# Clifford Algebra and Gabor filter for color image texture characterization

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***Abstract***

**The first texture descriptors are proposed in 1973 by Haralick[1] and in 1975 by Marie Galloway[2] are still used today for image classification or segmentation in various domains. The majority of these features are defined for gray level images. Many papers have proposed different approaches among them, a parallel study of color and gray level texture characterization when other combined the two groups of features by defining joint features[3][4][5]. Combining features or defining them jointly are outperformed [6] even though these features impose some constraints such as the color system, measurement similarity methods. The aim of this paper is to use Clifford Algebra to represent multi-component images and to propose a color image texture.**

***Index Terms***: Clifford Algebra – Gabor filter – Color image – Texture analysis.

## INTRODUCTION

Image texture characterization has two main objectives: firstly, pattern recognition and secondly image classification and segmentation. To achieve these, one may find out the features to be used to characterize each pixel of the target region or the input image. Many research works have been done to improve the results but most of them are based on gray scale images and the texture is fully characterized. Fortunately, Mäenpäa et al. in [6] concluded that combining color and texture features give better results for images' classification. To process the color image, one needs to transform it before applying the techniques and features defined for 2D images. These transformations often constrained to the appropriate color system to use, the measurements similarity and the classification methods. Clifford Algebra is now well used in image processing and it gives a formidable way to represent multi-components images. To decompose easily a color image for its texture characterization, we use Clifford Algebra property presented in [7] to perform Clifford Fourier transform. We exploit the decomposition to which we apply a bank of Gabor filters to propose texture feature descriptors.

The paper is organized as follow: In the first section, we make a glance on Clifford Algebra and show how a color image can be decomposed. In the second section, we present Gabor's filters while, in the third section we draw up texture feature extraction block diagram. Finally we present in the section four experimental results and then conclude this work with perspective works in section five.

## I. CLIFFORD ALGEBRA AND COLOR IMAGE DECOMPOSITION

Clifford Algebra is a particular Algebra which extends the vector space to a $2^n$ dimensional linear space by defining a bilinear form that satisfies the following properties[8]:

- Scalars commute with everything: $\alpha v = v\alpha$ where $\alpha$ is a scalar and $v$ a vector.



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- Vector x obey to xx = $\mathcal{Q}$(x) , where $\mathcal{Q}$ is the quadratic form associated to the bilinear form.
- Algebraic properties: geometric product is linear in both factors (associative and distributive other addition operator) but not commutative.

Geometric Algebra contains notions about scalars, vectors, bivectors, nvector, pseudo-scalars, and so on. We deal in this Algebra with multi-vectors. The geometric product of two vectors is referred to a bivector. The product of a bivector and a vector gives a trivector, and so on. The Figure 1 gives an illustration of bivector and trivector.

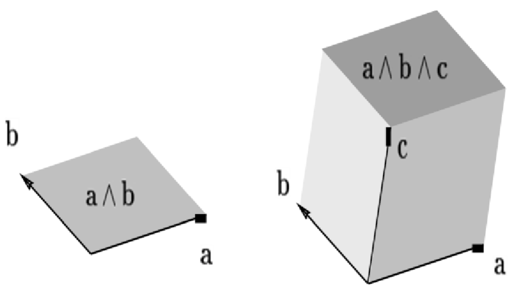



Fig. 1. From left to right bivector and trivector representation.

A multivector is a directed area or volume of n-dimensional space.

Let Cl (p; q) be a finite dimensional vector space of signature (p; q) over a ring K.

- p is the number of vectors with positive square in the basis of the space;
- q is the number vectors with negative square in the basis of the space.

The number n = p+q is the dimension of the vector space.

When we deal with 3D images, we could use Cl(3,0) in which vectors of the basis are : $e_0$, $e_1$, $e_2$, $e_3$. Each vector has $e_\alpha^2 = 1$ where α=1, 2, 3.

The following elements $e_0$, $e_1$, $e_2$, $e_3$, $e_1e_2$, $e_1e_3$, $e_2e_3$, $e_1e_2e_3$ referre to blades and are used to form the orthogonal basis of the space. Each of the forth elements $e_1e_2e_3$, $e_1e_2$, $e_1e_3$, $e_2e_3$ has its square equal to -1.

The operators used in this Algebra are mainly the geometric product, the inner product and the outer product that we define in the following subsections.

*A. Inner product*

This operator we denote (·) extends the common dot product. Let a and b be vectors in Cl(3,0).

$$a \cdot b = \tfrac{1}{2}(ab + ba) \qquad (1)$$

$$a \cdot b = b \cdot a. \qquad (2)$$

*B. Outer product*

This operator we denote (∧) extends the common cross product. Let a and b be vectors in Cl(3,0).

$$a \wedge b = \tfrac{1}{2}( ab - ba ) = -b \wedge a. \qquad (3)$$

*C. Geometric product*

This operator combines the inner and the outer product. Let a and b be vectors in Cl(3,0).

$$ab = a \cdot b + a \wedge b. \qquad (4)$$

*D. Projection and rejection*

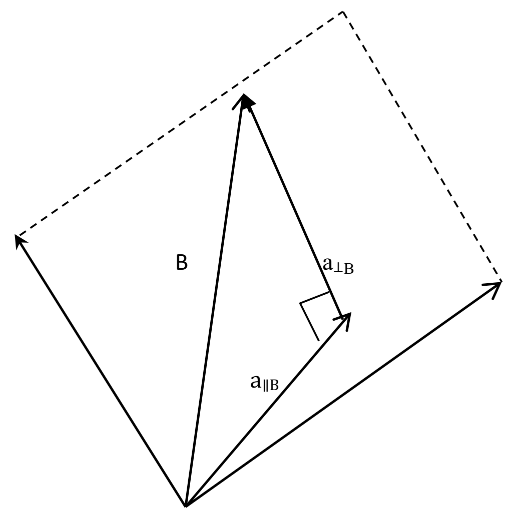



Fig. 2. Projection and rejection of the vector a onto B

Let B be a bivector and a a vector. The vector a can be decomposed into two parts (see Fig. 2.):

$$a = a_{\parallel B} + a_{\perp B} \qquad (5)$$

where $a_{\parallel B}$ is collinear to B and called the projection of a onto B and $a_{\perp B}$ is orthogonal to B and called the rejection of a from B.

The following properties can easily be obtained

$$a_{\parallel B} = a.B \qquad (6)$$

$$a_{\perp B} = a \wedge B \qquad (7)$$

### E. Color image decomposition

Let us recall that a color image when represented in RGB color system, is composed of three channels, each giving the pixel value of the red, green and blue colors respectively. Geometrically, a color image is represented in 3D space.

In Clifford Algebra, a color image f is a set of vectors belonging to Cl(3,0) space. So any image vector f(x,y) can be expressed as

$$f(x,y)= x_1e_1 + x_2e_2+x_3e_3. \quad (8)$$

Moreover, any vector can be decomposed (according to the projection plane), into parallel part and orthogonal part to the plane. Specifically, let B be the bivector. The color image can be written in the form

$$f(x,y)= f_{\parallel B}(x,y)+f_{\perp B}(x,y) \quad (9)$$

For practical image decomposition, Mennesson in [7] demonstrated that

$$f(x) = c\,[f(x) \cdot c + f(x){\cdot}cB\,]\,B + v\,[f(x) \cdot v + f(x){\cdot}vI_4B\,]\,I_4B \quad (10)$$

where x a vector, c and v orthogonal vectors, B a bivector, f(x) is the image and $I_4B$ is a bivector i.e. the dual of B.

## II. GABOR FILTER

Since 1946, Gabor in [9] has proposed an alternative aimed on locally analysis of 1D signal such a way that it is possible to study the local features of 1D signal. Going from this opening, Daugman in [10] extends this possibility to 2D image analysis and proposed a family of functions that are generally expressed as

$$e^{-Ax^2+Bxy+Cy^2+Dx+Ey+F} \quad (11)$$

where $B^2 < 4AC$ and D, E, F are complex numbers.

Moreover he even established the fundamental lower limit that must be satisfied as a constraint in 2D dimension:

$$(\Delta x)(\Delta y)(\Delta u)(\Delta v) \geq \frac{1}{16\pi^2}. \quad (12)$$

The family of functions that satisfy this constraint can be written in spatial and frequency domain respectively:

$$f(x,y)= e^{-\pi[(x-x_0)^2\alpha^2+(y-y_0)^2\beta^2]} \times e^{-2\pi i[u_0(x-x_0)+v_0(y-y_0)]}\ , \quad (13)$$

$$F(u,v)= e^{-\pi[(u-u_0)^2/\alpha^2+(v-v_0)^2/\beta^2]} \times e^{-2\pi i[x_0(u-u_0)+y_0(v-v_0)]} \quad (14)$$

A variant of this general family of the spatial domain functions given in [5] is the one we use in this paper. Its expression is given by equation

$$G(x,y,f,\theta)= \frac{1}{\sqrt{\pi\sigma 1\sigma 2}}\exp\left(\frac{-1}{2}\left(\frac{R_1^2}{\sigma_1^2}+\frac{R_2^2}{\sigma_2^2}\right)\right).\exp(i(f_x x + f_y y)) \quad (15)$$

where

$R_1 = x\cos\theta + y\sin\theta$, $R_2 = -x\sin\theta + y\cos\theta$, $\sigma_1 = \frac{c_1}{f}$, $\sigma_2 = \frac{c_2}{f}$, $f_x = f\cos\theta$, $f_y = f\sin\theta$. $c_1$, $c_2$ are constants

In [11] Zheng et al. proposed how different parameters could be selected.

## III. BLOCK DIAGRAM OF COLOR IMAGE TEXTURE CHARACTERIZATION

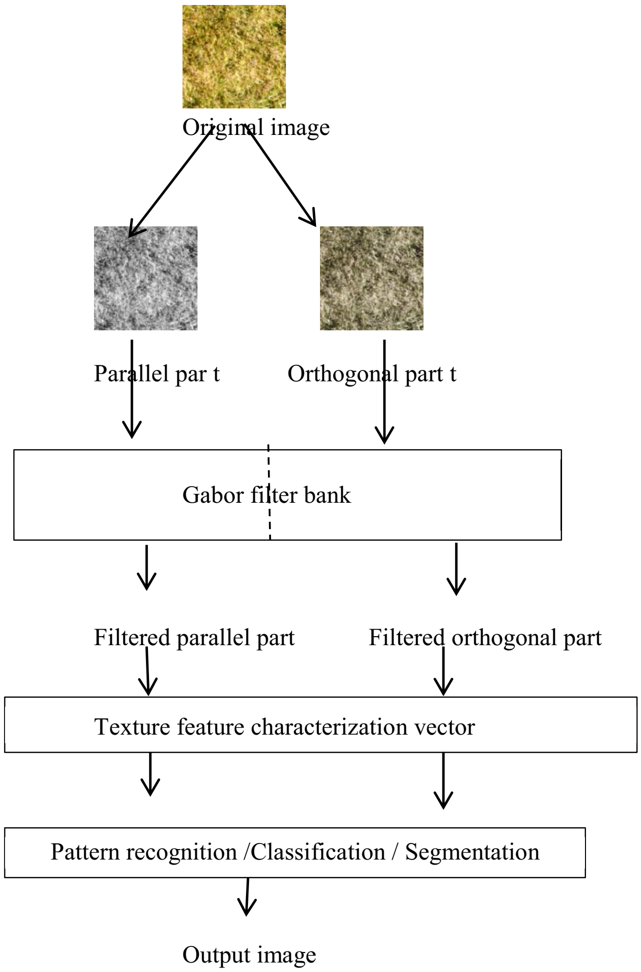



Fig. 3. Block diagram of color image texture characterization.

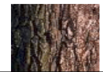
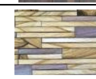
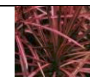
The characterization vector depends on the energy matrix variance of each target image component. Let us denote VC this vector.

$$VC = (var_{\parallel}\ var_{\perp}). \tag{16}$$

## IV. RESULTS

Our first experiment consists of implementing the correlation between the parallel part and the orthogonal part of a color image. We have chosen four different texture types from VisTex database[1]

TABLE I: CORRELATION COEFFICIENT OF FIVE TARGET IMAGES

| Number | Texture type | Target images | Correlation coefficient |
|---|---|---|---|
| 1 | bark |  | 0.4191 |
| 2 | |  | 0.0936 |
| 3 | brick |  | 0.3379 |
| 4 | |  | 0.1150 |
| 5 | Grass |  | 0.4578 |
| 6 | |  | 0.0982 |
| 7 | Leaves |  | 0.3754 |
| 8 | |  | 0.2395 |

The second experiment concerns the texture characterization vector.

TABLE II: FEATURE VECTOR VALUES

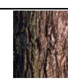
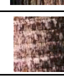
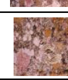
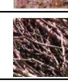
| Number | Target images | Parallel part variance ($var_{\parallel}$) | Orthogonal part variance($var_{\perp}$) |
|---|---|---|---|
| 1 |  | 53.9315 | 0.0222 |
| 2 |  | 75.8185 | 0.0124 |
| 3 |  | 17.4257 | 0.0315 |
| 4 |  | 19.7778 | 0.0527 |
| 5 |  | 100.2253 | 0.0222 |

Table I shows that the degree of correlation between parallel part and orthogonal part depends on the texture. Nevertheless, the highest correlation coefficient is 0.4578 when the lowest is 0.0936. We conclude that we shall take into account the two part of our images for their characterization.

[1] cdb.paradice-insight.us/corpora/Corpus

Table II presents the feature vector values obtained by carrying the second experiment. One can observe that parallel parts variances are more relevant and can be used as discriminant of different image textures even though the orthogonal part variances are less discriminant. For a general texture image characterization, one shall use both orthogonal part and parallel part variances.

## V. CONCLUSION AND PERSPECTIVES

We have used Clifford Algebra properties to decompose color 3D image and then applied Gabor filter bank to extract two groups of texture descriptors. We have seen that the parallel part and the orthogonal part are few correlated. The discrimination vector we have proposed is relevant and can be used for image classification. In further work, we will compare the efficiency of this geometrical decomposition of color image to the existing approaches.

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