

Turbo Packet Combining Strategies for the MIMO-ISI ARQ Channel

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Abstract

This paper addresses the issue of efficient turbo packet combining techniques for coded transmission with a Chase-type automatic repeat request (ARQ) protocol operating over a multiple-input–multiple-output (MIMO) channel with intersymbol interference (ISI). First of all, we investigate the outage probability and the outage-based power loss of the MIMO-ISI ARQ channel when optimal *maximum a posteriori* (MAP) turbo packet combining is used at the receiver. We show that the ARQ delay (i.e., the maximum number of ARQ rounds) does not completely translate into a diversity gain. We then introduce two efficient turbo packet combining algorithms that are inspired by minimum mean square error (MMSE)-based turbo equalization techniques. Both schemes can be viewed as low-complexity versions of the optimal MAP turbo combiner. The first scheme is called *signal-level* turbo combining and performs packet combining and multiple transmission ISI cancellation jointly at the signal-level. The second scheme, called *symbol-level* turbo combining, allows ARQ rounds to be separately turbo equalized, while combining is performed at the filter output. We conduct a complexity analysis where we demonstrate that both algorithms have almost the same computational cost as the conventional log-likelihood ratio (LLR)-level combiner. Simulation results show that both proposed techniques outperform LLR-level combining, while for some representative MIMO configurations, signal-level combining has better ISI cancellation capability and achievable diversity order than that of symbol-level combining.

Index Terms

Automatic repeat request (ARQ) mechanisms, multiple-input–multiple-output (MIMO), intersymbol interference (ISI), outage probability, turbo equalization, minimum mean square error (MMSE).

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I. INTRODUCTION

A. Research Motivation

Hybrid-automatic repeat request (ARQ) protocols and multiple-input-multiple-output (MIMO) play a key role in the evolution of current wireless systems toward high data rate wireless broadband standards [1]. While MIMO techniques allow the space and time diversities of the multi-antenna channel to be translated into diversity and/or multiplexing gains [2], hybrid-ARQ mechanisms exploit the ARQ delay, i.e., the maximum number of ARQ transmission rounds, to reduce the frame error rate (FER) and therefore increase the system throughput [3], [4].

In the last few years, special interest has been paid to the joint design of the transmission combiner (also referred to as “*packet combiner*”) and the signal processor (detection and/or equalization) receiver. Combining schemes targeting a joint design approach were first proposed by Samra and Ding for single antenna systems operating over intersymbol interference (ISI) channels [5]–[8], and are called transmission combining with integrated equalization (IEQ). In particular, it was shown in [8] that, when concatenated with an outer code, IEQ performs better than the iterative combining scheme introduced by Doan and Narayanan [9]. In iterative combining, multiple copies of the same packet are independently interleaved and combining is performed by iterating between multiple equalizers before channel decoding. The IEQ concept was then extended to MIMO systems with flat fading to jointly perform co-antenna interference (CAI) cancellation and transmission combining [10]–[12]. In parallel, several other MIMO ARQ architectures exploiting the high degree of freedom in the design of the MIMO ARQ transmitter were proposed (e.g. [13]–[20]). Turbo coded ARQ schemes with iterative minimum mean square error (MMSE) frequency domain equalization (FDE) for single carrier transmission over broadband channel were proposed for direct sequence code division multiple access (DS-CDMA) and MIMO systems in [21] and [22], [23], respectively.

Recently, in a seminal paper by El Gamal *et al.* [24], the diversity-multiplexing tradeoff¹ of the MIMO ARQ flat fading channel was characterized, and was referred to as diversity-multiplexing-delay tradeoff. The authors proved that the ARQ delay presents an important source of diversity even when the channel is constant over ARQ transmission rounds, a scenario referred to as long-term static channel. In particular, it was shown that operating over such a channel with a large ARQ delay results in a flat diversity-multiplexing tradeoff. This means that one can achieve full diversity and multiplexing gains

¹A fundamental tool for the design of space-time coding/multiplexing architectures initially proposed by Zheng and Tse for flat fading [25], and later extended to frequency selective fading [26]–[28].

if large ARQ windows are allowed. The diversity–multiplexing–delay tradeoff was then investigated in the case of delay-sensitive services and block-fading MIMO channels in [29] and [30], respectively.

B. In this Paper

Motivated by the IEQ concept [8] and the results in [24], we investigate efficient IEQ-aided packet combining strategies for coded transmission with hybrid–ARQ operating over MIMO-ISI channels. Our main objective is to reduce the number of ARQ rounds required to correctly decode a data packet while keeping the receiver complexity (computational load and memory requirements) affordable. In our design, packet combining is performed at each ARQ round by exchanging soft information in an iterative (turbo) fashion between the *soft packet combiner* and the soft-input–soft-output (SISO) decoder. We refer to this combining family as “*turbo packet combining*”.

We focus on space–time bit-interleaved coded modulation (ST-BICM) transmitter schemes with Chase-type ARQ, i.e., the data packet is entirely retransmitted. The choice of ST-BICM is motivated by the simplicity of this coding scheme, and the efficiency of its iterative decoding (ID) receiver in achieving high diversity and coding gains over block-fading MIMO-ISI channels [31]–[36]. Our work is still valid for other space–time codes (STCs). Note that some practical systems employ hybrid–ARQ with incremental redundancy (IR). In IR-type ARQ, retransmissions only carry portions of the data packet. It presents an efficient technique for increasing the system throughput while keeping the error performance acceptable. In this paper, we restrict our work to Chase-type ARQ. Turbo combining techniques for broadband MIMO transmission with IR-type ARQ are left for future investigations.

First of all, we derive the optimal *maximum a posteriori* (MAP) turbo packet combining algorithm² that makes use of all diversities available in the MIMO-ISI ARQ channel to perform transmission combining. The turbo packet combining strategies we introduce in this paper can be seen as low-complexity sub-optimal techniques of the MAP combining algorithm. An important ingredient in MAP turbo combining is an analogy between multiple transmissions and antennas, and which consists of considering ARQ rounds as virtual receive antennas. This allows the ARQ delay, i.e., maximum number of ARQ rounds, to be translated into receive diversity. We then analyze the outage performance of the MIMO-ISI ARQ channel. This analysis allows us to know how the ARQ delay influences the outage probability of the MIMO ARQ system. It also serves as a theoretical foundation for the turbo packet combiners we propose in this paper. We also investigate the outage-based power loss due to multiple

²In this paper, optimality refers to the exploitation of delay, space, time, and multipath diversities of the MIMO-ISI ARQ channel to combine multiple transmissions.

transmission rounds. This analysis establishes that in the outage region of interest (corresponding to an outage between 10^{-2} and 10^{-3}) the power loss due to ARQ is below 0.25dB.

The next step in our work corresponds to the derivation of two turbo packet combining strategies for the MIMO-ISI ARQ channel. Both techniques are inspired by the unconditional MMSE turbo equalization schemes of [34] and [37]. The first algorithm, named *signal-level* turbo packet combining, presents a low-complexity version of MAP turbo combining. It performs packet combining and equalization using signals from all transmission rounds. In contrast to what was initially stated in [38], we show that the computational complexity of this scheme is less sensitive to the number of ARQ rounds. Moreover, we provide an optimized implementation where it is not necessary for the receiver to store all signal vectors and channel matrices. The second combining scheme, namely, *symbol-level* turbo combining, performs soft equalization separately for each round, and combines multiple transmissions at the level of filter outputs. It has the same computational complexity and fewer memory requirements compared with the first scheme. We also show that receiver requirements (computational complexity and memory) of both turbo combining schemes are almost similar to those of conventional log-likelihood ratio (LLR)-level combining, where extrinsic LLRs corresponding to multiple transmissions are simply added together before SISO decoding. Finally, we provide numerical simulations for some MIMO configurations demonstrating the superior performance of the proposed algorithms compared with LLR-level combining, and the significant gains they offer with respect to both the outage probability and the matched filter bound (MFB).

Throughout the paper, the following notation is used. Superscript \top denotes transpose, and H denotes Hermitian transpose. $\mathbb{E}[\cdot]$ is the mathematical expectation of the argument (\cdot). When \mathbf{X} is a square matrix, $\det(\mathbf{X})$ denotes the determinant of \mathbf{X} . For each complex vector $\mathbf{x} \in \mathbb{C}^N$, $\text{diag}\{\mathbf{x}\}$ is the $N \times N$ diagonal matrix whose diagonal entries are the elements of \mathbf{x} . \mathbf{I}_N is the $N \times N$ identity matrix, and $\mathbf{0}_{N \times Q}$ denotes an all zero $N \times Q$ matrix. \otimes is the Kronecker product, and $j = \sqrt{-1}$.

The following sections of the paper are organized as follows. In Section II, we provide a description of the MIMO ARQ system model and introduce some assumptions considered in this paper. In Section III, we derive the structure of the optimal MAP turbo combining scheme, and analyze the outage probability and the outage-based power loss of the considered MIMO ARQ system. Section IV details the structure of the proposed combining schemes and discusses complexity issues. Numerical results are provided in Section V. The paper is concluded in Section VI.

II. SYSTEM MODEL AND ASSUMPTIONS

We consider a multi-antenna link operating over a frequency selective fading channel and using an ARQ protocol at the upper layer. The transmitter and the receiver are equipped with N_T transmit and N_R receive antennas, respectively. The MIMO-ISI channel is composed of L taps (index $l = 0, \dots, L-1$). Each data stream is encoded with the aid of a ρ -rate channel encoder, interleaved using a semi-random interleaver Π , then modulated and space-time multiplexed over the N_T transmit antennas. This presents a ST-BICM coding scheme. The mapping function that relates each set of M coded and interleaved bits $b_{1,t,i}, \dots, b_{M,t,i}$ to a symbol $s_{t,i}$ that belongs to the constellation set \mathcal{S} is denoted $\varphi : \{0, 1\}^M \rightarrow \mathcal{S}$, where $t = 1, \dots, N_T$, and $i = 0, \dots, T-1$ are the transmit antenna and the channel use indices, respectively, and $M = \log_2 |\mathcal{S}|$. The $N_T \times T$ symbol matrix corresponding to the entire frame is denoted

$$\mathbf{S} \triangleq [\mathbf{s}_0, \dots, \mathbf{s}_{T-1}] \in \mathcal{S}^{N_T \times T}, \quad (1)$$

$$\mathbf{s}_i \triangleq [s_{1,i}, \dots, s_{N_T,i}]^\top \in \mathcal{S}^{N_T} \quad (2)$$

is the vector of transmitted symbols at time instant i . The rate of this transmission scheme is therefore $R = \rho M N_T$. When the transmitter receives a negative acknowledgment (NACK) message due to an erroneously decoded block, subsequent transmission rounds occur until the packet is correctly received or a preset maximum number of rounds, i.e., ARQ delay, K is reached. The round index is denoted $k = 1, \dots, K$. Reception of a positive acknowledgment (ACK) indicates a successful decoding and the transmitter moves on to the next block message. We suppose that the signaling channel carrying the one bit ACK/NACK feedback message is error free. In addition, we assume perfect packet error detection (typically, using a cyclic redundancy check (CRC) code). Therefore, a decoding failure corresponds to an erroneous decoding outcome after K rounds. We focus on Chase-type ARQ mechanisms, i.e., the symbol matrix \mathbf{S} is completely retransmitted. Both puncturing and mapping diversity, i.e., optimization of the mapping function over transmission rounds, are not investigated in this paper, and are left for future contributions. We use a zero padding (ZP) sequence $\mathbf{0}_{N_T \times L}$ to prevent inter-block interference (IBI). The ST-BICM scheme with ARQ is depicted in Fig. 1. a. The MIMO-ISI channel is assumed to be quasi-static block fading, i.e., constant over a frame that spans T channel use and independently changes from round to round. This scenario corresponds to the so-called short-term static channel case where ARQ transmission rounds see different and independent channel realizations [24]. The long-

term static channel corresponds to the case where the channel is constant over all rounds related to the transmission of the same information block, i.e., $\mathbf{H}_l^{(k)} = \mathbf{H}_l \forall k \in \{1, \dots, K\}$. Note that in orthogonal frequency division multiplexing (OFDM) broadband wireless systems, the ARQ channel is rather short-term static because frequency hopping is used to mitigate ISI. While in time division multiplexing (TDM)-based systems, the channel dynamic can be either short or long-term static depending on the Doppler spread. In addition, we suppose that the channel profile, i.e., number of paths and power distribution, is identical for at least K consecutive rounds. This is a reasonable assumption for slowly time-varying wireless fading channels because the channel profile dynamic is mainly related to the shadowing effect. At the k th round, the channel impulse response is represented by the $N_R \times N_T$ complex matrices $\mathbf{H}_0^{(k)}, \dots, \mathbf{H}_{L-1}^{(k)}$ corresponding respectively to taps $0, \dots, L-1$, and whose entries are zero-mean circularly symmetric Gaussian $h_{r,t,l}^{(k)} \sim \mathcal{CN}(0, \sigma_l^2)$, where $h_{r,t,l}^{(k)}$ denotes the (r, t) th element of matrix $\mathbf{H}_l^{(k)}$. The total energy of taps $l = 0, \dots, L-1$ is normalized to one, i.e., $\sum_{l=0}^{L-1} \sigma_l^2 = 1$. Therefore, the channel energy per receive antenna $r = 1, \dots, N_R$ is

$$\sum_{l=0}^{L-1} \sum_{t=1}^{N_T} \mathbb{E} \left[\left| h_{r,t,l}^{(k)} \right|^2 \right] = N_T. \quad (3)$$

We suppose that no channel knowledge is available at the transmitter. Equal power transmission turns out to be the best power allocation strategy. In addition, under the assumption of infinitely deep interleaving, and by normalizing the symbol energy to one, we get

$$\mathbb{E} [\mathbf{s}_i \mathbf{s}_i^H] = \mathbf{I}_{N_T}. \quad (4)$$

At the k th round, after down-conversion and sampling at the symbol rate, the baseband complex received signal on the r th antenna and at time instant i is

$$y_{r,i}^{(k)} = \sum_{l=0}^{L-1} \sum_{t=1}^{N_T} h_{r,t,l}^{(k)} s_{t,i-l} + n_{r,i}^{(k)}, \quad (5)$$

where $n_{r,i}^{(k)}$ is the noise on the r th antenna, and $\mathbf{n}_i^{(k)} \triangleq [n_{1,i}^{(k)}, \dots, n_{N_R,i}^{(k)}]^\top \sim \mathcal{CN}(\mathbf{0}_{N_R \times 1}, \sigma^2 \mathbf{I}_{N_R})$.

III. OPTIMAL TURBO PACKET COMBINING AND OUTAGE ANALYSIS

In this section, we provide a brief description of the structure of the turbo packet combining concept we propose in this paper, and introduce the optimal MAP turbo combiner. We also investigate the outage probability and the outage-based transmit power loss then provide a numerical analysis.

A. General Architecture and Optimal Turbo Combining

The turbo packet combining strategies we propose in this paper allow decoding of a data packet transmitted over multiple MIMO-ISI channels in an iterative (turbo) fashion through the exchange of extrinsic information between the soft packet combiner and the SISO decoder. The main difference with conventional LLR-based packet combining is that multiple transmissions are combined before the computation of the soft information using a SISO packet combiner, while in LLR-level combining the soft outputs of different ARQ rounds are simply added together before channel decoding. The general block diagram is depicted in Fig. 1. b. Let N denote the number of turbo iterations performed between the combiner and the decoder at the k th round (index $n = 1, \dots, N$), and

$$\begin{aligned} \boldsymbol{\phi}_{t,i,n}^e \triangleq [\phi_{1,t,i,n}^e, \dots, \phi_{M,t,i,n}^e]^\top \in \mathbb{R}^M, \\ (t, i) \in \{1, \dots, N_T\} \times \{0, \dots, T-1\} \end{aligned} \quad (6)$$

denote the vectors of extrinsic log-likelihood ratio (LLR) values generated by the soft combiner at iteration n . $\phi_{m,t,i,n}^e$ is the extrinsic information related to coded and interleaved bit $b_{m,t,i}$ at turbo iteration n . We similarly define *a priori* vectors

$$\boldsymbol{\phi}_{t,i,n}^a \triangleq [\phi_{1,t,i,n}^a, \dots, \phi_{M,t,i,n}^a]^\top \in \mathbb{R}^M,$$

available at the input of the soft combiner at iteration n . For the sake of notation simplicity, the round index is not used in LLRs. At the n th iteration of the k th round, the soft packet combiner makes use of the $N_T T$ *a priori* vectors $\phi_{1,0,n}^a, \dots, \phi_{N_T, T-1, n}^a$ and received signals to combine transmissions corresponding to rounds $1, \dots, k$, and compute extrinsic vectors $\phi_{1,0,n}^e, \dots, \phi_{N_T, T-1, n}^e$. These extrinsic LLRs are de-interleaved and sent to the SISO decoder to compute *a posteriori* information about useful bits and extrinsic LLRs about coded bits. The generated extrinsic information is then interleaved and fed back to the soft combiner to serve as *a priori* information $\phi_{1,0,n+1}^a, \dots, \phi_{N_T, T-1, n+1}^a$ at next iteration $n+1$. Note that the feedback of a NACK message does not necessarily mean that all information bits are erroneous. Therefore, extrinsic information generated by the SISO decoder during the last iteration of round $k-1$ can be used as *a priori* information at the first iteration of round k .³

Now, let us focus on the optimal soft packet combiner that allows the exploitation of all diversities, i.e., space, time, multipath, and retransmission, present in the MIMO-ISI ARQ channel to iteratively compute extrinsic information about coded and interleaved bits at the k th round. First, let us introduce

³Generally speaking, iterative processing at round k will help correct information bits erroneously decoded during round $k-1$, while the LLR values of other bits remain the same.

$$\mathbf{y}_i^{(k)} \triangleq \left[y_{1,i}^{(k)} \cdots y_{N_R,i}^{(k)} \right]^\top \quad (7)$$

that groups the signals received at time instant i of the k th round (5). We assume that the signals received at rounds $1, \dots, k$ (i.e., $\mathbf{y}_0^{(1)}, \dots, \mathbf{y}_{T-1}^{(k)}$) and their corresponding channel responses (i.e., $\mathbf{H}_0^{(1)}, \dots, \mathbf{H}_{L-1}^{(k)}$) are available at the receiver. Note that this assumption may present an important limiting factor (in addition to the computational complexity) for implementing the optimal turbo combiner, since all signals and channel responses have to be stored in the receiver. The low-complexity signal-level turbo combining strategy we introduce in Section IV relaxes this condition by using two recursions for keeping signals and channel matrices of previous rounds. At the n th turbo iteration of round k , the optimal soft combiner computes extrinsic LLR about coded and interleaved bit $b_{m,t,i}$ according to the MAP criterion as,

$$\phi_{m,t,i,n}^e = \log \frac{\Pr \left\{ \mathbf{y}^{(k)} \mid b_{m,t,i} = 1; \mathbf{H}_0^{(1)}, \dots, \mathbf{H}_{L-1}^{(k)}, \text{ a priori LLRs} \right\}}{\Pr \left\{ \mathbf{y}^{(k)} \mid b_{m,t,i} = 0; \mathbf{H}_0^{(1)}, \dots, \mathbf{H}_{L-1}^{(k)}, \text{ a priori LLRs} \right\}}, \quad (8)$$

where

$$\mathbf{y}^{(k)} \triangleq \left[\mathbf{y}_{T-1}^{(1)\top}, \dots, \mathbf{y}_{T-1}^{(k)\top}, \dots, \mathbf{y}_0^{(1)\top}, \dots, \mathbf{y}_0^{(k)\top} \right]^\top \in \mathbb{C}^{kN_R T}. \quad (9)$$

Note that this vector representation is of a great importance because it allows us to view each transmission round as a source of an additional set of virtual N_R receive antennas. Therefore, ARQ diversity translates into space diversity (i.e., virtual receive antennas). The signal vector $\mathbf{y}^{(k)}$ corresponding to the transmission of matrix \mathbf{S} over k MIMO-ISI channels can be expressed as,

$$\mathbf{y}^{(k)} = \mathbf{H}^{(k)} \mathbf{s} + \mathbf{n}^{(k)}, \quad (10)$$

where $\mathbf{H}^{(k)}$ is a block Toeplitz matrix, and

$$\mathbf{H}^{(k)} \triangleq \begin{bmatrix} \boxed{\begin{matrix} \mathbf{H}_0^{(1)} \\ \vdots \\ \mathbf{H}_0^{(k)} \end{matrix}} & \cdots & \boxed{\begin{matrix} \mathbf{H}_{L-1}^{(1)} \\ \vdots \\ \mathbf{H}_{L-1}^{(k)} \end{matrix}} & & \\ & & \ddots & & \\ & & & \boxed{\begin{matrix} \mathbf{H}_0^{(1)} \\ \vdots \\ \mathbf{H}_0^{(k)} \end{matrix}} & \cdots & \boxed{\begin{matrix} \mathbf{H}_{L-1}^{(1)} \\ \vdots \\ \mathbf{H}_{L-1}^{(k)} \end{matrix}} \end{bmatrix}_{kN_R T \times N_T T}, \quad (11)$$

$$\mathbf{s} \triangleq [\mathbf{s}_{T-1}^\top, \dots, \mathbf{s}_0^\top]^\top \in \mathcal{S}^{N_T T}, \quad (12)$$

$$\mathbf{n}^{(k)} \triangleq [\mathbf{n}_{T-1}^{(1)\top}, \dots, \mathbf{n}_{T-1}^{(k)\top}, \dots, \mathbf{n}_0^{(1)\top}, \dots, \mathbf{n}_0^{(k)\top}]^\top \in \mathbb{C}^{kN_R T}. \quad (13)$$

With respect to (10), extrinsic LLR given by (8) can now be expressed as,

$$\phi_{m,t,i,n}^e = \log \frac{\sum_{\mathbf{s} \in \mathcal{S}_{m,t,i}^1} \exp \left\{ -\frac{1}{2\sigma^2} \|\mathbf{y}^{(k)} - \mathbf{H}^{(k)}\mathbf{s}\|^2 + \sum_{(m',t',i') \neq (m,t,i)} \varphi_{m'}^{-1}(x_{t',i'}) \phi_{m',t',i',n}^a \right\}}{\sum_{\mathbf{s} \in \mathcal{S}_{m,t,i}^0} \exp \left\{ -\frac{1}{2\sigma^2} \|\mathbf{y}^{(k)} - \mathbf{H}^{(k)}\mathbf{s}\|^2 + \sum_{(m',t',i') \neq (m,t,i)} \varphi_{m'}^{-1}(x_{t',i'}) \phi_{m',t',i',n}^a \right\}}, \quad (14)$$

where $\mathcal{S}_{m,t,i}^b = \{\mathbf{s} \in \mathcal{S}^{N_T T} \mid \varphi_m^{-1}(s_{t,i}) = b\}$, $b = 0, 1$.

B. Outage Probability and Outage-Based Transmit Power Loss

It is well known that for non-ergodic channels, i.e., block fading quasi-static channels, outage-probability P_{out} [39]–[41] is regarded as a meaningful tool for performance evaluation because it provides a lower bound on the block error rate (BLER) [42, p. 187]. The outage probability is defined as the probability that the mutual information, as a function of the channel realization and the average signal to noise ratio (SNR) γ per receive antenna, is below the transmission rate R . Mutual information rates of quasi-static frequency selective fading MIMO channel have been investigated in [43], [44].

1) *Outage Probability* : To derive the outage probability of the considered MIMO ARQ system, we use the *renewal theory* [45] which was first used by Zorzi and Rao to analyze the performance of ARQ protocols [46]. Recently, it was also used by [24], [47] to evaluate the performance of ARQ systems operating over wireless flat fading channels. Let \mathcal{A}_k denote the event that an ACK message is fed back at round k , and \mathcal{E}_k the event that the ARQ system is in outage at round k . Under the assumption of perfect packet error detection and error-free ACK/NACK feedback, and by applying the *renewal theory*, the outage probability for a given SNR γ and target rate R is given as

$$P_{out}^R(\gamma) = \Pr \{ \mathcal{E}_K, \bar{\mathcal{A}}_1, \dots, \bar{\mathcal{A}}_{K-1} \}. \quad (15)$$

Note that a Chase-type ARQ mechanism with an ARQ delay K can be viewed as a repetition coding scheme where K parallel sub-channels are used to transmit one symbol message [42, p. 194]. Therefore, (15) can be expressed as

$$P_{out}^R(\gamma) = \Pr \left\{ \frac{1}{K} I(\mathbf{s}; \mathbf{y}^{(K)} \mid \mathbf{H}^{(K)}, \gamma) < R, \bar{\mathcal{A}}_1, \dots, \bar{\mathcal{A}}_{K-1} \right\}. \quad (16)$$

The virtual $KN_R \times N_T$ MIMO-ISI communication model at the K th ARQ round is

$$\begin{bmatrix} \mathbf{y}_i^{(1)} \\ \vdots \\ \mathbf{y}_i^{(K)} \end{bmatrix} = \sum_{l=0}^{L-1} \begin{bmatrix} \mathbf{H}_l^{(1)} \\ \vdots \\ \mathbf{H}_l^{(K)} \end{bmatrix} \mathbf{s}_{i-l} + \begin{bmatrix} \mathbf{n}_i^{(1)} \\ \vdots \\ \mathbf{n}_i^{(K)} \end{bmatrix},$$

and the mutual information $I(\mathbf{s}; \mathbf{y}^{(K)} | \mathbf{H}^{(K)}, \gamma)$ in (16) can therefore be expressed in the case of i.i.d circularly symmetric complex Gaussian channel inputs as in [43], i.e.,

$$I(\mathbf{s}; \mathbf{y}^{(K)} | \mathbf{H}^{(K)}, \gamma) = \frac{1}{T} \sum_{i=0}^{T-1} \log_2 \left(\det \left(\mathbf{I}_{KN_R} + \frac{\gamma}{N_T} \mathbf{\Lambda}_i^{(K)} \mathbf{\Lambda}_i^{(K)H} \right) \right), \quad (17)$$

where $\mathbf{\Lambda}_i^{(K)}$ is the discrete Fourier transform (DFT) of the K th round $KN_R \times N_T$ virtual MIMO-ISI channel at the i th frequency bin, i.e.,

$$\mathbf{\Lambda}_i^{(K)} = \sum_{l=0}^{L-1} \begin{bmatrix} \mathbf{H}_l^{(1)} \\ \vdots \\ \mathbf{H}_l^{(K)} \end{bmatrix} \exp \left\{ -j \frac{2\pi}{T} il \right\}. \quad (18)$$

2) *Outage-Based Transmit Power Loss*: To compare the outage probability performance of different ARQ configurations that operate at the same rate R but use different ARQ delays, we consider a short-term power constraint scenario where the same power Γ is used for all transmission rounds, i.e., the k th round transmit power is $\Gamma_k = \Gamma \forall k$. We evaluate the power loss incurred by multiple transmission rounds due to link outage. Note that system performance can be improved when a power control algorithm is jointly used with packet combining (typically, a long-term power constraint scenario), but this is beyond the scope of this paper. The average SNR present in the outage expression (16) is therefore given as

$$\gamma = \Gamma \frac{N_T}{\sigma^2}. \quad (19)$$

Let p count the number of information blocks, $q = 1, \dots, p$ denote the block index, and \mathcal{T}_q the number of rounds used for transmitting block q . Therefore, for a given ARQ delay K , average SNR γ , and rate R , the average transmit power is

$$\begin{aligned} \Gamma_{avg} &= \lim_{p \rightarrow \infty} \frac{\sum_{q=1}^p \mathcal{T}_q}{p} \Gamma \\ &= \mathbb{E}[\mathcal{T} | K, \gamma, R] \Gamma. \end{aligned} \quad (20)$$

This indicates that an ARQ protocol with an ARQ delay K and operating with rate R at average SNR γ incurs an *outage-based transmit power loss* of $10 \log_{10}(\mathbb{E}[\mathcal{T} | K, \gamma, R])$ compared with an ARQ

with $K = 1$ round (i.e., no retransmissions).

C. Outage Analysis

In the following subsection we investigate, using simulations, both the outage probability and the outage-based transmit power loss for some MIMO-ISI ARQ configurations. This will serve as a theoretical foundation for the performance evaluation of turbo packet combiners which we will introduce in the next subsection. Let us consider a MIMO-ISI channel with $L = 2$ taps and equally distributed power, i.e., $\sigma_0^2 = \sigma_1^2 = \frac{1}{2}$. We use Monte Carlo simulations to evaluate the outage probability (16) of the considered ARQ system. We choose $T = 256$ channel use. At each round k , a $N_R \times N_T$ MIMO-ISI channel $\mathbf{H}_0^{(k)}$ and $\mathbf{H}_1^{(k)}$ is generated, and the mutual achievable rate after k rounds is computed using (17). If the target rate R is not reached and $k < K$, the system moves on to the next round $k + 1$. The ARQ process is stopped and another is started, either because of system outage (i.e., the achievable rate after K rounds is below R) or non-outage (i.e., the achievable rate is greater than R after round $k \leq K$).

In Fig. 2. a, we plot the outage probability as a function of the ARQ delay K for the two path MIMO-ISI channel with two transmit and two receive antennas ($N_T = N_R = 2$), and a target rate $R = 2$. The ARQ diversity gain, due to the short-term static channel dynamic, clearly appears when $K = 2$. For instance, a gain of approximately 1dB is achieved at $5 * 10^{-3}$ outage compared with the case of $K = 1$ (i.e., no ARQ). When $K = 3$, the outage probability performance is similar to that of $K = 2$. Fig. 2. b, shows the outage curves for $N_T = 4$ and $N_R = 2$ with a target rate $R = 4$. We notice that as in the previous configuration, $K = 2$ and $K = 3$ have the same outage performance, while the overall diversity gain is more important than that corresponding to $N_T = N_R = 2$ (i.e., outage curve slopes are steeper than those of the first configuration). Note that the stacking procedure (9) relative to the optimal MAP-based turbo combiner creates kN_R virtual receive antennas after k rounds, but not all these virtual antennas will translate into a receive diversity, because the target rate R has to be maintained as it can be seen from the expression of the achievable information rate in (16). This justifies the outage performance saturation after $K = 2$. This issue was recently addressed in [24] for flat fading MIMO ARQ channels, and it was demonstrated that the diversity gain does not linearly increase with increase of the ARQ delay K .⁴

⁴In [24, Theorem 2], the authors demonstrated that for the case of a short-term static flat fading MIMO ARQ channel, the optimal diversity gain is $d^*(r_e, K) = Kf\left(\frac{r_e}{K}\right)$ $0 \leq r_e < \min\{N_T, N_R\}$, where r_e is the multiplexing gain and f is the piecewise linear function connecting the points $(x, (N_T - x)(N_R - x))$ for $x = 0, \dots, \min\{N_T, N_R\}$.

In Fig. 3, we present the outage-based transmit power loss for the considered MIMO configurations. We observe that in the region of low SNR, the outage-based loss is significant for both $K = 2$ and $K = 3$. When the outage probability is below $< 10^{-2}$ (the region corresponding to FER values typically required in practical systems), the transmit power loss is below 0.25dB. This indicates that in the corresponding SNR region, blocks are mainly error-free during the first transmission, and only a small number of frames require additional rounds.

Motivated by these theoretical results, in the next section we design a class of reduced complexity MMSE-based turbo combiners.

IV. LOW COMPLEXITY MMSE-BASED TURBO PACKET COMBINING

It is obvious that the complexity of the MAP turbo combining technique presented in Subsection III-A is exponential in the number of transmit antennas and channel use. In this section, we introduce two low-complexity turbo packet combining techniques using the MMSE criterion, and analyze their computational cost and memory requirements.

A. Signal-Level Turbo Combining

Let us recall the MAP turbo combiner block communication model (10) with a block length $\kappa = \kappa_1 + \kappa_2 + 1 \ll T$, where κ_1 and κ_2 are the lengths of the forward and backward filters, respectively. The corresponding $kN_R\kappa \times N_T(\kappa + L - 1)$ sliding-window (around channel use i) communication model after k rounds is similar to (10), and is given as,

$$\underline{\underline{\mathbf{y}}}_i^{(k)} = \underline{\underline{\mathbf{H}}}_i^{(k)} \underline{\underline{\mathbf{s}}}_i + \underline{\underline{\mathbf{n}}}_i^{(k)}, \quad (21)$$

where

$$\underline{\underline{\mathbf{y}}}_i^{(k)} \triangleq \left[\mathbf{y}_{i+\kappa_1}^{(1)\top}, \dots, \mathbf{y}_{i+\kappa_1}^{(k)\top}, \dots, \mathbf{y}_{i-\kappa_2}^{(1)\top}, \dots, \mathbf{y}_{i-\kappa_2}^{(k)\top} \right]^\top \in \mathbb{C}^{kN_R\kappa}, \quad (22)$$

$$\underline{\underline{\mathbf{n}}}_i^{(k)} \triangleq \left[\mathbf{n}_{i+\kappa_1}^{(1)\top}, \dots, \mathbf{n}_{i+\kappa_1}^{(k)\top}, \dots, \mathbf{n}_{i-\kappa_2}^{(1)\top}, \dots, \mathbf{n}_{i-\kappa_2}^{(k)\top} \right]^\top \in \mathbb{C}^{kN_R\kappa}, \quad (23)$$

$$\underline{\underline{\mathbf{s}}}_i \triangleq \left[\mathbf{s}_{i+\kappa_1}^\top, \dots, \mathbf{s}_{i-\kappa_2-L+1}^\top \right]^\top \in \mathcal{S}^{N_T(\kappa+L-1)}, \quad (24)$$

and $\underline{\underline{\mathbf{H}}}_i^{(k)} \in \mathbb{C}^{kN_R\kappa \times N_T(\kappa+L-1)}$ is defined similarly to (11).

To compute, at the n th iteration extrinsic information $\phi_{m,t,i,n}^e$ about bit $b_{m,t,i}$, using signals received during rounds $1, \dots, k$, we jointly (over all rounds) cancel soft ISI in a parallel interference cancellation (PIC) fashion. This yields a soft ISI-free signal vector $\tilde{\underline{\underline{\mathbf{y}}}}_{i|(t,n)}^{(k)} \in \mathbb{C}^{kN_R\kappa}$ expressed as,

$$\tilde{\underline{\underline{\mathbf{y}}}}_{i|(t,n)}^{(k)} \triangleq \underline{\underline{\mathbf{y}}}_i^{(k)} - \underline{\underline{\mathbf{H}}}_i^{(k)} \tilde{\underline{\underline{\mathbf{s}}}}_{i|(t,n)}, \quad (25)$$

where $\tilde{\underline{\mathbf{s}}}_i|_{(t,n)}$ is the conditional average of symbol vector $\underline{\mathbf{s}}_i$ with zero at the $(\kappa_1 N_T + t)$ th position,

$$\tilde{\underline{\mathbf{s}}}_i|_{(t,n)} \triangleq \mathbb{E} [\underline{\mathbf{s}}_i \mid \phi_{m',t',i',n}^a : (t', i') \neq (t, i)]. \quad (26)$$

The components of $\tilde{\underline{\mathbf{y}}}_{\underline{\mathbf{i}}|_{(t,n)}}^{(k)}$ are then combined using an unconditional MMSE filter to produce the scalar input $\xi_{t,i,n}^{(k)}$ for the soft demapper. Applying the matrix inversion lemma [48] similarly to [37, eq. 6], we can write the output of the unconditional MMSE filter as,

$$\xi_{t,i,n}^{(k)} = \zeta_{t,n}^{(k)} \mathbf{e}_t^\top \underline{\underline{\mathbf{H}}}^{(k)H} \mathbf{A}_n^{(k)-1} \tilde{\underline{\mathbf{y}}}_{\underline{\mathbf{i}}|_{(t,n)}}^{(k)}, \quad (27)$$

where

$$\mathbf{A}_n^{(k)} = \underline{\underline{\mathbf{H}}}^{(k)} \underline{\underline{\mathbf{\Xi}}}_n \underline{\underline{\mathbf{H}}}^{(k)H} + \sigma^2 \mathbf{I}_{kN_R\kappa} \in \mathbb{C}^{kN_R\kappa \times kN_R\kappa}, \quad (28)$$

$$\underline{\underline{\mathbf{\Xi}}}_n = \mathbf{I}_{\kappa+L-1} \otimes \tilde{\underline{\underline{\mathbf{\Xi}}}}_n \in \mathbb{C}^{N_T(\kappa+L-1) \times N_T(\kappa+L-1)}, \quad (29)$$

$$\tilde{\underline{\underline{\mathbf{\Xi}}}}_n \triangleq \text{diag} \{ \tilde{\sigma}_{1,n}^2, \dots, \tilde{\sigma}_{N_T,n}^2 \}, \quad (30)$$

$$\mathbf{e}_t \triangleq \begin{bmatrix} 0, \dots, 0, 1, 0, \dots, 0 \\ \underbrace{\hspace{10em}}_{\kappa_1 N_T + t - 1} \quad \underbrace{\hspace{10em}}_{(\kappa_2 + L) N_T - t} \end{bmatrix}^\top \in \mathbb{C}^{N_T(\kappa+L-1)}, \quad (31)$$

$$\zeta_{t,n}^{(k)} = \left(1 + (1 - \tilde{\sigma}_{t,n}^2) \mathbf{e}_t^\top \underline{\underline{\mathbf{H}}}^{(k)H} \mathbf{A}_n^{(k)-1} \underline{\underline{\mathbf{H}}}^{(k)} \mathbf{e}_t \right)^{-1}, \quad (32)$$

and $\tilde{\sigma}_{t,n}^2$ is the unconditional variance at iteration n of symbols $\{s_{t,i}\}_{i=0}^{T-1}$ transmitted over antenna t ,

$$\tilde{\sigma}_{t,n}^2 = \frac{1}{T} \sum_{i=0}^{T-1} \mathbb{E} [|s_{t,i} - \tilde{s}_{t,i,n}|^2 \mid \phi_{m,t,i,n}^a : m = 1, \dots, M], \quad (33)$$

$$\tilde{s}_{t,i,n} \triangleq \mathbb{E} [s_{t,i} \mid \phi_{m,t,i,n}^a : m = 1, \dots, M] \quad (34)$$

is the conditional average of symbol $s_{t,i}$ at iteration n .

Combining the soft PIC (25) and unconditional MMSE filtering (27) steps, and after some matrix manipulations, we can write the soft demapper input $\xi_{t,i,n}^{(k)}$ as,

$$\xi_{t,i,n}^{(k)} = \mathbf{F}_{t,n}^{(k)} \mathbf{z}_i^{(k)} - \mathbf{B}_{t,n}^{(k)} \tilde{\underline{\mathbf{s}}}_i|_{(t,n)}. \quad (35)$$

$\mathbf{F}_{t,n}^{(k)}$ and $\mathbf{B}_{t,n}^{(k)}$ are the forward and backward filters corresponding to antenna t at the n th iteration,

$$\mathbf{F}_{t,n}^{(k)} = (\sigma^2 + (1 - \tilde{\sigma}_{t,n}^2) \mathbf{e}_t^\top \underline{\underline{\mathbf{\Lambda}}}_n^{(k)} \underline{\underline{\mathbf{\Upsilon}}}_n^{(k)} \mathbf{e}_t)^{-1} \mathbf{e}_t^\top \underline{\underline{\mathbf{\Lambda}}}_n^{(k)}, \quad (36)$$

$$\mathbf{B}_{t,n}^{(k)} = \mathbf{F}_{t,n}^{(k)} \underline{\underline{\mathbf{\Upsilon}}}_n^{(k)}. \quad (37)$$

$\underline{\underline{\mathbf{\Lambda}}}_n^{(k)}$, $\underline{\underline{\mathbf{z}}}_i^{(k)}$, and $\underline{\underline{\mathbf{\Upsilon}}}_n^{(k)}$ are given as

$$\Lambda_n^{(k)} = \mathbf{I}_{N_T(\kappa+L-1)} - \Upsilon^{(k)} (\Upsilon^{(k)} + \sigma^2 \Xi_n^{-1})^{-1}, \quad (38)$$

$$\begin{cases} \underline{\mathbf{z}}_i^{(k)} &= \underline{\mathbf{z}}_i^{(k-1)} + \underline{\mathbf{H}}^{(k)H} \underline{\mathbf{y}}_i^{(k)} \\ \underline{\mathbf{z}}_i^{(0)} &= \mathbf{0}_{N_T(\kappa+L-1) \times 1}, \end{cases} \quad (39)$$

$$\begin{cases} \Upsilon^{(k)} &= \Upsilon^{(k-1)} + \underline{\mathbf{H}}^{(k)H} \underline{\mathbf{H}}^{(k)} \\ \Upsilon^{(0)} &= \mathbf{0}_{N_T(\kappa+L-1) \times N_T(\kappa+L-1)}. \end{cases} \quad (40)$$

$\underline{\mathbf{H}}^{(k)}$ and $\underline{\mathbf{y}}_i^{(k)}$ are the block Toeplitz matrix and signal output of the sliding-window communication model at round k , respectively, and are given as,

$$\underline{\mathbf{H}}^{(k)} \triangleq \begin{bmatrix} \mathbf{H}_0^{(k)} & \cdots & \mathbf{H}_{L-1}^{(k)} & & \\ & \ddots & & \ddots & \\ & & \mathbf{H}_0^{(k)} & \cdots & \mathbf{H}_{L-1}^{(k)} \end{bmatrix}_{N_R \kappa \times N_T(\kappa+L-1)}, \quad (41)$$

$$\underline{\mathbf{y}}_i^{(k)} \triangleq [\mathbf{y}_{i+\kappa_1}^{(k)\top}, \dots, \mathbf{y}_{i-\kappa_2}^{(k)\top}]^\top \in \mathbb{C}^{N_R \kappa}. \quad (42)$$

$$\underline{\mathbf{y}}_i^{(k)} = \underline{\mathbf{H}}^{(k)} \underline{\mathbf{s}}_i + \underline{\mathbf{n}}_i^{(k)}, \quad (43)$$

$$\underline{\mathbf{n}}_i^{(k)} \triangleq [\mathbf{n}_{i+\kappa_1}^{(k)\top}, \dots, \mathbf{n}_{i-\kappa_2}^{(k)\top}]^\top \in \mathbb{C}^{N_R \kappa}. \quad (44)$$

Recursions (39) and (40) are easily obtained by invoking (22) and the general structure (11). Details about the derivation of (35) are omitted because of space limitation. Assuming the conditional soft demapper input is Gaussian, i.e., $(\xi_{t,i,n}^{(k)} | s_{t,i}) \sim \mathcal{N}(\alpha_{t,n}^{(k)}, \delta_{t,n}^{(k)2})$, extrinsic information $\phi_{m,t,i,n}^{e[Sig]}$ can be computed as,

$$\phi_{m,t,i,n}^{e[Sig]} = \log \frac{\sum_{s \in \mathcal{S}_m^1} \exp \left\{ -\frac{1}{2\delta_{t,n}^{(k)2}} \left| \xi_{t,i,n}^{(k)} - \alpha_{t,n}^{(k)} s \right|^2 + \sum_{m' \neq m} \varphi_{m'}^{-1}(s) \phi_{m',t,i,n}^a \right\}}{\sum_{s \in \mathcal{S}_m^0} \exp \left\{ -\frac{1}{2\delta_{t,n}^{(k)2}} \left| \xi_{t,i,n}^{(k)} - \alpha_{t,n}^{(k)} s \right|^2 + \sum_{m' \neq m} \varphi_{m'}^{-1}(s) \phi_{m',t,i,n}^a \right\}}, \quad (45)$$

where

$$\begin{cases} \alpha_{t,n}^{(k)} &= \mathbf{B}_{t,n}^{(k)} \mathbf{e}_t \\ \delta_{t,n}^{(k)2} &= \left(1 - \alpha_{t,n}^{(k)}\right) \alpha_{t,n}^{(k)}, \end{cases} \quad (46)$$

and $\mathcal{S}_m^b = \{s \in \mathcal{S} \mid \varphi_m^{-1}(s) = b\}$. The signal-level combining algorithm is summarized in Table I.

Note that the forward-backward filtering structure (35) together with recursions (39) and (40) present the core part of the proposed algorithm, and allow a reduced computational complexity and an optimized implementation. Indeed, equations (39) and (40) allow to use at each ARQ round all signals and channel matrices corresponding to previous rounds $k - 1, \dots, 1$ without being required to be explicitly stored in the receiver. This is performed in a recursive fashion using modified versions of the sliding window input and matrix (i.e., $\underline{\mathbf{H}}^{(k)H} \underline{\mathbf{y}}_i^{(k)}$ and $\underline{\mathbf{H}}^{(k)H} \underline{\mathbf{H}}^{(k)}$, respectively) at round k .

B. Symbol-Level Turbo Combining

In this combining scheme, we propose to perform equalization separately for each round k based on the communication model (43). Then, soft combining is conducted at the level of unconditional MMSE filter outputs: The output at iteration n of round k is combined with the outputs obtained at the last iteration of previous rounds $k - 1, \dots, 1$. As in the previous subsection, let $\check{\xi}_{t,i,n}^{(k)}$ denote the filter output ⁵ at iteration n of round k , and $\left(\check{\xi}_{t,i,n}^{(k)} \mid s_{t,i} \right) \sim \mathcal{N} \left(\check{\alpha}_{t,n}^{(k)}, \check{\delta}_{t,n}^{(k)2} \right)$. The soft demapper, which has a vector input in this case, computes extrinsic information $\phi_{m,t,i,n}^{e[\text{Symb}]}$ as,

$$\phi_{m,t,i,n}^{e[\text{Symb}]} = \log \frac{\sum_{s \in \mathcal{S}_m^1} \exp \left\{ -\frac{1}{2} \left(\check{\xi}_{t,i,n}^{(k)} - s \check{\alpha}_{t,n}^{(k)} \right)^H \Delta_{t,n}^{(k)-1} \left(\check{\xi}_{t,i,n}^{(k)} - s \check{\alpha}_{t,n}^{(k)} \right) + \sum_{m' \neq m} \varphi_{m'}^{-1}(s) \phi_{m',t,i,n}^a \right\}}{\sum_{s \in \mathcal{S}_m^0} \exp \left\{ -\frac{1}{2} \left(\check{\xi}_{t,i,n}^{(k)} - s \check{\alpha}_{t,n}^{(k)} \right)^H \Delta_{t,n}^{(k)-1} \left(\check{\xi}_{t,i,n}^{(k)} - s \check{\alpha}_{t,n}^{(k)} \right) + \sum_{m' \neq m} \varphi_{m'}^{-1}(s) \phi_{m',t,i,n}^a \right\}},$$

where

$$\check{\xi}_{t,i,n}^{(k)} \triangleq \left[\check{\xi}_{t,i,N}^{(1)}, \dots, \check{\xi}_{t,i,N}^{(k-1)}, \check{\xi}_{t,i,n}^{(k)} \right]^T \in \mathbb{C}^k, \quad (47)$$

$$\check{\alpha}_{t,n}^{(k)} \triangleq \left[\check{\alpha}_{t,N}^{(1)}, \dots, \check{\alpha}_{t,N}^{(k-1)}, \check{\alpha}_{t,n}^{(k)} \right]^T \in \mathbb{C}^k, \quad (49)$$

and $\Delta_{t,n}^{(k)}$ is the covariance matrix of $\left(\check{\xi}_{t,i,n}^{(k)} \mid s_{t,i} \right)$ which can be approximated as (assuming residual ISI plus noise terms at different rounds are independent),

$$\Delta_{t,n}^{(k)} \approx \text{diag} \left\{ \check{\delta}_{t,N}^{(1)2}, \dots, \check{\delta}_{t,N}^{(k-1)2}, \check{\delta}_{t,n}^{(k)2} \right\}. \quad (50)$$

The algorithm is summarized in Table II.

C. Complexity Analysis

In this subsection, we focus on the analysis of the computational cost of forward and backward filters as well as the memory requirements for the proposed algorithms. The other steps are similar and have the same complexity for both algorithms. We also provide comparisons with the conventional LLR-level combining technique.

⁵The forward and backward filters can be easily derived using the equations in the previous subsection and assuming $k = 1$.

In the case of signal-level turbo combining, the computation of forward and backward filters involves, at each round k and iteration n , one inversion of a $N_T(\kappa + L - 1) \times N_T(\kappa + L - 1)$ matrix (i.e., matrix $\Upsilon^{(k)} + \sigma^2 \Xi_n^{-1}$ in eq. (38)) for computing $\Lambda_n^{(k)}$, and whose cost is $\mathcal{O}(N_T^3 \kappa^3)$ (assuming $\kappa \gg L$, and neglecting the cost of obtaining $\Xi_n^{-1} = \mathbf{I}_{\kappa+L-1} \otimes \tilde{\Xi}_n^{-1}$ since $\tilde{\Xi}_n$ is diagonal). This indicates that the computational complexity of the signal-level combining scheme is less sensitive to k . The number of rounds only influences the number of additions required for obtaining vectors $\{\underline{\mathbf{z}}_i^{(k)}\}_{0 \leq i \leq T-1}$ and matrix $\Upsilon^{(k)}$, according to (39) and (40), respectively. The cost of these steps is

$$\Delta N_{Add} = N_T^2 (\kappa + L - 1)^2 + N_R \kappa T \quad (51)$$

for each round $k > 1$. Note that the number of operations required for obtaining $\underline{\mathbf{H}}^{(k)H} \underline{\mathbf{H}}^{(k)}$ and $\underline{\mathbf{H}}^{(k)H} \underline{\mathbf{y}}_i^{(k)}$ in not considered in (51) since symbol-level combining also involves the same operations. Therefore, the computational cost of forward and backward filters is almost the same for both combining algorithms. Note that the significant reduction in the complexity of the signal-level combining scheme (with respect to the dimensionality of the sliding-window model (21) used by the algorithm) is due to recursion (40) which consists of writing $\underline{\mathbf{H}}^{(k)H} \underline{\mathbf{H}}^{(k)}$ as the sum $\sum_{u=1}^k \underline{\mathbf{H}}^{(u)H} \underline{\mathbf{H}}^{(u)}$.

Memory requirements for the two proposed schemes are determined by the update steps Tables I. **1.1** and II. **1.3**. For the signal-level combining technique, a $N_T(\kappa + L - 1) \times N_T(\kappa + L - 1)$ complex matrix is required to accumulate channel matrices $\underline{\mathbf{H}}^{(k)H} \underline{\mathbf{H}}^{(k)}$ according to (40) (and therefore generating $\Upsilon^{(k)}$), in addition to a $N_T(\kappa + L - 1) \times T$ complex matrix that serves to accumulate signal vectors $\{\underline{\mathbf{z}}_i^{(k)}\}_{i=0}^{T-1}$ using (39). Note that these two recursions, i.e., (39) and (40), avoid the storage of all signals and channel matrices as in MAP turbo combining. In the case of symbol-level combining, only N_T complex matrices of size $K \times T$ and two $K \times N_T$ complex matrices are required to store filter outputs and their corresponding parameters, i.e., symbol gains and residual ISI plus thermal noise variances. Therefore, signal-level combining requires slightly more memory than its symbol-level counterpart, because only two or three ARQ rounds are considered (according to the outage analysis in Subsection III-C) and in general $\kappa \gg L$.

Finally, note that in the case of conventional LLR-level combining, soft equalization is separately performed for each ARQ round exactly as in symbol-level combining, while extrinsic LLRs are added together before decoding. This translates into $N_T M T N$ real additions at each round, and a real vector of size $N_T M T$ to combine extrinsic values. Therefore, the three combining strategies have similar implementation requirements. They slightly differ in the number of additions and storage memory.

V. NUMERICAL RESULTS

In this section, we provide simulated BLER and throughput performance for the proposed turbo packet combining techniques presented in Section IV. Considering some representative MIMO configurations, our main focus is to demonstrate that the signal-level turbo combining approach has better ISI cancellation capability and diversity gain than the symbol-level approach. We also show that both techniques provide better performance than conventional LLR-level combining.

A. Simulation Settings

In all simulations, we use an ST-BICM scheme composed of a 64-state $\frac{1}{2}$ -rate convolutional code with polynomial generators $(133_8, 171_8)$. The length of the code frame is 1800 bits including tail bits. We consider either quadrature phase shift keying (QPSK) or 16-state quadrature amplitude modulation (QAM) depending on the target rate R of the ST-BICM code. The MIMO-ISI channel has the same profile as in Subsection III-C, i.e., two equal power taps. With respect to the outage analysis in Section III, we consider a ARQ delay $K = 2$. We verified, with simulations, that for the considered ST-BICM code, the improvement in BLER performance is only incremental when $K > 2$. Note that in [38], only a four-state code is used, and performance results are reported with a maximum number of rounds $K = 3$. Simulations are carried out as in Subsection III-C, i.e., the transmission of an information block is stopped and the system moves on to the next block when an ACK message is received or the decoding outcome is erroneous after round $K = 2$.

Note that the benefits of an ARQ mechanism appear in the region of low to moderate SNR, where multiple transmissions are required to help correct packets erroneously received after the first round. For high SNR values, ARQ may not be needed because most packets are correct after the first transmission. Therefore, we focus our analysis on the SNR region where BLER values, after the first round, are between 1 and 10^{-1} . In this region, an ARQ protocol is essential to have reliable communication. Our main goal is to analyze the ISI cancellation capability and the achieved diversity order for the proposed turbo combining schemes. We, therefore, evaluate the BLER performance per ARQ round. We also evaluate the throughput improvement offered by the proposed schemes. The SNR appearing in all figures is per symbol per receive antenna. For both schemes, we consider five turbo iterations for decoding an information block at each transmission. We compare the resulting performance with the outage probability and the MFB. Note that for the purpose of fair comparison, the computation of the outage performance does not take into account the rate distortion as in (16). The MFB curves

are obtained for each transmission assuming perfect ISI cancellation and maximum ratio combining (MRC) of all time, space, multipath, and delay diversity branches.

B. Analysis

First we consider an ST-BICM code with $N_T = 2$ and QPSK signaling. This corresponds to a rate $R = 2$. The number of receive antennas is $N_R = 2$, and the filter length is $\kappa = 9$ ($\kappa_1 = \kappa_2 = 4$) for all combiners. Fig. 4 compares the BLER performance for the signal-level, symbol-level, and LLR-level combining with the MFB and the outage probability. For both signal and symbol-level turbo combining, the performance improvement after the second ARQ round is very significant compared with LLR-level combining. The signal-level combining scheme is shown to achieve the MFB while the symbol-level scheme presents approximately a gap of 1dB compared with the MFB. This means that signal-level combining has higher ISI cancellation capability than symbol-level combining. This result is due to the fact that in signal-level combining, each ARQ round is considered as a set of virtual N_R receive antennas. This allows the ARQ delay diversity to be efficiently exploited. On the other hand, both proposed schemes are shown to achieve the asymptotic slope of the outage probability.

Now, we turn to ST-BICM codes with rate $R = 4$. Firstly, we consider a configuration similar to that of the previous case but using 16-QAM modulation. The filter length is kept equal to $\kappa = 9$. The BLER performance is reported in Fig. 5. In this scenario, the signal-level scheme clearly outperforms both the LLR-level and the symbol-level schemes. Indeed, the gap between the latter and the MFB is about 2.25dB. Both proposed techniques asymptotically achieve the diversity gain of the MIMO ARQ channel. In Fig. 6, we examine a ST-BICM code with $N_T = 4$, QPSK signaling, and $N_R = 2$. Note that this type of “unbalanced” configuration, i.e., more transmit than receive antennas, is suitable for the forward link. The filter length is increased to $\kappa = 13$ ($\kappa_1 = \kappa_2 = 6$) for all schemes. The signal-level combining technique is shown to achieve BLER performance close to the MFB (the gap is less than 0.5dB), while both the LLR-level and the symbol-level techniques have a degraded probability of error (the gap between the symbol-level and the MFB is more than 3dB at $2 * 10^{-2}$ BLER). It is also important to note that signal-level combining manifests itself in almost achieving the diversity gain while it is shown that symbol-level combining fails to do so. This is mainly due to the fact that, at the second ARQ round, the signal-level scheme constructs a 4×4 virtual MIMO-ISI channel for ISI cancellation and symbol detection, while the MIMO configuration remains unbalanced in the case of symbol-level combining. In Fig. 7, we compare the throughput performance of the three algorithm for

the 4×2 configuration. It is shown that signal-level combining offers higher throughput. Also, note that while the MFB achieves the maximum throughput of 4bit/s/Hz, the proposed techniques saturate around 2bit/s/Hz because most of the packets received in the first ARQ round are erroneous.

Finally, note that in practical systems, channel estimation presents the bottle-neck that causes performance loss. In [38], we evaluated the BLER performance for a low-rate ST-BICM code (typically, $N_T = N_R = 2$, and $R = 2$) with imprecise channel estimates and using signal-level turbo packet combining. We have shown that when MMSE channel estimation is performed in a turbo fashion together with turbo packet combining (i.e., channel is iteratively re-estimated at each ARQ round using both pilot symbols and soft LLRs), the performance loss is less than 0.5dB when $K = 2$, and does not exceed 1dB when the ARQ delay is increased to $K = 3$. Also, we have shown that even for the case of short-term static dynamic, turbo channel estimation can offer attractive BLER performance without requiring the re-transmission of the pilot sequence since channel estimation in subsequent ARQ rounds can rely only on soft LLRs.

VI. CONCLUSION

In this paper, we considered the design of efficient turbo packet combining schemes for MIMO ARQ protocols operating over frequency selective channels. First of all, we derived the structure of the optimal MAP packet combiner that exploits all the diversities available in the MIMO-ISI ARQ channel to perform transmission combining. Inspired by [24], [47], we then investigated the outage probability and the outage-based power loss for Chase-type MIMO ARQ protocols operating over ISI channels. Then, we introduced two MMSE-based turbo combining schemes that exploit the delay diversity to perform transmission combining. The signal-level scheme considers an ARQ round as a set of virtual receive antennas and performs packet combining jointly with ISI cancellation. The symbol-level scheme separately equalizes multiple transmissions, while combining is performed at the level of filter outputs. We showed that both combining schemes have computational complexities similar to that of the conventional LLR-level combining. Finally, we presented simulation results that demonstrated that signal-level combining provides better BLER and throughput performance than that of symbol-level and LLR-level combining.

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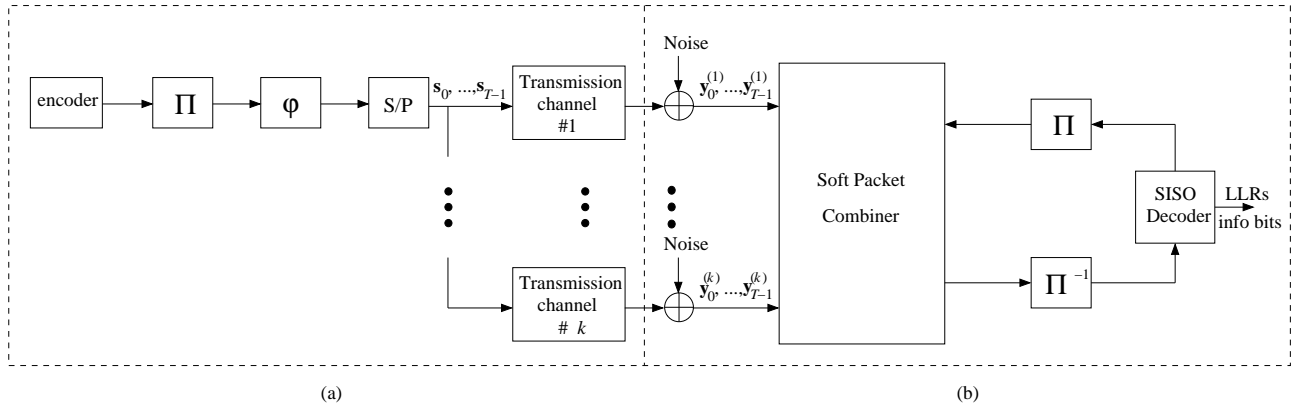


Fig. 1. ST-BICM diagram with ARQ and turbo packet combining: (a) transmitter, (b) receiver.

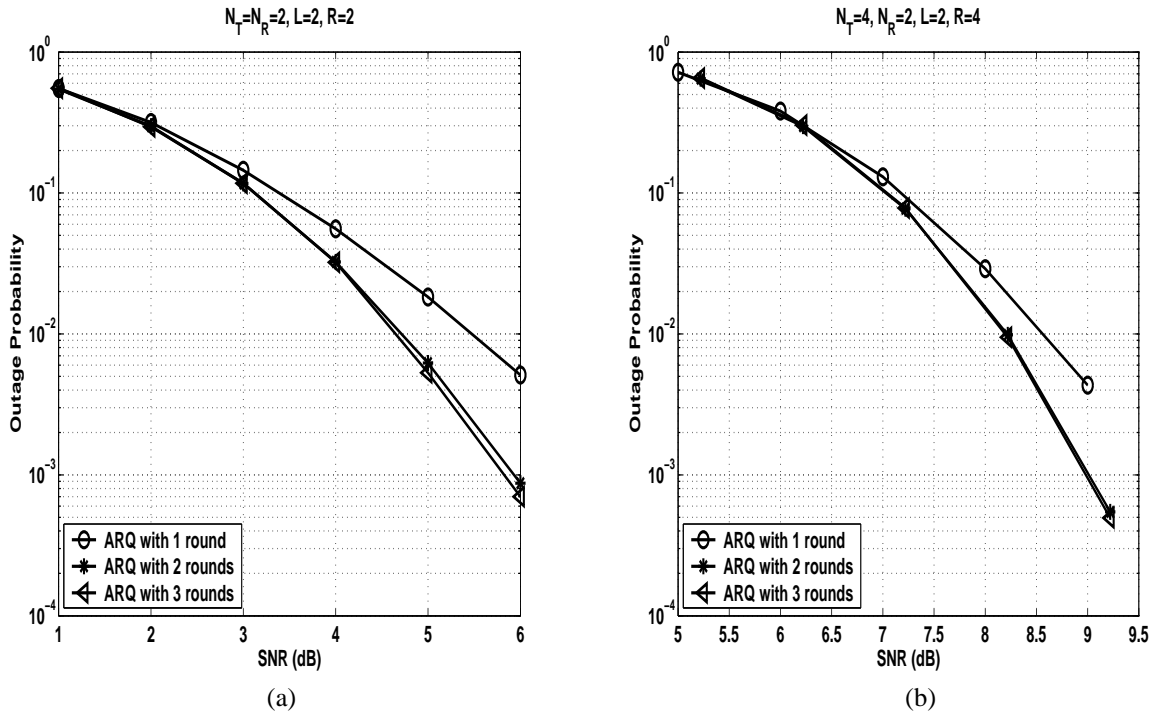


Fig. 2. Outage probability versus the maximum number of rounds K for $L = 2$ taps, $N_R = 2$, and: (a) $N_T = 2, R = 2$, (b) $N_T = 4, R = 4$

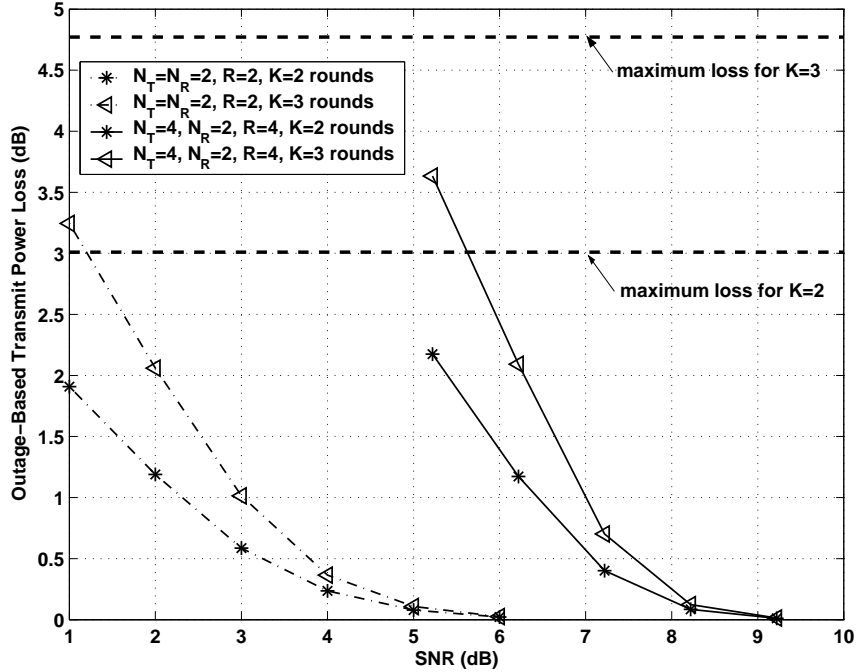


Fig. 3. Outage-based transmit power loss for $N_T = N_R = 2, R = 2$, and $N_T = 4, N_R = 2, R = 4$

TABLE I

SUMMARY OF THE SIGNAL-LEVEL TURBO PACKET COMBINING ALGORITHM

0. Initialization

Initialize $\Upsilon^{(0)}$ and $\{\mathbf{z}_i^{(0)}\}_{i=0}^{T-1}$ with $\mathbf{0}_{N_T(\kappa+L-1)}$ and vectors $\mathbf{0}_{N_T(\kappa+L-1) \times 1}$, respectively.

1. Combining at round k

1.1. Update $\{\mathbf{z}_i^{(k)}\}_{i=0}^{T-1}$ and $\Upsilon^{(k)}$ according to (39) and (40).

1.2. For $n = 1, \dots, N$

1.2.1. Compute: conditional symbol averages and unconditional variances using (34) and (33).

1.2.2. Compute: $\Lambda_n^{(k)}$ using (38).

1.2.3. For $t = 1, \dots, N_T$

1.2.3.1. Compute: $\mathbf{F}_{t,n}^{(k)}, \mathbf{B}_{t,n}^{(k)}, \alpha_{t,n}^{(k)}$, and $\delta_{t,n}^{(k)2}$ using (36), (37), and (46).

1.2.3.2. For each $i = 0, \dots, T-1$, compute the soft demapper input $\xi_{t,i,n}^{(k)}$ according to (35).

1.2.3.3. For each $m = 1, \dots, M$, compute extrinsic information $\phi_{m,t,i,n}^{e[Sig]}$ using (45).

1.2.4. End 1.2.3.

1.3. End 1.2.

TABLE II
SUMMARY OF THE SYMBOL-LEVEL TURBO PACKET COMBINING ALGORITHM

0. Initialization

Initialize $\{\check{\xi}_{t,i}\}_{i=0}^{T-1}$, $\check{\alpha}_t$, and $\check{\delta}_t^2$ with empty vectors for $t = 1, \dots, N_T$.

1. Combining at round k

1.1. For $n = 1, \dots, N$

1.1.1. Compute: conditional symbol averages and unconditional variances using (34) and (33).

1.1.2. For $t = 1, \dots, N_T$

1.1.2.1. Compute: forward and backward filters, $\check{\alpha}_{t,n}^{(k)}$, and $\check{\delta}_{t,n}^{(k)2}$ as in Subsection IV-A.

1.1.2.2. For each $i = 0, \dots, T - 1$, compute the filter output $\check{\xi}_{t,i,n}^{(k)}$.

1.1.2.3. For each $m = 1, \dots, M$, compute extrinsic information $\phi_{m,t,i,n}^{[Symbol]}$ using (47).

1.2.3. End **1.1.2.**

1.2. End **1.1.**

1.3. Update: $\{\check{\xi}_{t,i} := [\check{\xi}_{t,i} \ \check{\xi}_{t,i,N}^{(k)}]\}_{i=0}^{T-1}$, $\check{\alpha}_t := [\check{\alpha}_t \ \check{\alpha}_{t,N}^{(k)}]$, and $\check{\delta}_t^2 := [\check{\delta}_t^2 \ \check{\delta}_{t,N}^{(k)2}]$ for $t = 1, \dots, N_T$.

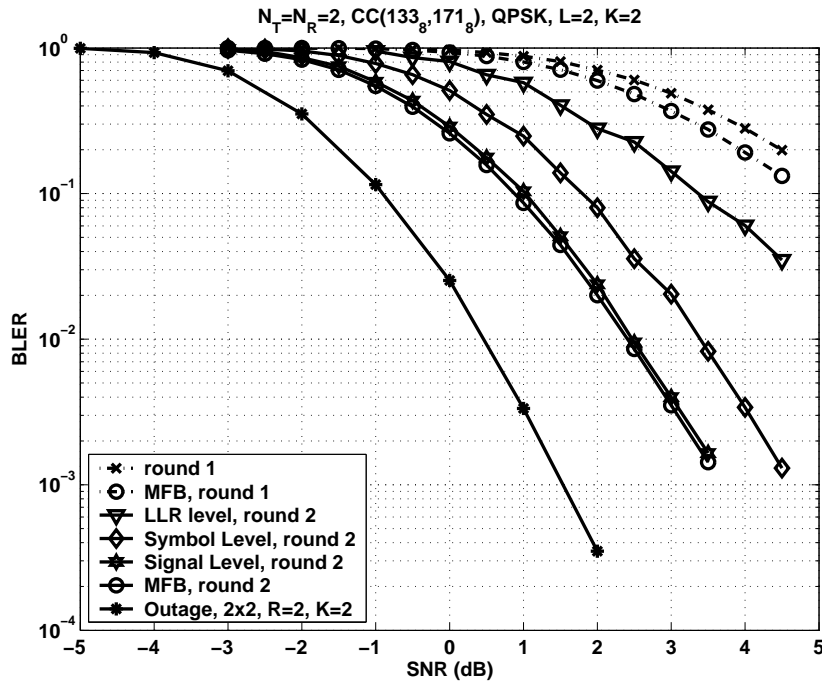


Fig. 4. BLER performance comparison for $N_T = N_R = 2$, CC(133₈, 171₈), QPSK, $K = 2$ rounds, and $L = 2$ taps.

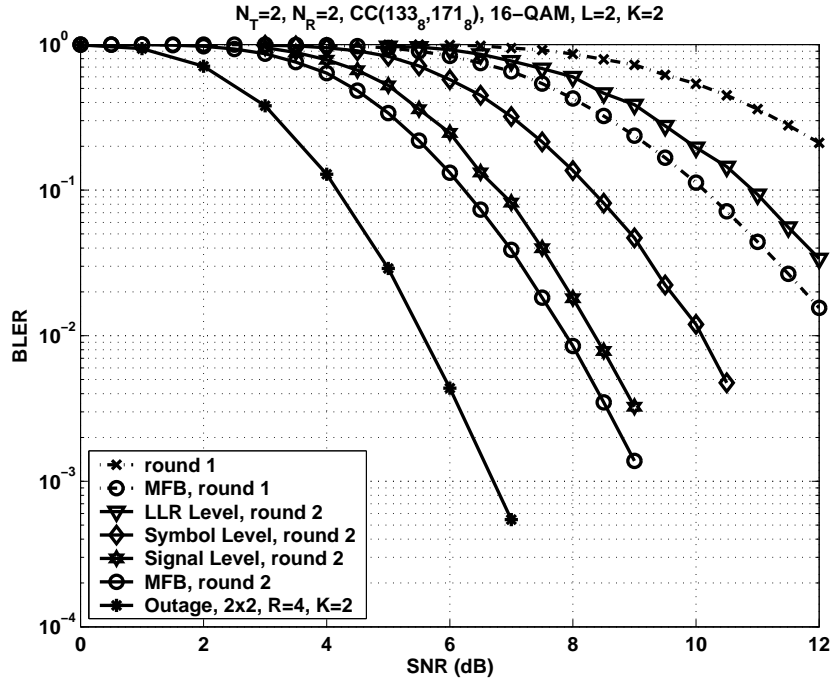


Fig. 5. BLER performance comparison for $N_T = N_R = 2$, $CC(133_8, 171_8)$, 16-QAM, $K = 2$ rounds, and $L = 2$ taps.

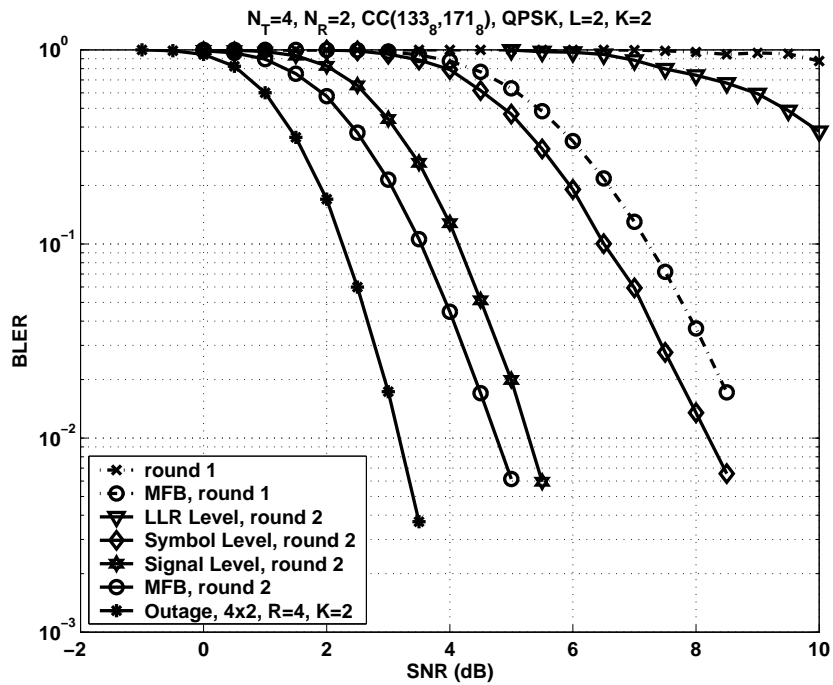


Fig. 6. BLER performance comparison for $N_T = 4$, $N_R = 2$, $CC(133_8, 171_8)$, QPSK, $K = 2$ rounds, and $L = 2$ taps.

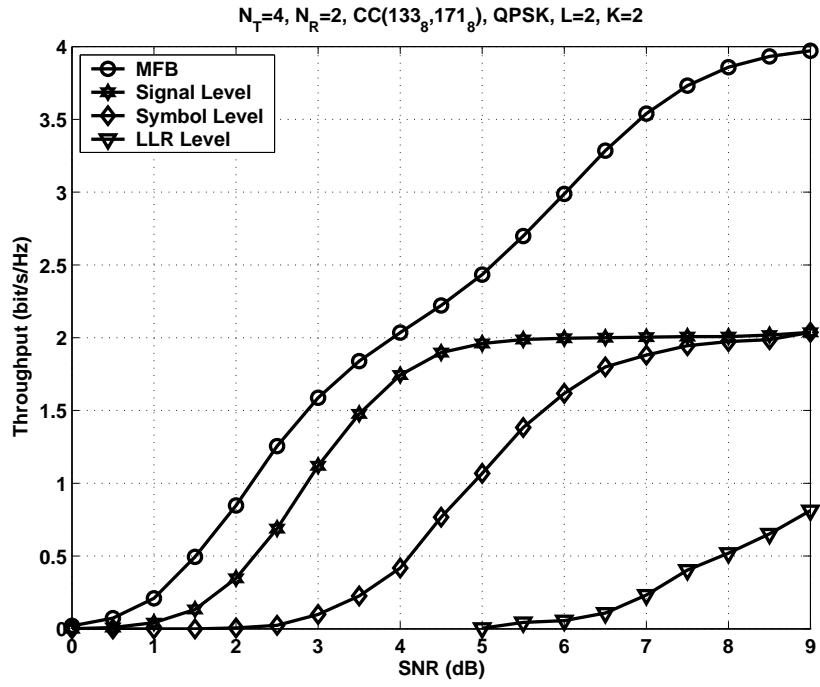


Fig. 7. Throughput performance comparison for $N_T = 4$, $N_R = 2$, $CC(133_8, 171_8)$, QPSK, $K = 2$ rounds, and $L = 2$ taps.