Robust Sliding Mode Control for Discrete Stochastic Systems with Mixed Time-delays, Randomly Occurring Uncertainties and Nonlinearities

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***Abstract*—This paper investigates the robust sliding mode control (SMC) problem for a class of uncertain nonlinear stochastic systems with mixed time-delays. Both the sector-like nonlinearities and the norm-bounded uncertainties enter into the system in randomly ways, and such randomly occurring uncertainties (ROUs) and randomly occurring nonlinearities (RONs) obey certain mutually uncorrelated Bernoulli distributed white noise sequences. The mixed time-delays consist of both the discrete and the distributed delays. The time-varying delays are allowed in state. By employing the idea of delay-fractioning and constructing a new Lyapunov-Krasovskii functional, sufficient conditions are established to ensure the stability of the system dynamics in the specified sliding surface by solving certain semidefinite programming problem. A full-state feedback SMC law is designed to guarantee the reaching condition. A simulation example is given to demonstrate the effectiveness of the proposed SMC scheme.**

***Index Terms*—Sliding mode control, randomly occurring uncertainties (ROUs), randomly occurring nonlinearities (RONs), discrete time-delays, infinite distributed delays.**

I. Introduction

In the past two decades, SMC has become one of the most active branches of control theory that has found successful applications in a variety of engineering systems such as robot manipulators, aircrafts, electrical motors and automotive engines [4], [6], [14], [16]. Also, considerable research attention has been devoted to the theoretical research on SMC problems for different systems. For example, the concept of SMC has been widely employed for uncertain systems [2], [8], [23], stochastic systems [7], [15], [19] as well as fuzzy systems [12], [22]. It should be pointed out that most results mentioned above have been concerned with continuous-time systems. Recently, many important results have been reported on the SMC problem for *discrete-time* systems, see [1], [5], [10], [20], [21]. In [5], a discrete-time sliding mode reaching condition and the concept of quasi-sliding mode have been thoroughly investigated, which have later been applied in [20], [21] to address the SMC problems for a class of uncertain time-delay systems.

It is well known that time-delays are frequently encountered in many engineering systems. The existence of time delays may cause undesirable dynamic behaviors such as oscillation and instability. According to the occurrence way of time-delays, they can be generally classified into two types: discrete delays and distributed delays. Over the past decades, much effort has been made to address the SMC problem for time-delay systems, see e.g. [11], [20], [21]. It is worth mentioning that most of results are applicable to continuous systems only, and the relevant results for discrete systems with mixed delays (i.e., both discrete and distributed) have been very few. Note that the distributed delays in the discrete-time setting is an emerging concept that has been proposed in [18] for complex networks. Such a situation gives us the initial motivation for establishing a unified framework in order to handle the mixed time-delays for discrete-time systems by using the SMC scheme.

Nonlinearities and uncertainties serve as two important kinds of complexities for system modeling. In the networked world nowadays, the nonlinear disturbances and the parameter uncertainties may be subject to random changes in environmental circumstances, for instance, network-induced random failures and repairs of components, sudden environmental disturbances etc. Therefore, both the nonlinearities and the uncertainties may occur in a probabilistic way with certain types and intensity, which is particularly true in a networked environment. Very recently, in [18], the concept of RONs has been introduced to model the randomly occurring nonlinear functions for complex networks, but ROUs have not yet received adequate research attention. It is, therefore, our aim in this paper to shorten such a gap by applying the delay-fractioning approach (see e.g. [13]), for handling the SMC problem with ROUs, RONs and mixed time-delays.

Motivated by the above discussion, we deal with the robust SMC problem for a class of discrete stochastic systems with

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ROUs, RONs and mixed time-delays. By using the delay-fractioning approach, a sufficient condition is presented to ensure the stability of the sliding mode dynamics by means of the feasibility of a certain semidefinite programming problem with an equality constraint. A computational algorithm is used to convert the original nonconvex problem into a minimization problem, and an SMC law is synthesized. The main contributions of this paper can be highlighted as follows: *(i) the concepts of ROUs, RONs and mixed time-delays are, for the first time, introduced together for the SMC problem in order to reflect a more realistic environment; (ii) the delay-fractioning approach as well as a new Lyapunov-Krasovskii functional is applied, for the first time, to design the SMC law with hope to reduce the possible conservatism caused by the time-delays; and (iii) intensive stochastic analysis is carried out to account for the random nature of the appearance of the uncertainties and nonlinearities*.

II. Problem Formulation and Preliminaries

Consider the following class of uncertain nonlinear stochastic systems:

$$\begin{cases} x_{k+1} = (A+\alpha_k \Delta A)x_k + (A_d + \beta_k \Delta A_d)x_{k-d_k} \\ \qquad\qquad + B(u_k + f(x_k)) + C\displaystyle\sum_{p=1}^{+\infty} \mu_p x_{k-p} \\ \qquad\qquad + \displaystyle\sum_{i=1}^{d} \gamma_k^i D g_i(x_k) + E\sigma(x_k, x_{k-d_k})\omega_k \\ x_k = \varphi_k, \quad \forall\, k \in \mathbb{Z}^- \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^q$ is the control input, $f(x_k)$ denotes the unknown nonlinear function that is bounded in terms of Euclidean norm, i.e., there exists a known scalar function $\rho_f(x_k)$ such that $\|f(x_k)\| \le \rho_f(x_k)$, and ω_k is a one-dimensional, zero-mean Gaussian white noise sequence on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with $\mathbb{E}\{\omega_k^2\} = 1$. A, B, A_d, C, D and E are known matrices, and φ_k is a given initial condition.

The nonlinear function $\sigma(\cdot,\cdot)$ satisfies

$$\sigma^T(x,y)\sigma(x,y) \le \rho_1 x^T x + \rho_2 y^T y, \quad \forall\, x, y \in \mathbb{R}^n \tag{2}$$

where $\rho_1 > 0$ and $\rho_2 > 0$ are known scalars.

The real-valued matrices ΔA and ΔA_d represent the norm-bounded parameter uncertainties of the following structure

$$[\Delta A \;\; \Delta A_d] = [H_a F \;\; H_d F]N, \tag{3}$$

where H_a, H_d and N are known constant matrices, and F is an unknown matrix function satisfying $F^T F \le I$.

The stochastic variables $\alpha_k \in \mathbb{R}$ and $\beta_k \in \mathbb{R}$ are Bernoulli distributed white sequences taking values on either 0 or 1 with

$$\begin{aligned} \text{Prob}\{\alpha_k = 1\} = \alpha, \quad \text{Prob}\{\alpha_k = 0\} = 1-\alpha, \\ \text{Prob}\{\beta_k = 1\} = \beta, \quad \text{Prob}\{\beta_k = 0\} = 1-\beta, \end{aligned} \tag{4}$$

where $\alpha \in [0,1]$ and $\beta \in [0,1]$ are known constants.

For each $1 \le i \le d$, the nonlinear function $g_i : \mathbb{R}^n \to \mathbb{R}^n$ stands for the mismatched external nonlinearity that satisfies the following sector-bounded condition:

$$[g_i(x) - F_{1i}x]^T [g_i(x) - F_{2i}x] \le 0, \quad \forall x \in \mathbb{R}^n \tag{5}$$

where F_{1i} and F_{2i} are known real constant matrices, and $F_i = F_{1i} - F_{2i}$ is symmetric positive definite matrix.

The stochastic variables $\gamma_k^i \in \mathbb{R}$ $(1 \le i \le d)$, which account for the phenomena of multiple RONs, are another series of Bernoulli distributed white sequences taking values on 0 or 1 with

$$\text{Prob}\{\gamma_k^i = 1\} = \gamma^i, \quad \text{Prob}\{\gamma_k^i = 0\} = 1 - \gamma^i, \tag{6}$$

where $\gamma^i \in [0,1]$ are known constants. Furthermore, the constant $\mu_p \ge 0$ $(p = 1, 2, \ldots)$ satisfies the following convergence condition

$$\bar{\mu} = \sum_{p=1}^{\infty} \mu_p \le \sum_{p=1}^{\infty} p\mu_p < +\infty. \tag{7}$$

Remark 1: The random variables α_k and β_k are introduced to characterize the phenomenon of the ROUs. Such a description is more suitable for reflecting parameter variations of a random nature especially in the network transmission. On the other hand, it is customary that the sector-like description of the nonlinearity g_i in (5) is said to belong to sectors $[F_{1i}, F_{2i}]$ [9], which is more general than the usually used Lipschitz-type functions, see e.g. [17], [18]. By employing such a description, it would be possible to reduce the conservatism of the results caused by quantifying the nonlinearities via the convex optimization technique.

Remark 2: In this paper, the random variables γ_k^i $(i = 1, 2, \ldots, d)$ are used to model the probability distribution of the nonlinear functions in system (1). Together with condition (5), each g_i enters into the system in a random way according to an individual Bernoulli distribution. This description can reflect the fact that the multiple RONs can appear or disappear in a probabilistic way due to unpredictable changes of the environmental circumstances.

Before proceeding, we make the following assumptions.

Assumption 1: The parameter uncertainties ΔA and ΔA_d are bounded in terms of Euclidean norm.

Assumption 2: The positive integer d_k describes the discrete time-varying delay that satisfies:

$$d_m \le d_k \le d_M \tag{8}$$

where d_m and d_M are known positive integers representing the lower and upper bounds of the time-delay, respectively. The lower bound of delay d_m can always be described by $d_m = \tau m$, where τ and m are integers.

Assumption 3: a) The stochastic variables α_k, β_k and γ_k^i are mutually uncorrelated random variables which are also unrelated with ω_k. b) The stochastic variables α_k, β_k and γ_k^i $(i = 1, 2, \ldots, d)$ are independent of $\mathcal{F}$.

III. Design of SMC

In this section, a sufficient condition is presented to ensure the stability of sliding mode dynamics, and an SMC law is synthesized to drive the state trajectories of system (1) onto the pre-specified sliding surface.

A. Sliding surface

Firstly, a discrete-time switching function is constructed as follows:

$$s_k = Gx_k - GAx_{k-1} \tag{9}$$

where G is to be designed such that GB is nonsingular and $G\mathfrak{B} = 0$, where $\mathfrak{B} = \left[\begin{array}{ccc} C & D & E \end{array} \right]$. In this paper, we choose $G = B^T P$ with $P > 0$ to confirm the nonsingularity of GB, and B is assumed to be of full column rank.

When the state trajectories of the system (1) enter into the ideal quasi-sliding mode $s_{k+1} = s_k = 0$, the equivalent control law of the sliding motion can be obtained,

$$u_k^{eq} \;\; = \;\; -(GB)^{-1}G\mathscr{A}_k - f(x_k) \tag{10}$$

where $\mathscr{A}_k = \alpha\Delta Ax_k + (A_d + \beta\Delta A_d)x_{k-d_k}$.

Substituting (10) as u_k into (1), we obtain the sliding mode dynamics as follows:

$$\left\{ \begin{aligned} x_{k+1} = {} & \mathcal{A}_k - B(GB)^{-1}G\mathscr{A}_k + (\alpha_k - \alpha)\Delta Ax_k \\ & + (\beta_k - \beta)\Delta A_d x_{k-d_k} + C\sum_{p=1}^{+\infty} \mu_p x_{k-p} \\ & + \sum_{i=1}^{d} \gamma_k^i Dg_i(x_k) + E\sigma(x_k, x_{k-d_k})\omega_k, \end{aligned} \right. \tag{11}$$

where $\mathcal{A}_k = (A + \alpha\Delta A)x_k + (A_d + \beta\Delta A_d)x_{k-d_k}$.

B. Performance of the sliding motion

Before proceeding, we introduce the following lemmas.

Lemma 1: For any real vectors a, b and matrix $P > 0$ of appropriate dimensions, the following inequality holds

$$a^T b + b^T a \le a^T Pa + b^T P^{-1} b. \tag{12}$$

Lemma 2: Let $Q = Q^T$, N and H be real matrices of compatible dimensions with F satisfying $F^T F \le I$. Then $Q + NFH + H^T F^T N^T < 0$ if and only if there exists a scalar $\varepsilon > 0$ such that $Q + \varepsilon NN^T + \varepsilon^{-1} H^T H < 0$ or, equivalently

$$\left[\begin{array}{ccc} Q & \varepsilon N & H^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{array} \right] < 0. \tag{13}$$

The following theorem gives a sufficient condition to guarantee the robustly asymptotic mean-square stability of the sliding mode dynamics (11).

Theorem 1: Consider the system (11) and the sliding surface described by (9). For a given scalar $\varrho \in (0, 1)$, the sliding mode dynamics (11) is robustly asymptotically mean-square stable if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $S_1 > 0$, $S_2 > 0$, $T > 0$, real matrices $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$, and scalars $\lambda^* > 0$, $\varepsilon > 0$, $\varphi > 0$ satisfying

$$\left[\begin{array}{cccccc} \Phi_{11} & \Phi_{12} & \vartheta_1\mathcal{X} & \vartheta_2\mathcal{Y} & \Phi_{15} & \varepsilon\Phi_{16}^T \\ * & \Phi_{22} & 0 & 0 & \Phi_{25} & 0 \\ * & * & -\varrho P & 0 & 0 & 0 \\ * & * & * & -\varrho P & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon I \end{array} \right] < 0, \tag{14}$$

$$\left[\begin{array}{cccccc} \Phi_{11} & \Phi_{12} & \vartheta_1\mathcal{X} & \vartheta_2\mathcal{Z} & \Phi_{15} & \varepsilon\Phi_{16}^T \\ * & \Phi_{22} & 0 & 0 & \Phi_{25} & 0 \\ * & * & -\varrho P & 0 & 0 & 0 \\ * & * & * & -\varrho P & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon I \end{array} \right] < 0, \tag{15}$$

$$E^T PE \le \lambda^* I, \tag{16}$$

$$B^T P\mathfrak{B} = 0, \tag{17}$$

where

$$\Phi_{11} = \mathrm{sym}(\Pi_1 + \hat{h}^2\sum_{i=1}^{d} r_i\Xi_1^T PD\Xi_{gi} + \hat{h}^2\Xi_1^T PC\Xi_{g\mu}) + \Pi_2 + \Pi_3,$$

$$\Phi_{12} = \left[\begin{array}{cccc} \sqrt{2}\hat{h}\Xi_1^T P & 2\hat{h}\Xi_2^T PB & 0 & 0 \end{array} \right],$$

$$\Phi_{22} = \mathrm{diag}\{-P, -B^T PB, -P, -P\},$$

$$\Phi_{15}^T = \left[\begin{array}{cccccc} 0 & r_1\hat{h}^2\alpha H_a^T PD & r_2\hat{h}^2\alpha H_a^T PD & \ldots & r_d\hat{h}^2\alpha H_a^T PD & \hat{h}^2\alpha H_a^T PC \\ 0 & r_1\hat{h}^2\beta H_d^T PD & r_2\hat{h}^2\beta H_d^T PD & \ldots & r_d\hat{h}^2\beta H_d^T PD & \hat{h}^2\beta H_d^T PC \end{array} \right],$$

$$\Phi_{16} = \left[\begin{array}{cccc} N & 0_{n_N\times mn} & 0_{n_N\times n} & 0_{n_N\times(d+2)n} \\ \hline 0_{n_N\times n} & 0_{n_N\times mn} & N & 0_{n_N\times(d+2)n} \end{array} \right],$$

$$\Phi_{25}^T = \left[\begin{array}{cc} \mathscr{X}_{11} & \mathscr{X}_{12} \end{array} \right],$$

$$\mathscr{X}_{11} = \left[\begin{array}{cc} \sqrt{2}\hat{h}\alpha H_a^T P & 2\hat{h}\alpha H_a^T PB \\ \sqrt{2}\hat{h}\beta H_d^T P & 2\hat{h}\beta H_d^T PB \end{array} \right],$$

$$\mathscr{X}_{12} = \left[\begin{array}{cc} \sqrt{\bar{\alpha}}\hat{h}H_a^T P & 0 \\ 0 & \sqrt{\bar{\beta}}\hat{h}H_d^T P \end{array} \right],$$

$$\Xi_1 = \left[\begin{array}{cccc} A & 0_{n\times mn} & A_d & 0_{n\times(d+2)n} \end{array} \right],$$

$$\Xi_2 = \left[\begin{array}{ccc} 0_{n\times(m+1)n} & A_d & 0_{n\times(d+2)n} \end{array} \right],$$

$$\Xi_{gi} = \left[\begin{array}{cccc} 0_{n\times(m+i+2)n} & I_{n\times n} & 0_{n\times(d-i)n} & 0_{n\times n} \end{array} \right],$$

$$\Xi_{g\mu} = \left[\begin{array}{cc} 0_{n\times(m+d+3)n} & I_{n\times n} \end{array} \right],$$

$$\Pi_1 = \left[\begin{array}{ccc} \mathcal{X} & \mathcal{Y} & \mathcal{Z} \end{array} \right] \times \left[\begin{array}{c} I_{n\times n} \quad 0_{n\times(m+1)n} \quad -I_{n\times n} \quad 0_{n\times(d+1)n} \\ \hline 0_{n\times mn} \quad I_{n\times n} \quad -I_{n\times n} \quad 0_{n\times(d+2)n} \\ \hline 0_{n\times(m+1)n} \quad I_{n\times n} \quad -I_{n\times n} \quad 0_{n\times(d+1)n} \end{array} \right],$$

$$\Pi_2 = W_R^T \mathcal{R} W_R, \;\; \mathcal{R} = \mathrm{diag}\{R, -R\}, \;\; \Pi_3 = W_\Theta^T \Theta W_\Theta,$$

$$W_\Theta = \left[\begin{array}{c} I_{n\times n} \quad 0_{n\times(m+d+3)n} \\ \hline 0_{n\times mn} \quad I_{n\times n} \quad 0_{n\times(d+3)n} \\ \hline 0_{n\times(m+1)n} \quad I_{n\times n} \quad 0_{n\times(d+2)n} \\ \hline 0_{n\times(m+2)n} \quad I_{n\times n} \quad 0_{n\times(d+1)n} \\ \hline 0_{dn\times(m+3)n} \quad I_{dn\times dn} \quad 0_{dn\times n} \\ \hline 0_{n\times(m+d+3)n} \quad I_{n\times n} \end{array} \right],$$

$$W_R = \left[\begin{array}{c} I_{mn\times mn} \quad 0_{mn\times(d+4)n} \\ \hline 0_{mn\times n} \quad I_{mn\times mn} \quad 0_{mn\times(d+3)n} \end{array} \right],$$

$$\Theta = \left[\begin{array}{cc} \aleph_{11} & \aleph_{12} \\ * & \aleph_{22} \end{array} \right],$$

$$\aleph_{11} = \mathrm{diag}\{\Theta_{11}, -S_1, \hat{h}^2\lambda^*\rho_2 I - Q, -S_2\},$$

$$\aleph_{12}^T = \left[\begin{array}{cccc} \Theta_{15}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right],$$

$$
\begin{aligned}
&\aleph_{22} = \begin{bmatrix} \Theta_{55} & \hat{h}^2\Theta_{56} \\ * & 2\hat{h}^2 C^T P C - \frac{1}{\bar{\mu}} T \end{bmatrix}, \\
&\Theta_{11} = (2\varrho\hbar - 1)P + \hat{h}^2\lambda^*\rho_1 I + (d_M - d_m + 1)Q \\
&\qquad\quad + S_1 + S_2 + \bar{\mu}T - \varphi\sum_{i=1}^{d}\tilde{F}_i, \\
&\Theta_{15} = \begin{bmatrix} \varphi\bar{F}_1 & \varphi\bar{F}_2 & \cdots & \varphi\bar{F}_d \end{bmatrix}, \\
&\Theta_{55} = \begin{bmatrix} \Theta_{55}^{(ij)} \end{bmatrix}, \ \ (i,j = 1,2,\ldots,d) \\
&\Theta_{55}^{(ii)} = \hat{h}^2((\gamma^i)^2 + \gamma^i) D^T P D - 2\varphi I, \\
&\Theta_{55}^{(ij)} = 2\gamma^i\gamma_j\hat{h}^2 D^T P D, \ (i \neq j) \\
&\Theta_{56}^T = \begin{bmatrix} r_1 I & r_2 I & \ldots & r_d I \end{bmatrix} C^T P D, \\
&\tilde{F}_i = F_{1i}^T F_{2i} + F_{2i}^T F_{1i}, \ \bar{F}_i = F_{1i}^T + F_{2i}^T, \\
&\hbar = 2d_M - d_m, \ \ \hat{h} = \sqrt{1 + 2\varrho\hbar}, \ \ \vartheta_1 = \sqrt{d_M}, \\
&\vartheta_2 = \sqrt{d_M - d_m}, \ \ \bar{\alpha} = \alpha(1-\alpha), \ \ \bar{\beta} = \beta(1-\beta)
\end{aligned}
$$

and n_N is the number of row in matrix N. $\mathrm{sym}(\Im)$ represents $\Im + \Im^T$.

Proof: Choose the following Lyapunov-Krasovskii functional candidate for system (11):

$$
V_k = \sum_{i=1}^{6} V_k^i \tag{18}
$$

where

$$
\begin{aligned}
V_k^1 &= x_k^T P x_k \\
V_k^2 &= \sum_{l=k-d_k}^{k-1} x_l^T Q x_l + \sum_{j=-d_M+1}^{-\tau m}\sum_{l=k+j}^{k-1} x_l^T Q x_l \\
V_k^3 &= \sum_{l=k-d_m}^{k-1} x^T(l) S_l x_l + \sum_{l=k-d_M}^{k-1} x_l^T S_2 x_l \\
V_k^4 &= \sum_{l=k-\tau}^{k-1} \Gamma_l^T R \Gamma_l \\
V_k^5 &= \varrho\sum_{j=-d_M+1}^{0}\sum_{l=k-1+j}^{k-1} \eta_l^T P \eta_l + \varrho\sum_{j=-d_M+1}^{-\tau m}\sum_{l=k-1+j}^{k-1} \eta_l^T P \eta_l \\
V_k^6 &= \sum_{p=1}^{+\infty}\mu_p\sum_{l=k-p}^{k-1} x_l^T T x_l \\
\eta_l &= x_{l+1} - x_l, \ \Gamma_l = \mathrm{col}\{x_l, x_{l-\tau}, \ldots, x_{l-(m-1)\tau}\}
\end{aligned}
$$

with $P > 0$, $Q > 0$, $R > 0$, $S_1 > 0$, $S_2 > 0$, $T > 0$ being matrices to be determined. Here, $\mathrm{col}\{\cdots\}$ denotes a vector column with blocks given by the vectors in $\{\cdots\}$.

By using Lemma 1 and noting Assumption 3, we have

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_k^1\} \\
\leq\ &\mathbb{E}\Bigg\{\xi_k^T\Bigg[2\hat{\Xi}_1^T P\hat{\Xi}_1 + 2\hat{\Xi}_1^T P C \Xi_{g\mu} + 2\sum_{i=1}^{d} r_i\hat{\Xi}_1^T P D \Xi_{gi} \\
&+4\hat{\Xi}_2^T G^T (GB)^{-1} G\hat{\Xi}_2 + 2\Xi_{g\mu}^T C^T P C \Xi_{g\mu} \\
&+2\sum_{i=1}^{d} r_i \Xi_{g\mu}^T C^T P D \Xi_{gi} + \bar{\alpha}\hat{\Xi}_3^T P\hat{\Xi}_3 + \bar{\beta}\hat{\Xi}_4^T P\hat{\Xi}_4\Bigg]\xi_k \\
&+2\Bigg[\sum_{i=1}^{d}\gamma^i D g_i(x_k)\Bigg]^T P\Bigg[\sum_{i=1}^{d}\gamma^i D g_i(x_k)\Bigg] \\
&+\sum_{i=1}^{d}\gamma^i(1-\gamma^i) g_i^T(x_k) D^T P D g_i(x_k) \\
&+\sigma^T(x_k, x_{k-d_k}) E^T P E \sigma(x_k, x_{k-d_k}) - x_k^T P x_k\Bigg\}
\end{aligned} \tag{19}
$$

where

$$
\begin{aligned}
\xi_k^T &= \Big[\ \Gamma_k^T \quad x_{k-d_m}^T \quad x_{k-d_k}^T \quad x_{k-d_M}^T \quad g^T(x_k) \quad \sum_{p=1}^{+\infty}\mu_p x_{k-p}^T\ \Big], \\
g(x_k) &= \begin{bmatrix} g_1^T(x_k) & g_2^T(x_k) & \cdots & g_d^T(x_k) \end{bmatrix}^T, \\
\hat{\Xi}_1 &= \begin{bmatrix} A + \alpha\Delta A & 0_{n\times mn} & A_d + \beta\Delta A_d & 0_{n\times(d+2)n} \end{bmatrix}, \\
\hat{\Xi}_2 &= \begin{bmatrix} \alpha\Delta A & 0_{n\times mn} & A_d + \beta\Delta A_d & 0_{n\times(d+2)n} \end{bmatrix}, \\
\hat{\Xi}_3 &= \begin{bmatrix} \Delta A & 0_{n\times(m+d+3)n} \end{bmatrix}, \\
\hat{\Xi}_4 &= \begin{bmatrix} 0_{n\times(m+1)n} & \Delta A_d & 0_{n\times(d+2)n} \end{bmatrix},
\end{aligned}
$$

and $\bar{\alpha}$, $\bar{\beta}$, $\Xi_{g\mu}$, Ξ_{gi} are defined in Theorem 1 (below (17)).

On the other hand, it can be derived that

$$
\begin{aligned}
&\sum_{i=2}^{6}\mathbb{E}\{\Delta V_k^i\} \\
\leq\ &\mathbb{E}\Bigg\{x_k^T((d_M - d_m + 1)Q + S_1 + S_2 + \bar{\mu}T)x_k \\
&-x_{k-d_k}^T Q x_{k-d_k} - x_{k-d_m}^T S_1 x_{k-d_m} - x_{k-d_M}^T S_2 x_{k-d_M} \\
&+\Gamma_k^T R\Gamma_k - \Gamma_{k-\tau}^T R\Gamma_{k-\tau} + 2\varrho\hbar x_{k+1}^T P x_{k+1} \\
&+2\varrho\hbar x_k^T P x_k - \sum_{l=k-d_M}^{k-1}\eta_l^T\varrho P\eta_l - \sum_{l=k-d_k}^{k-\tau m-1}\eta_l^T\varrho P\eta_l \\
&-\sum_{l=k-d_M}^{k-d_k-1}\eta_l^T\varrho P\eta_l - \frac{1}{\bar{\mu}}\Bigg[\sum_{p=1}^{+\infty}\mu_p x_{k-p}\Bigg]^T \\
&\times T\Bigg[\sum_{p=1}^{+\infty}\mu_p x_{k-p}\Bigg]\Bigg\}
\end{aligned} \tag{20}
$$

It follows from (2) and (16) that

$$
\begin{aligned}
&\sigma^T(x_k, x_{k-d_k}) E^T P E \sigma(x_k, x_{k-d_k}) \\
\leq\ &\lambda^*(\rho_1 x_k^T x_k + \rho_2 x_{k-d_k}^T x_{k-d_k}).
\end{aligned} \tag{21}
$$

For each $1 \leq i \leq d$, (5) is equivalent to

$$
\begin{bmatrix} x_k \\ g_i(x_k) \end{bmatrix}^T \begin{bmatrix} -\tilde{F}_i & \bar{F}_i \\ * & -2I \end{bmatrix} \begin{bmatrix} x_k \\ g_i(x_k) \end{bmatrix} \geq 0, \tag{22}
$$

where matrices $\tilde{F}_i$ and $\bar{F}_i$ are defined in Theorem 1 (below (17)). Furthermore, according to the definition of η_l, for any matrices $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ with appropriate dimensions, the following equations always hold:

$$0 = 2\xi_k^T \mathcal{X}\bigg[x_k - x_{k-d_M} - \sum_{l=k-d_M}^{k-1} \eta_l\bigg], \tag{23}$$

$$0 = 2\xi_k^T \mathcal{Y}\bigg[x_{k-\tau m} - x_{k-d_k} - \sum_{l=k-d_k}^{k-\tau m-1} \eta_l\bigg], \tag{24}$$

$$0 = 2\xi_k^T \mathcal{Z}\bigg[x_{k-d_k} - x_{k-d_M} - \sum_{l=k-d_M}^{k-d_k-1} \eta_l\bigg]. \tag{25}$$

Noting (19)-(25), we have

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k\} \\
\leq\ & \mathbb{E}\bigg\{\xi_k^T\bigg[\bigg(\frac{d_k-d_m}{d_M-d_m}\bigg)\bigg(\Pi_1+\Pi_1^T+\Pi_2+\Pi_3 \\
&+2\hat{h}^2\hat{\Xi}_1^T P\hat{\Xi}_1 + 2\hat{h}^2\sum_{i=1}^{d} r_i\hat{\Xi}_1^T PD\Xi_{gi} + 2\hat{h}^2\hat{\Xi}_1^T PC\Xi_{g\mu} \\
&+4\hat{h}^2\hat{\Xi}_2^T G^T(GB)^{-1}G\hat{\Xi}_2 + \bar{\alpha}\hat{h}^2\hat{\Xi}_3^T P\hat{\Xi}_3 + \bar{\beta}\hat{h}^2\hat{\Xi}_4^T P\hat{\Xi}_4 \\
&+d_M\mathcal{X}(\varrho P)^{-1}\mathcal{X}^T + (d_M-d_m)\mathcal{Y}(\varrho P)^{-1}\mathcal{Y}^T\bigg) \\
&+\bigg(\frac{d_M-d_k}{d_M-d_m}\bigg)\bigg(\Pi_1+\Pi_1^T+\Pi_2+\Pi_3+2\hat{h}^2\hat{\Xi}_1^T P\hat{\Xi}_1 \\
&+2\hat{h}^2\sum_{i=1}^{d} r_i\hat{\Xi}_1^T PD\Xi_{gi} + 2\hat{h}^2\hat{\Xi}_1^T PC\Xi_{g\mu} + 4\hat{h}^2\hat{\Xi}_2^T G^T \\
&\times(GB)^{-1}G\hat{\Xi}_2 + \bar{\alpha}\hat{h}^2\hat{\Xi}_3^T P\hat{\Xi}_3 + \bar{\beta}\hat{h}^2\hat{\Xi}_4^T P\hat{\Xi}_4 \\
&+d_M\mathcal{X}(\varrho P)^{-1}\mathcal{X}^T + (d_M-d_m)\mathcal{Z}(\varrho P)^{-1}\mathcal{Z}^T\bigg)\bigg]\xi_k\bigg\},
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
\Sigma_1 &= \Big[\varrho P\eta_l + \mathcal{X}^T\xi_k\Big]^T(\varrho P)^{-1}\Big[\varrho P\eta_l + \mathcal{X}^T\xi_k\Big] \\
\Sigma_2 &= \Big[\varrho P\eta_l + \mathcal{Y}^T\xi_k\Big]^T(\varrho P)^{-1}\Big[\varrho P\eta_l + \mathcal{Y}^T\xi_k\Big] \\
\Sigma_3 &= \Big[\varrho P\eta_l + \mathcal{Z}^T\xi_k\Big]^T(\varrho P)^{-1}\Big[\varrho P\eta_l + \mathcal{Z}^T\xi_k\Big]
\end{aligned}$$

and Π_1, Π_2 and Π_3 are defined in Theorem 1 (below (17)).

By using Lemma 2 and applying Schur complement, it follows from (14) and (15) that $\mathbb{E}\{\Delta V_k\} < 0$ holds, and therefore the mean-square asymptotic stability of the sliding mode dynamics (11) can be confirmed. This completes the proof. ■

Remark 3: Comparing with the conventional Lyapunov-Krasovskii functional for delay systems, (18) exhibits two extra terms, V_k^4 and V_k^5, both of which exists for particular reasons. Specifically, V_k^4 is adjusted to reduce the conservatism in response to the need of delay-fractioning for discrete systems, and V_k^5 is there to fit the SMC framework where a so-called "weighting" scalar parameter $\varrho \in (0,1)$ is enforced. Such a parameter is exploited to reflect both the delay-fractioning approach and parameter uncertainties, and its value can be determined *a priori* to help the feasibility study of (14)-(17) in Theorem 1.

C. Computational Algorithm

Note that there exists an equality constraint condition (17) $B^T P\mathfrak{B} = 0$ in Theorem 1. Based on the algorithm presented in [11], this constraint can be equivalently converted into $tr[(B^T P\mathfrak{B})^T B^T P\mathfrak{B}] = 0$. By introducing an inequality $(B^T P\mathfrak{B})^T B^T P\mathfrak{B} \leq \gamma I$ ($\gamma > 0$ is a sufficiently small scalar) and using Schur Complement, we have

$$\begin{bmatrix} -\gamma I & \mathfrak{B}^T PB \\ B^T P\mathfrak{B} & -I \end{bmatrix} \leq 0. \tag{27}$$

Hence, the original nonconvex feasibility problem is now converted into a problem of finding the global solution to the following minimization problem:

$$\begin{aligned} &\min \gamma \\ &\text{subject to (14)-(16) and (27).} \end{aligned} \tag{28}$$

D. Reachability Analysis

In this part, an SMC law is proposed and the reachability analysis of the specified sliding surface is conducted.

By considering the inequality reaching condition in [5],

$$\begin{cases} \Delta s_k \leq & -\kappa U \mathrm{sgn}[s_k] - \kappa V s_k, \quad \text{if } s_k > 0 \\ \Delta s_k \geq & -\kappa U \mathrm{sgn}[s_k] - \kappa V s_k, \quad \text{if } s_k < 0 \end{cases} \tag{29}$$

where κ denotes the sampling period, $U = \mathrm{diag}\{\mu_1, \mu_2, \ldots, \mu_q\} \in \mathbb{R}^{q\times q}$, $V = \mathrm{diag}\{\nu_1, \nu_2, \ldots, \nu_q\} \in \mathbb{R}^{q\times q}$, and $\mu_i > 0$, $\nu_i > 0$ are properly chosen constants satisfying $0 < 1 - \kappa\nu_i < 1$ $(i = 1, 2, \ldots, q)$.

Notice that ΔA, ΔA_d and $f(x_k)$ are all bounded in terms of Euclidean norm, let $\Delta_a(k) := \alpha_k G\Delta A x_k$, $\Delta_d(k) := G(A_d + \beta_k\Delta A_d)x_{k-d_k}$ and $\Delta_f(k) := GBf(x_k)$, then there exist $\underline{\delta}_a^i$, $\overline{\delta}_a^i$, $\underline{\delta}_d^i$, $\overline{\delta}_d^i$, $\underline{\delta}_f^i$ and $\overline{\delta}_f^i$ $(i = 1, 2, \cdots, q)$ satisfying

$$\underline{\delta}_a^i \leq \delta_a^i(k) \leq \overline{\delta}_a^i, \ \ \underline{\delta}_d^i \leq \delta_d^i(k) \leq \overline{\delta}_d^i, \ \ \underline{\delta}_f^i \leq \delta_f^i(k) \leq \overline{\delta}_f^i \tag{30}$$

where $\delta_a^i(k)$, $\delta_d^i(k)$ and $\delta_f^i(k)$ are the ith elements in $\Delta_a(k)$, $\Delta_d(k)$ and $\Delta_f(k)$, respectively. Then, by defining

$$\begin{aligned}
\widehat{\Delta}_a &= \begin{bmatrix} \widehat{\delta}_a^1 & \widehat{\delta}_a^2 & \cdots & \widehat{\delta}_a^q \end{bmatrix}^T, \ \ \widehat{\delta}_a^i = \frac{\overline{\delta}_a^i + \underline{\delta}_a^i}{2}, \\
\widetilde{\Delta}_a &= \mathrm{diag}\Big\{\widetilde{\delta}_a^1, \widetilde{\delta}_a^2, \ldots, \widetilde{\delta}_a^q\Big\}, \quad \widetilde{\delta}_a^i = \frac{\overline{\delta}_a^i - \underline{\delta}_a^i}{2}, \\
\widehat{\Delta}_d &= \begin{bmatrix} \widehat{\delta}_d^1 & \widehat{\delta}_d^2 & \cdots & \widehat{\delta}_d^q \end{bmatrix}^T, \ \ \widehat{\delta}_d^i = \frac{\overline{\delta}_d^i + \underline{\delta}_d^i}{2}, \\
\widetilde{\Delta}_d &= \mathrm{diag}\Big\{\widetilde{\delta}_d^1, \widetilde{\delta}_d^2, \ldots, \widetilde{\delta}_d^q\Big\}, \quad \widetilde{\delta}_d^i = \frac{\overline{\delta}_d^i - \underline{\delta}_d^i}{2}, \\
\widehat{\Delta}_f &= \begin{bmatrix} \widehat{\delta}_f^1 & \widehat{\delta}_f^2 & \cdots & \widehat{\delta}_f^q \end{bmatrix}^T, \ \ \widehat{\delta}_f^i = \frac{\overline{\delta}_f^i + \underline{\delta}_f^i}{2}, \\
\widetilde{\Delta}_f &= \mathrm{diag}\Big\{\widetilde{\delta}_f^1, \widetilde{\delta}_f^2, \ldots, \widetilde{\delta}_f^q\Big\}, \quad \widetilde{\delta}_f^i = \frac{\overline{\delta}_f^i - \underline{\delta}_f^i}{2},
\end{aligned} \tag{31}$$

we are now ready to present the design scheme of the robust SMC law.

Theorem 2: Assume that the minimization problem (28) is solvable. For the system (1) with sliding surface (9), where

$G = B^T P$ and P is the solution of (28), if the SMC law is given as follows

$$u(k) = -(GB)^{-1}(\kappa U \mathrm{sgn}[s_k] + \kappa V s_k - s_k + (\widehat{\Delta}_a + \widehat{\Delta}_d + \widehat{\Delta}_f) + (\widetilde{\Delta}_a + \widetilde{\Delta}_d + \widetilde{\Delta}_f)\mathrm{sgn}[s_k]), \quad (32)$$

then the discrete-time sliding mode reaching condition of system (1) with specified sliding surface (9) is satisfied.

Proof: Together with (32) and (9), we have

$$\begin{aligned}\Delta s_k = & -\kappa U \mathrm{sgn}[s_k] - \kappa V s_k + \Delta_a(k) - (\widehat{\Delta}_a + \widetilde{\Delta}_a \mathrm{sgn}[s_k]) \\ & + \Delta_d(k) - (\widehat{\Delta}_d + \widetilde{\Delta}_d \mathrm{sgn}[s_k]) \\ & + \Delta_f(k) - (\widehat{\Delta}_f + \widetilde{\Delta}_f \mathrm{sgn}[s_k]). \quad (33)\end{aligned}$$

It follows easily from (31) that (29) holds, and then proof of this theorem is complete. ■

Remark 4: In the stochastic model presented in this paper, there are four main aspects that complicate the design of SMC, i.e., ROUs, RONs, infinite distributed delays as well as the delay-fractioning approach. In our main results, all these four aspects have been explicitly reflected, where the occurrence probabilities α and β are there for the ROUs, the occurrence probabilities γ^i $(i = 1, 2, \ldots, d)$ and constant matrices F_{ij} $(i = 1, 2;\ j = 1, 2, \ldots, d)$ quantify the multiple randomly occurring sector-like nonlinearities, the constant $\bar{\mu}$ accounts for infinite distributed delays, and the new Lyapunov-Krasovskii functional (18) stems from the discrete-time delay-fractioning idea.

IV. An Illustrative Example

Following [3], [17], we consider the SMC problem for an F-404 aircraft engine system. Setting the sampling time $T = 1.2s$, we obtain the following discretized nominal system matrix

$$A = \begin{bmatrix} 0.2504 & 0 & 0.3919 \\ 0.0570 & 0.6188 & -0.0616 \\ 0.0502 & 0 & 0.1262 \end{bmatrix}.$$

In the F-404 aircraft engine model, x_k^1 and x_k^2 represent the horizontal position and x_k^3 is the altitude of the aircraft. The control inputs u_k^1 and u_k^2 are the engine thrust and flight path angle, respectively. The movement of the aircraft is affected by the wind that acts as a stochastic disturbance ω_k. To this end, other parameters are given as:

$$A_d = \begin{bmatrix} 0.03 & 0 & -0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.01 \end{bmatrix}, \ B = \begin{bmatrix} 0.1817 & 0.4286 \\ 0.1597 & 0.793 \\ 0.1138 & 0.0581 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.03 & 0.015 & -0.01 \\ 0.02 & 0.03 & 0 \\ 0.02 & 0.025 & -0.01 \end{bmatrix}, \ H = \begin{bmatrix} 0.01 \\ 0.02 \\ 0.005 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.025 & 0.01 & 0 \\ 0 & -0.03 & 0 \\ 0.04 & 0.035 & -0.01 \end{bmatrix}, \ H_d = \begin{bmatrix} 0.02 \\ 0.03 \\ 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.015 & 0 & -0.01 \\ 0.01 & 0.015 & 0 \\ 0.02 & 0.025 & -0.01 \end{bmatrix}, \ N^T = \begin{bmatrix} 0.2 \\ 0.1 \\ 0 \end{bmatrix}.$$

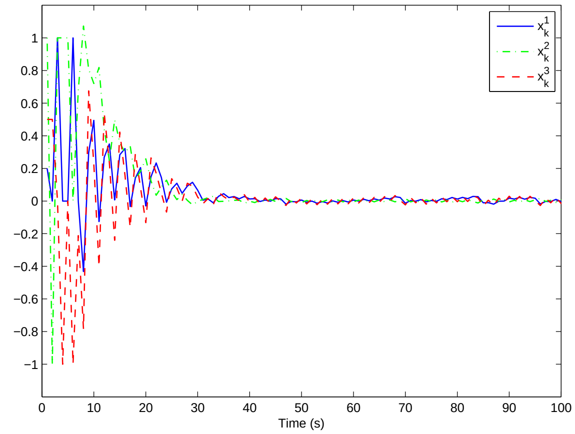

Fig. 1. The trajectory of state x_k (m)

Let

$$\begin{aligned} & F = \sin(0.6k), \\ & f(x_k) = \begin{bmatrix} 0.4\sin(x_k^1 x_k^3) & 0.3\sin(x_k^2) \end{bmatrix}^T, \\ & \sigma(x_k, x_{k-d_k}) = 0.5x_k + 0.5x_{k-d_k}, \\ & g_1(x_k) = 0.5(F_{11} + F_{21})x_k + 0.5(F_{21} - F_{11})\sin(x_k)x_k, \\ & g_2(x_k) = 0.5(F_{12} + F_{22})x_k + 0.5(F_{22} - F_{12})\cos(x_k)x_k, \end{aligned}$$

where

$$\begin{aligned} & \sin(x_k) := \mathrm{diag}\{\sin(x_k^1), \sin(x_k^2), \sin(x_k^3)\}, \\ & \cos(x_k) := \mathrm{diag}\{\cos(x_k^1), \cos(x_k^2), \cos(x_k^3)\}, \\ & F_{11} = F_{12} = \mathrm{diag}\{0.4, 0.5, 0.8\}, \\ & F_{21} = F_{22} = \mathrm{diag}\{0.3, 0.2, 0.6\}. \end{aligned}$$

Set $\alpha = 0.75$, $\beta = 0.78$, $\gamma_1 = 0.86$ and $\gamma_2 = 0.82$. Assume that the time-varying delay d_k satisfies $3 \le d_k \le 5$. Moreover, choosing the constants $\mu_p = 2^{-3-p}$, we can easily see that the convergence condition (7) holds.

Setting $m = 1$ and $\varrho = 1.2 \times 10^{-4}$ in (18) and solving the minimization problem (28), we obtain

$$P = \begin{bmatrix} 0.4676 & -0.0765 & -0.1145 \\ -0.0765 & 0.1880 & -0.0459 \\ -0.1145 & -0.0459 & 0.3346 \end{bmatrix},$$

and $\gamma = 6.9607 \times 10^{-5}$ (hence the equality constraint in Eq. (17) is considered to be achieved). It follows from Theorem 2 that the desired SMC controller (32) can be described by all known parameters. The simulation results are shown in Figs. 1-3. Among them, Fig. 1 shows the response of system state by taking $\mu_j = \nu_j = 0.001$ $(j = 1, 2)$. The responses of sliding surface s_k and control input u_k are shown in Figs. 2-3, respectively. The simulation results have confirmed our theoretical results. It would be interesting to look into the possibility of carrying out real-time experiments on flight control systems with actual engines in the future.

V. Conclusions

The robust SMC problem for a class of discrete mixed time-delays stochastic system with ROUs and RONs has been

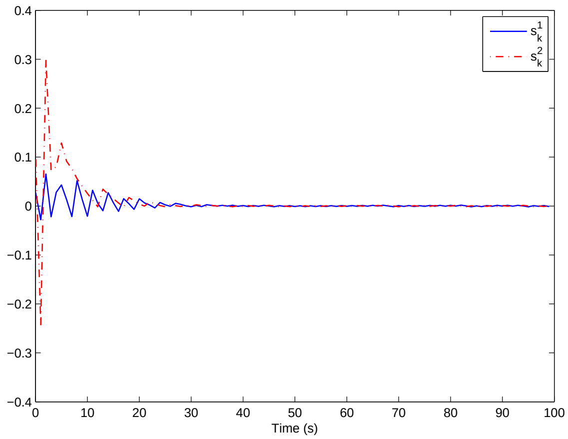

Fig. 2. The trajectory of sliding variable s_k

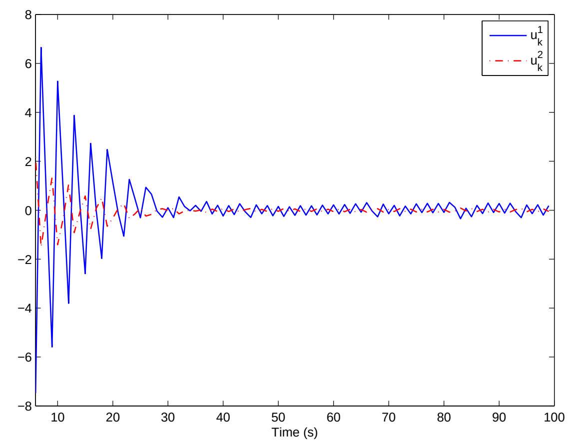

Fig. 3. The control signals u_k^1 (kN) and u_k^2 (deg)

considered in this paper. By making use of delay-fractioning approach and constructing a new Lyapunov-Krasovskii functional, a sufficient condition has been derived to ensure the stability of the sliding mode dynamics, and an SMC law has been designed such that the state trajectory of the system starting from any initial state is globally driven onto the specified sliding surface. Further research topics include the SMC problem for networked control system with plant output delays and input delays.

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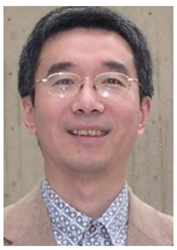

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