

a) M_1: Wiener process with linear drift: This process is the special case of a Diffusion process when the drift and the variance are not time dependent (μ, σ are constant). This Wiener process (or drifted Brownian motion) which is also a Lévy process is suitable for fluctuating degradation records linearly increasing in time. It will be referred as M_1 in Section V.

The RUL cumulative distribution function (cdf) for a drifted Brownian motion given the observation value $X_t = x$ at the observation time t are given as follows [2]:

$$\begin{aligned} F_{RUL_{(x,t)}}(u) \quad &= \quad \Phi\left(\frac{-L+\mu u + x}{\sigma\sqrt{u}}\right) \\ &+ e^{2\frac{\mu}{\sigma^2}(L-x)}\Phi\left(\frac{-L-\mu u + x}{\sigma\sqrt{u}}\right) \end{aligned} \tag{14}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

b) M_2: Wiener process with time-dependent drift: This process is the particular case of a diffusion process when the degradation process is exponentially increasing in time ($\mu(t) = at^b$). It will be referred as M_2 in Section V. In this case, the ratio between its drift and diffusion is not a constant and also depends on the time. Therefore, it is difficult to derive the explicit expression of the RUL distribution. Its evaluation requires solving a non-singular Volterra Integral Equation. It can be done numerically, see e.g. [10].

2) Diffusion process with purely time-dependent drift and diffusion (M_3): This process is the particular case of a diffusion process when $\mu(t)$ and $\sigma(t)$ are time dependent functions independent of X_t). This is suitable for a degradation process including random walks with time-dependent drift and diffusion terms. In this paper, we consider a special case of the purely time dependent drift and diffusion Brownian motion: $\mu(t) = cat^b$ and $\sigma(t) = \sqrt{2at^b}$. It will be denoted as M_3 in Section V. As the power-law drift is proportional with drift, according to [17], the RUL CDF of the process at time t given a degradation level at time t, $X_t = x$, is derived:

$$\begin{aligned} F_{RUL_{(x,t)}}(u) = \Phi\left(\frac{-L-c\gamma(u+t,t)+x}{\sqrt{2\gamma(u+t,t)}}\right) \\ + e^{c(L-x)}\Phi\left(\frac{-L-c\gamma(u+t,t)+x}{\sqrt{2\gamma(u+t,t)}}\right) \end{aligned} \tag{15}$$

where $\gamma(u+t,t)$ is given by Eq.(12).

B. Gamma process

The increments of a gamma process (X_t), are independent and such that for $s < t$ $X_t - X_s \sim Ga(\alpha(t) - \alpha(s), \beta)$ with the transition probability density function from $X_s = y$ to $X_t = x$ as follows:

$$p(x,t|y,s) = \frac{\beta^{\alpha(t)-\alpha(s)}}{\Gamma(\alpha(t)-\alpha(s))}(x-y)^{\alpha(t)-\alpha(s)-1}e^{-(x-y)\beta} \tag{16}$$

where the shape function $\alpha(t)$ is an increasing function defined on $\mathbb{R}^+$. Γ is the Euler's Gamma function. and β the scale parameter. The mean and variance values of X_t are given by:

$$\mathbb{E}[X_t] = \frac{\alpha(t)}{\beta}, \ \text{Var}[X_t] = \frac{\alpha(t)}{\beta^2} \tag{17}$$

The choice of $\alpha(.)$ and β allows to model various deterioration behaviors from almost deterministic to very chaotic. Note that, based on the form of $\alpha(t)$, the Gamma process can be:

- Homogeneous if $\alpha(t)$ is a linear function in t: $\alpha(t) = at$. This process is denoted M_4.
- Non-homogeneous if $\alpha(t)$ is a non-linear function: for instance $\alpha(t) = at^b$, $a > 0$, $b > 1$ for the process referred as M_5 in the following.

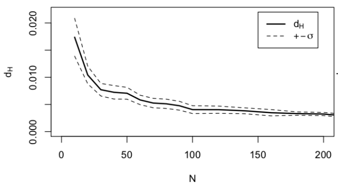
Given a degradation level $X_t = y$ at time t, the cumulative distribution of the remaining useful life of a gamma process is given by [21]:

$$F_{RUL_{(y,t)}}(u) = \frac{\Gamma(\alpha(u+t)-\alpha(t), (L-y_t)/\beta)}{\Gamma(\alpha(u+t)-\alpha(t))};$$
$$\Gamma(\cdot,\cdot) \text{ is the upper incomplete Gamma function.} \tag{18}$$

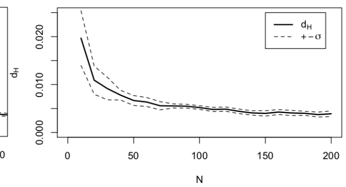
V. Numerical experiments

For simulation of a model selection procedure presented in subsection III-B, we generate a stochastic degradation path by one of the five models presented in Section IV ($M1$: drifted Brownian motion, $M2$: time-dependent-drifted Brownian motion, $M3$: time-and-diffusion-dependent drifted Brownian motion, $M4$: homogeneous gamma process, $M5$: non-homogeneous gamma process). Its parameters are chosen in order to have the mean degradation level 100 at time $t = 100 : x = 100$ with different values of coefficient of variation $vc \in [10\%, 30\%, 50\%]$ at time $t = 100$. The evolution of a degradation process from 0 to $T_m = 250$ is generated with the time step $\Delta t = 0.1$. Considering the failure threshold $L = 80$, the degradation data is observed until threshold $L_\tau = \tau \cdot L$ where $\tau = 0.7$. Based on data inspected by the Benchmark policy, we estimate parameters for every candidate models and then evaluate their corresponding measured values of the selection criterion (CR^s) and its normalized values, $\overline{CR}^s$. These values are considered as the reference values and used to assess the efficiency of other inspection policies characterized by different decision variables (a, b, c). Note that, for PAC criterion, we assess the probability that the estimated failure time belongs to the interval $[\epsilon^-, \epsilon^+]$ where $\epsilon = 0.01$.

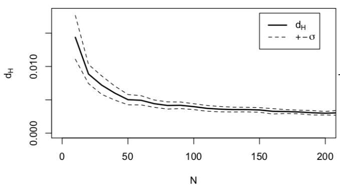
To eliminate the biased results induced by the random feature of degradation processes, we repeat the above model



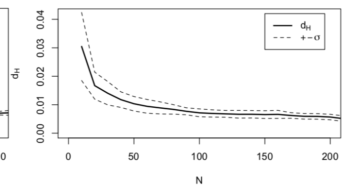
(a) M_1 and $vc = 30\%$



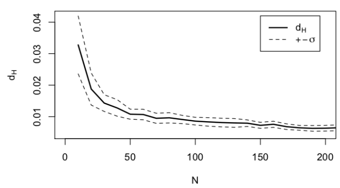
(b) M_2 and $vc = 30\%$



(c) M_3 and $vc = 30\%$



(d) M_4 and $vc = 30\%$



(e) M_5 and $vc = 30\%$

Fig. 2. Illustration of convergence of the optimal results with respect to number of simulations N

selection procedure N times. For each of the N simulations, the optimization criteria of the inspection policy j, $OC1_j$, $OC2_j$, are evaluated. Afterward the mean values of the results of N simulations are deduced. Based on these mean values, we obtain the Pareto optimal front for inspections policies according to different data types and values of the variance coefficient.

A. Discussion on the convergence for optimal results with respect to the number of simulation processes

In order to ensure the robustness of our results, we study the convergence for optimal results (objective function) with respect to the number of simulated processes N. However, since our problem is a bi-objective optimization problem, the solution is not given by a single point but rather by a set of efficient solutions, called the Pareto set. Therefore, the issue is to measure the difference between the Pareto fronts or to assess its convergence when the simulation number, N, is increasing. The following approach is proposed to address this problem.

- We firstly consider a large number $N_{\max}$ as the maximal simulation numbers due to the limits of simulation time. Then, we use its corresponding Pareto set as a reference set, noted $P_{N_{\max}} = \{y_1, ..., y_{n_{N_{\max}}}\}$ where $n_{N_{\max}}$ is the number of points on reference Pareto front $P_{N_{\max}}$.
- According to every different value of $N < N_{\max}$, we obtain a candidate Pareto set, noted $A_N = \{a_1, ..., a_{n_{A_N}}\}$ where n_{A_N} is the number of points on the Pareto front A_N.
- The distance between reference Pareto set $P_{N_{\max}}$ and candidate Pareto set A_N is evaluated and used to measure the convergency of the optimization results.

In order to measure the distance between two sets having different size, we propose to use the Hausdorff distance d_H which is well-known and commonly used in the field of evolutionary multi-objective optimization (EMO). It defines a metric in the mathematical sense on the set of compact subsets of the $\mathbb{R}^n$, therefore returns one value characterized information about the relation between two sets having different sizes, refer to [23].

In this purpose, first let use define $GD(A_N)$ and $IGD(A_N)$ respectively the Generational Distance and the Inverted Generational Distance from the candidate set A_N to the reference set $P_{N_{\max}}$ as follows:

$$GD(P_{N_{\max}}) = \frac{1}{n_{A_N}} \left(\sum_{i=1}^{n_{A_N}} d_i^p \right)^{1/p}$$

$$IGD(A_N) = \frac{1}{n_{N_{\max}}} \left(\sum_{j=1}^{n_{N_{\max}}} \tilde{d}_j^p \right)^{1/p} \qquad (19)$$

where d_i^p denotes the minimal p-norm distance from a_i to $P_{N_{\max}}$, i.e. $d_i^p = \min_{y_j \in P_{N_{\max}}} ||a_i - y_j||_{p-\text{norm}}$ and $\tilde{d}_j^p$ denotes the minimal p-norm distance from y_j to A_N, i.e. $\tilde{d}_j^p = \min_{a_i \in A_N} ||a_i - y_j||_{p-\text{norm}}$.
The Hausdorff distance is evaluated as follows:

$$d_H(A_N, P_{N_{\max}}) = \max(GD(P_{N_{\max}}), IGD(A_N)) \qquad (20)$$

After evaluating the Hausdorff distance (d_H) from a Pareto front A_N corresponding to N numbers of simulations to the reference Pareto front $P_{N_{\max}}$, we find that this Hausdorff distance could be considered as a convex decreasing function in N. In fact, it is slightly constant when $N > 80$, see Figure 2 for illustration. More over its standard deviation (σ) is also decreasing when simulation number N increases. For degradation data generated by different models M_1, M_2, ..., M_5 with coefficient of variation $vc = 30\%$, Figure 2 illustrate the variation of d_H with respect to N. Based on Figure 2 and other simulations carried out and not exposed in the paper, we consider that d_H will converge when N is sufficiently large. Therefore, for numerical experiments, based on obtained results, N is set to 100 in order to ensure the robustness of our results.

B. Pareto front of optimization of the condition based inspection policies for model selection

Figure 3 presents Pareto fronts of inspection policy optimization. In every sub-figure, the horizontal axis represents the inconsistency score (Eq.(10)) between selection result based on the Benchmark inspection policy and the one based on other inspection policies. The vertical axis represents the proportion between inspection numbers (Eq.(9)) of such inspection policy when compared with the observation points of the Benchmark policy. The coordinate of every black point in sub-figures characterizes the bi-dimensional objective function's value for every considered inspection policy. Each point is corresponding to an inspection policy characterized by the set of parameters (a, b, c). Note that all types of linear, concave, convex inspection policies are investigated at the same time.

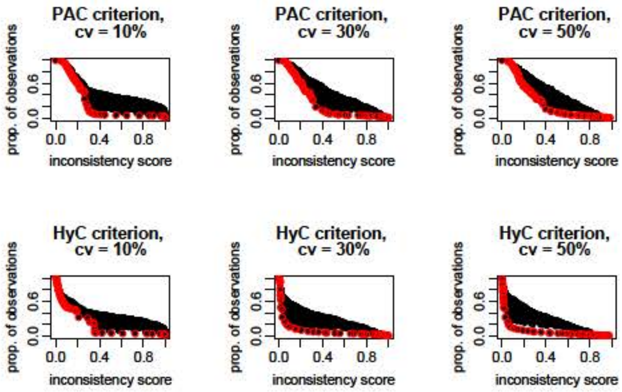


Fig. 3. Pareto front of optimization of degradation state based inspection policies for model selection based on data generated by $M1$

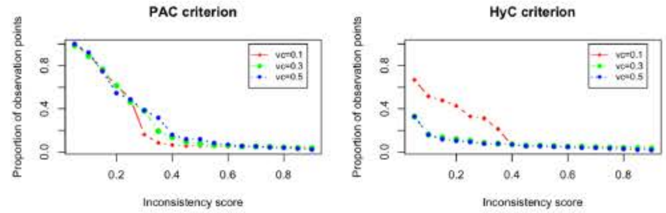
The Pareto front, characterized by red circles, represents the problem trade-offs of an objective function, that can not be improved in one dimension without being worsened in another. We find that the Pareto fronts of the optimal inspection policies have convex form, see Figure 3 for example. When the inconsistency score increases from 0 to 1, the proportion of observation numbers also decreases from 1 to 0. In other words, reducing the inconsistency score will require more inspection numbers.

For more details, we consider Figures 4 and 5 which illustrate the optimal results for data generated by different models M_i, $i \in \{1, \cdots, \}$ and different variance coefficients considering the two decision criterions.

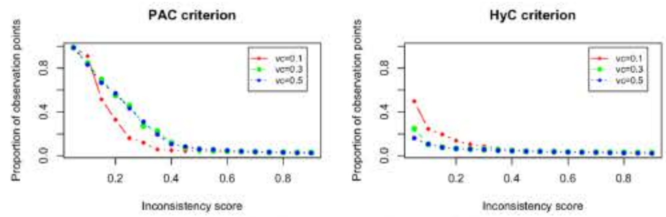
For data generated by diffusion type degradation processes, we find that:

- According to PAC criterion, it is difficult to ensure an acceptable inconsistency score for the model selection results based on RUL estimation accuracy when reducing the number of observations. Considering Figure 4(a) for example, it requires at least 60% of observation points of the Benchmark policy to guarantee that the inconsistency score of the result when comparing with the Benchmark policy one is less than 0.2.
- In the HyC criterion, taking into account the goodness-of-fit information of observation data in the evaluation of the RUL estimation accuracy allow us to guarantees the robustness of model selection results when reducing the number of observation, especially for a high coefficient variance. Indeed, considering Figure 4(b) for example when $vc = 0.5$ it requires less than 10% of observation points of the Benchmark policy to ensure that the inconsistency score of the result when comparing with the Benchmark policy one is less than 0.2.

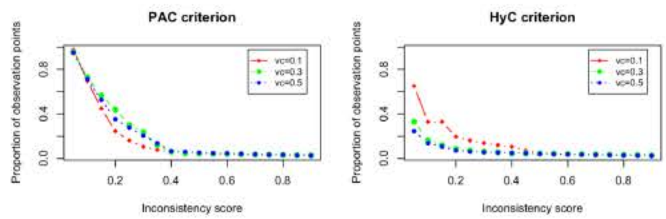
However, for data generated by Gamma degradation processes (see Figure 5), we find that the HyC do not allow us to guarantee the robustness of model selection results when reducing the number of observation, especially for a high coefficient variance. Indeed, it requires more observation numbers than the PAC criterion when vc is great. In detail, for data generated by Gamma homogeneous process ($M4$)



(a) Data generated by $M1$: Brownian process

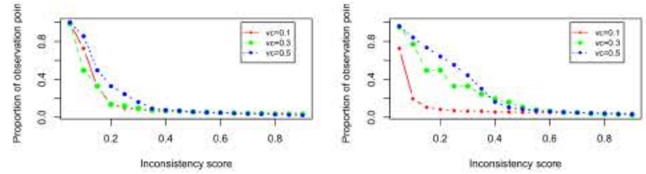


(b) Data generated by $M2$: Time-dependent-drifted-Brownian process

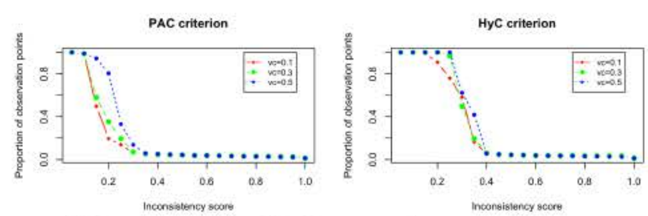


(c) Data generated by $M3$: Time-dependent-drifted-and-diffusion-Brownian process

Fig. 4. Requirement for proportion of observation points according to maximal inconsistency threshold of model selection result for diffusion type degradation data



(a) Data generated by $M5$: Gamma homogeneous process



(b) Data generated by $M6$: Gamma non-homogeneous process

Fig. 5. Requirement for proportion of observation points according to maximal inconsistency threshold of model selection result for Gamma process degradation data

with high variance coefficient ($vc = 0.5$), it requires less than 40% of observation points according to the PAC criterion for the threshold of 0.2 for the inconsistency score. However, according to the HyC criterion, more than 60% of inspection numbers is necessary to satisfy this threshold.

C. Discussion on the optimal inspection policies

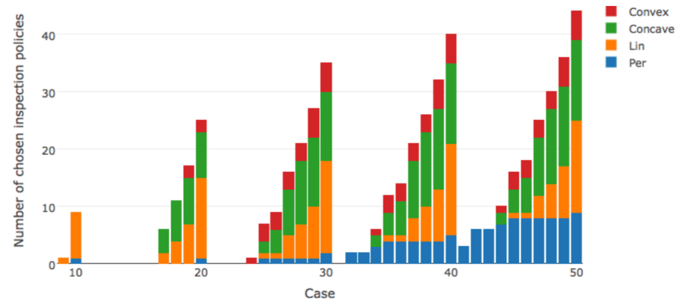
In the previous section, for each configuration, the optimal inspection policy with regards to number of inspections and closeness to the best choice is proposed. Nevertheless, the information on the type of this policy is not discussed. Indeed, it is not discussed that in each configuration, the proposed optimal policy corresponds to which percentage of optimal policies obtained in the overall N simulations. It is possible that the number of times that this policy is considered to be the best is very close to the number of times a second candidate is optimal. Moreover, it is possible that the second candidate can be implemented much easier than the first candidate without enormous loss of optimality. To explore these matters, in this section, the percentage of inspection policy types are given and analyzed.

In this section, we consider under every selection criteria, which inspection policies will be preferred for a given data set. In this purpose, numerous numerical examples are carried out: model selection is performed for data generated by 5 models M_i, $i \in \{1, 2, ..., 5\}$ with different values of $vc = [0.1, 03, 0.5]$. For this numerical example, we consider 50 couples of the inconsistency score threshold (IST) and the proportion of observation points threshold (POT) where $IST \in \{0.1, 0.2, ..., 0.5\}$ and $POT \in \{0.1, 0.2, ..., 1\}$. Thus, we obtain 50 sets of inspection policies. Each policy in these sets ensures that the corresponding inconsistency score and proportion of observation points are inferior to the coupling threshold of IST and POT.

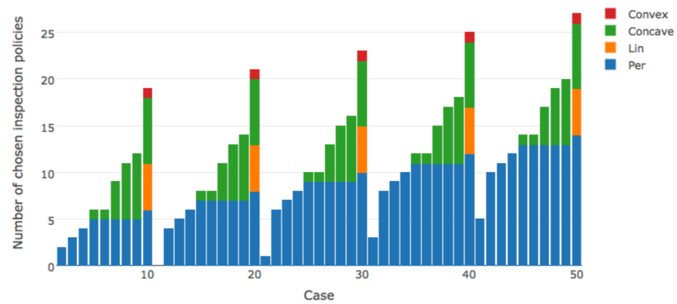
To illustrate these conclusions, we illustrate in Figure 6 the number of chosen inspection policies and the proportion of their type (Periodic, Linear, Concave, Convex) corresponding to 50 cases of couples (IST, POT) when data is generated by $M1$ with $vc = 0.3$. For the first 10 cases (Case 1 to Case 10), POT is respectively increasing from 0.1 to 1 with $IST = 0.1$. Next, for Case 11 to Case 20, POT is respectively increasing from 0.1 to 1 with $IST = 0.2$ and similar for next cases. We find that according to PAC, among chosen inspection policies, the linear and concave form are the two best types (see sub-figure 6.a) while according to HyC, the periodic inspections and concave form are preferred (see sub-figure 6.b). For other cases, the characteristics of the chosen policies are then summarized in Table I.

The results in Table I are summed up as follows.

- In general, according to each model selection criteria, each data set admits its optimal inspection policies. For example, according to PAC, the optimal non-periodic linear and concave inspection policies are approved for data generated by a Brownian process while the optimal periodic and non-periodic concave polices are preferred for the Gamma non-homogeneous model. On the other hand, according to HyC, the optimal concave and linear policies are favored for the Gamma model family while the periodic inspections are chosen for Wiener model family. Therefore, when we do not have the prior information about data type, periodic inspections are recommended.
- When IST and POT are small, the criteria lead to totally different results:
  - according to the PAC criterion, non-periodic inspections are generally preferred for degradation modeling of data generated by a Wiener process (see sub-figure 6.(a) for an example) while periodic inspections are favored for Gamma processes in most cases (see sub-figure 7.(a).
  - according to the HyC criterion, when IST is

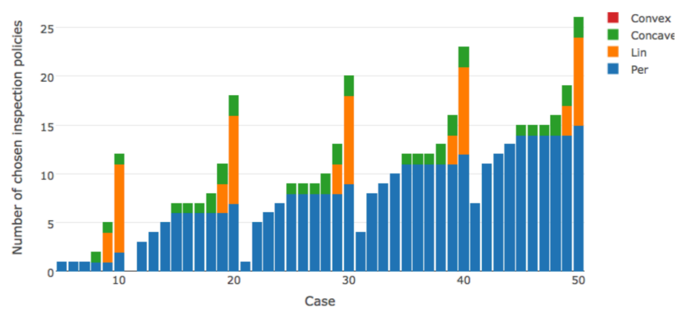


(a) PAC criteria

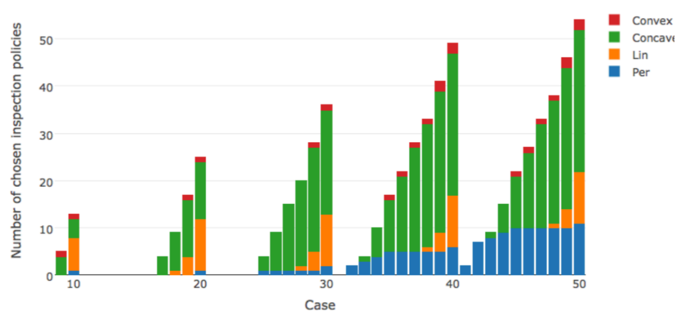


(b) HyC criteria

Fig. 6. Number of chosen inspection policies and their type proportions when data are generated by $M1$-Brownian process with $vc = 0.3$: Periodic (Per), Linear (Lin), Concave, Convex.



(a) PAC criteria



(b) HyC criteria

Fig. 7. Number of chosen inspection policies and their type proportions when data are generated by $M4$-Gamma process with $vc = 0.3$: Periodic (Per), Linear (Lin), Concave, Convex.

TABLE I
SUMMARY OF FEATURES OF THE OPTIMAL INSPECTION POLICIES FOR MODEL SELECTION

Criterion PAC :

	M1	M2	M3	M4	M5
Often optimal	Concave, Linear	Non-periodic	Convex, Linear	Periodic, Linear	Concave, Periodic
small (IST, POT)	Non-periodic	Non-periodic	Non-periodic	Periodic	Periodic
high variance vc	$T_1 \downarrow$ Periodic $\downarrow$	$T_1 \uparrow\downarrow$ Periodic $\downarrow$	$T_1 \uparrow\downarrow$ Periodic $\downarrow$	$T_1 \downarrow$ Periodic $\uparrow$	$T_1 \downarrow\uparrow$ Periodic $\downarrow\uparrow$

Criterion HyC :

Often optimal	Concave, Periodic	Concave, Periodic	Concave, Periodic	Concave, Linear	Concave, Linear
small (IST, POT)	Periodic	Periodic	Periodic	Non-periodic	Non-periodic
high variance vc	$T_1 \downarrow$ Periodic $\uparrow$	$T_1 \uparrow\downarrow$ Periodic $\uparrow$	$T_1 \uparrow\downarrow$ Periodic $\uparrow$	$T_1 \downarrow$ Periodic $\downarrow\uparrow$	$T_1 \uparrow$ Periodic $\downarrow\uparrow$

small, periodic inspections are generally preferred for degradation modeling of data generated by a Wiener process (see sub-figure 6.b for an example) while non-periodic inspections are favored for data generated by Gamma processes (see sub-figure 7.(b) for an example).

- When the variance coefficient vc is increasing,
  - the first inspection time T_1 arrives very early for data generated by a homogeneous Gamma process or a Wiener process, while no distinguished features can be derived for T_1 for data generated by other processes.
  - for Wiener or diffusion processes, periodic inspections are more often used according to HyC while non-periodic ones are more used according to PAC.
  - according to both PAC, periodic inspections are more preferred for Gamma homogeneous process while for the non-homogeneous Gamma process the periodic inspections not always favored.

D. Discussion of quality-index-based-filter

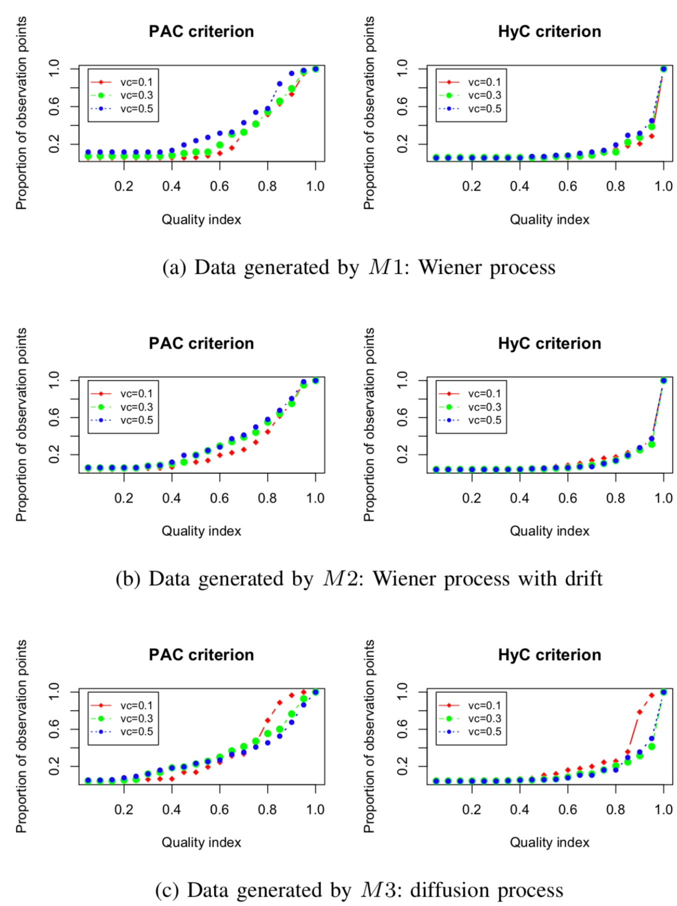
In the previous section, we have discussed the features of the chosen optimal inspection policies according to a couple of the inconsistency score and the proportion of observation point threshold (IST, POT). However, there could exist cases where the prognostic metrics (before normalization) are not good enough but the inconsistency scores of the selection results are acceptable. To avoid these unusual cases and to ensure their elimination in decision making, we propose in this section a quality-index-based-filter. This quality index is also the ratio between the prognostic accuracy metrics given by an inspection policy when comparing with the Benchmark policy.

Recall that given observation data acquired by the Benchmark policy, the CR_i^s is the selection criteria value before normalization for the ith candidate model. When considering Hybrid criterion (HyC), the kth model having the minimal value CR_i^s is the best candidate: $k = \arg\min_{(i \in M)}(CR_i^s)$. When considering Prognostic Accuracy criterion (PAC), the kth model having the maximal value of CR_i^s is chosen: $k = \arg\max_{(i \in M)}(CR_i^s)$. For model selection, the value of CR_k^s could be considered as a standard value. When we use an inspection policy (for example j-th policy), we hope that its value CR_k^j is close to the standard value, CR_k^s. Based on this concept, the quality index corresponding to jth inspection policy, QI_j is defined by :

$$
\begin{aligned}
QI_j &= \frac{CR_k^s}{CR_k^j}; \quad \text{if using } HyC \text{ criterion} \\
&= \frac{CR_k^j}{CR_k^s}; \quad \text{if using } PAC \text{ criterion} \qquad (21)
\end{aligned}
$$

$QI_j \in [0, 1]$, where 1 is the best value.

For a quality-index-based-filter, only inspection policies whose quality index is superior or equal than a threshold QIT are kept. Considering the inconsistency score threshold $IST = 0.5$, the minimal proportions of observation numbers according to different values of quality index threshold, $QIT \in [0.05, 1]$, are represented in Figure 8 and 9. We find that when QIT is increasing, the minimal required observation number is also increasing.

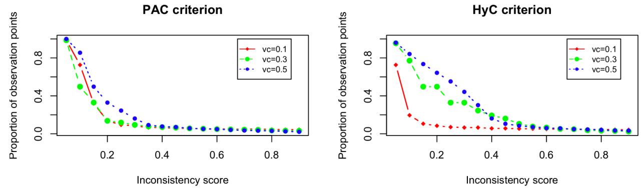


(a) Data generated by $M1$: Wiener process

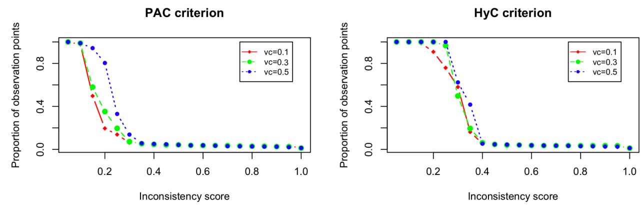
(b) Data generated by $M2$: Wiener process with drift

(c) Data generated by $M3$: diffusion process

Fig. 8. Requirement for proportion of observation points according to quality index of model selection result for Wiener degradation data

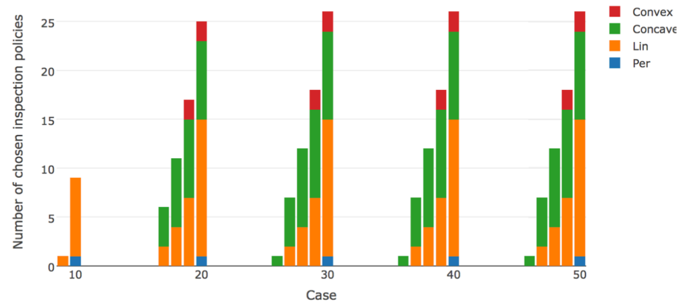


(a) Data generated by $M5$: Gamma homogeneous process

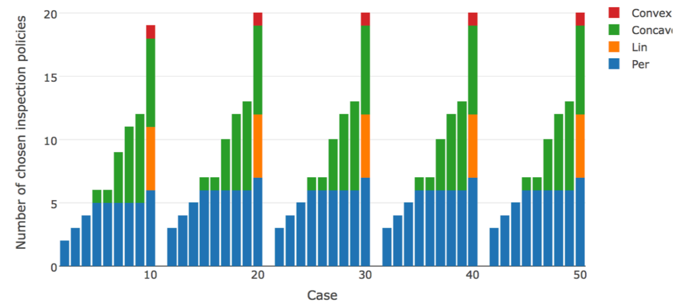


(b) Data generated by $M6$: Gamma non-homogeneous process

Fig. 9. Requirement for proportion of observation points according to quality index of model selection result for Gamma degradation data

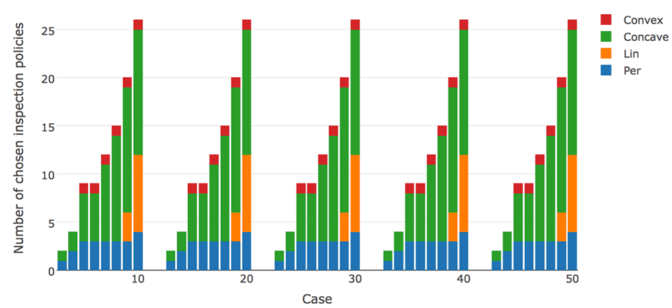


(a) PAC criteria

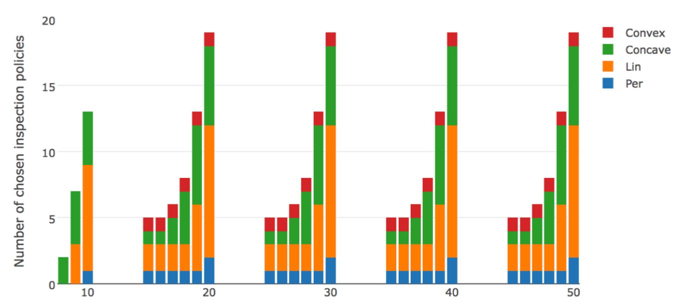


(b) HyC criteria

Fig. 10. According to $QIT = 0.8$, number of chosen inspection policies and their type proportions when data are generated by $M1$-Brownian process with $vc = 0.3$: Periodic (Per), Linear (Lin), Concave, Convex.



(a) PAC criteria



(b) HyC criteria

Fig. 11. According to $QIT = 0.8$, number of chosen inspection policies and their type proportions when data are generated by $M4$-Gamma process with $vc = 0.3$: Periodic (Per), Linear (Lin), Concave, Convex.

- For data generated by Wiener or diffusion processes,
  - taking into account the goodness-of-fit information of observation data (HyC) in the evaluation of the RUL estimation accuracy allows us to guarantee a high-quality index of prognostic metric even if the proportion of observation numbers is less than 0.2. Indeed, considering Figure 8(a) for example, when $QIT = 0.8$, according to PAC the minimal proportion of observation points when comparing with the Benchmark policy is more than 0.6 while it is less than 0.2 according to HyC.
  - for both criterions PAC and HyC, the effect of high variance coefficient is not significant.
- For data generated by Gamma processes, according to both PAC and HyC,
  - taking into account the goodness-of-fit information of observation data in the evaluation of the RUL estimation accuracy do not allow us to guarantee a high-quality index of prognostic metric when reducing the inspection numbers.
  - high variance coefficient strongly effects on the requirements of observation points to satisfy a quality index threshold. For example, see Figure 8(b), when $QIT = 0.8$, the proportion of observation points when comparing with the Benchmark policy is less than 0.4 at $vc = 0.1$ while it is equal to 1 at $vc = 0.5$.

Next, we discuss the type of the chosen optimal inspection policies after using the quality-index-based filter. For numerical examples, QIT is chosen as : $QIT = 0.8$. Similar to Section V-C, we consider 50 couples of (IST, POT). The numerical results are given in Figures 10 and 9. The results can be summed up as follows.

- For Wiener or diffusion processes,
  - according to PAC, in order to satisfy at the same time the requirements of the quality index and the inconsistency score, it requires at least 50% of the observation numbers of the Benchmark policy. For example, consider Figure 10.a, no inspection policy can satisfy the quality index requirement $QIT = 0.8$

TABLE II
CHOSEN OPTIMAL INSPECTION POLICIES FOR DATA GENERATED BY $M1$ WITH $vc = 0.3$

a	b	c	Inconsistency	Pro. of observation	Quality index
0	2	1	0.1	0.5	0.9
0.2	41	1	0.12	0.49	0.95
0.5	25	2	0.13	0.48	0.96
0.59	14	2	0.16	0.47	0.96
1	3	2	0.17	0.46	0.93
0.64	22	1	0.18	0.45	0.96
0.59	15	2	0.19	0.44	0.97
0.59	16	2	0.192	0.43	0.97

and help the manager to reduce more than 50% of the observation numbers.

  - according to HyC, when the requirement of proportion of observation points is small, $POT \leq 0.5$, the periodic inspection policies is often chosen. When POT is high, for example in Figure 10.b when $POT \geq 0.5$, the non-periodic inspection with concave form is the best policy.

- For Gamma process, when the variance coefficient is high, there is no inspection policy which satisties in the same time the requirements of the quality index, the inconsistency score and the proportion of observation points. Whereas, when the variance coefficient is small,
  - according to PAC, when the requirement of observation number is low, periodic inspection policies and non-periodic inspections with concave form are favored, see Figure 9.(a) for example.
  - according to HyC, non-periodic inspection policies (linear and concave function) is widely used, see Figure 9.(b).

In order to have a better understanding of the chosen optimal policies, we consider the case study $QIT = 0.8$, $IST = 0.2$, $POT = 0.5$. According to PAC criterion, there exists no inspection policy that satisfies at the same time ($QIT = 0.8$, $IST = 0.2$, $POT = 0.5$). Table II represents the chosen optimal inspection policies for model selection according hybrid criterion. The values of decision variables a, b and c are respectively characterized Among the chosen optimal inspection policies, there exist:

- one periodic inspection policy ($a = 0$, $c = 1$) with the constant period length $\Delta_{T_i} = 2$,
- two non-periodic policies with linear inspection function. For example, considering the policy characterized by $a = 0.2$, $b = 41$, $c = 1$, the first inspection time is $T_1 = 41$. At the inspection time, if the observed state is higher than $0.2 \cdot 41 = 8.2$, the duration until the next inspection is $\Delta_{\min}$. If not, it is evaluated by a linear function defined by Eq. (1).
- five non-periodic policies with concave inspection function. For example, considering the policy characterized by $a = 0.5$, $b = 25$, $c = 2$, the first inspection time is $T_1 = 25$. At the inspection time, if the observed state is higher than $0.5 \cdot 25 = 12.5$, the duration until the next inspection is $\Delta_{\min}$. If not, it is evaluated a concave function defined by Eq. (2).

VI. CONCLUSION

In this paper, we have presented a new methodology that allows decision-makers to improve an inspection policies for making it less expensive while ensuring the quality of information for model selection which can e.g. be used for failure prognostic. The selection procedure is based on the prognostic accuracy criterion (PAC) and the hybrid criterion (HyC) that takes into account the goodness-of-fit information of the observation data when evaluating the prognostic measure. Besides, we have investigated different types of inspection policies including periodic or non-periodic ones by considering multiples functions that are linear, concave or convex in the degradation state of the system. Among different competing policies, the optimal ones are chosen based on the multi-criteria optimization process that takes into account minimal requirements for the observation point ratio, and the inconsistency score with the quality index filter. The advantages and disadvantages of this methodology are discussed through numerous numerical examples for different types of degradation process, particularly diffusion and Gamma processes.

The paper also provides instructions to improve the selection of appropriate inspection policies according to the degradation process family as well as the evaluation criteria. In general, in case of total absence of prior information about the degradation process family (Wiener or Gamma for example), the periodic inspections could be encouraged to acquire data. We discussed, in addition, the impact of the data variance coefficient on the inspection policy optimization results. In details, high variance ratio significantly effects on the minimum requirements for observation number when considering Gamma family while its impacts are not important when examining Wiener models.

In the scope of this paper, we utilized the grid search algorithm to solve the multi-objective optimization problem. It can lead to long implementation times when investigating large search spaces for real work problems. In further works, an efficient method will be developed to reduce the optimization time. Moreover, the set of models can also be extended, and some other models such as Inverse Gaussian, Variance-Gamma or jump diffusion models can be included to capture the various characteristic of the system degradation process in practice.

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Khanh Nguyen received the Ph.D. degree in Automation and Production Engineering from Ecole Centrale de Nantes, France in 2012. From 2013 to 2015, she was a Postdoctoral Fellow at the French Institute of Science and Technology for Transport, Development and Networks (IFSTTAR). From 2016 to 2017, she was an Assistant Professor at the University of Technology of Troyes.

In September 2017, she joined the National Polytechnic Institute of Toulouse (Toulouse INP) as an Associate Professor. Her research interests include system reliability, maintenance optimization, safety assessment of railway safety related systems, and prognostic health management.

Mitra Fouladirad Professor at the Troyes University of Technology (UTT) is a member of the Systems Modelling and Dependability Laboratory - Operational Research, Applied Statistics and Simulation Centre. Her research interests focus on the construction of probabilistic decision indicators used for prediction and maintenance optimisation. In the framework of maintenance modelling she addressed the problem of condition-based maintenance strategies with dynamic on-line monitoring integration. She has authored more than 25 papers in international journals with peer-reviewed process and more than 70 international conferences.

Antoine Grall is a Professor of Mathematics, Reliability and Maintenance Engineering at Charles Delaunay Institute - UMR 6281, CNRS (Systems Modeling and Dependability Group), Troyes University of Technology. His current research interests include stochastic degradation modeling, lifetime prognosis, reliability of deteriorating control systems and mainly maintenance modeling and optimisation. He is especially interested in condition-based and predictive dynamic maintenance policies with integration of on-line monitoring information, He is member of the Editorial Board of Reliability Engineering & System Safety and of the Journal of Risk and Reliability.