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A Mobility Analysis Method of Closed-Chain Mechanisms with Over-constraints and Non-holonomic Constraints

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abstract : Mobility for a great portion of robot mechanisms having over-constraint and non-holonomic constraints has not been clearly identified. This work is to introduce a method of mobility analysis for such systems using the concept of representative screws and pseudo-joint. The pseudo-joint is employed to effectively represent the real motion trajectory due to the rolling contact of the wheel. To show the validity and effectiveness of the proposed method, mobility of various types of planar mobile robots having over-constraint and non-holonomic constrains are examined.

keywords: ***representative screw, pseudo joint, mobility***

I. Introduction

In general, to analyze mobility of mechanisms, the well-known Grubler's mobility formula has been used[1-4]. This method is often called as the zeroth-order mobility analysis. However, this formula is sometimes not adequate in directly computing the mobility of a number of mechanisms because Grubler's mobility formula assumes that the allowed motion space of all joints in one loop is not constrained by the joints in other loops. In over-constrained systems, this is not the case. Thus, the size of allowed motion space of joints in each of independent loops needs to be identified.

Screw approach, which is called, the first-order mobility analysis, provides a means of identifying the size of the allowed motion space of joints in a loop. However, the screw intrinsically represents only the infinitesimal characteristics of motion (i.e., the first-order kinematic characteristics). Thus, screw approach often fails to exactly represent the motion along finite displacements occurring in rolling contact. To identify the mobility in such cases, second-order mobility analysis of rolling motion trajectory needs to be examined. Rimon and Burdick[5] suggested a second-order mobility analysis method on gripping mechanisms, but the process requires either a significant computational or algebraic burden. Thus, mobility analysis for great portion of robot mechanisms with over-constrains or with non-holonomic constraints such as rolling motion of wheels have not been done completely up to now [6-8]. This is mainly because the available methods or procedures are either complicated or not easy to understand to use them in finding the mobility of such over-constrained mechanisms.

This paper is arranged as follows. Firstly, Grubler's mobility formula and its limitation of direct application to over-constrained mechanisms are discussed. Secondly, concept of representative screws and method of identifying them are described with an exemplary mechanism.[9] Lastly, mobility analyses of several planar mobile systems with nonholonomic constraints as well as ones with singular configuration, are conducted by representing the characteristics of shapes of the allowed motion trajectory of joints and rolling contacts as "pseudo-joints" and "pseudo-screws".

II. Grubler Formula and Its Applications

Grubler's mobility formula can be written in the following form[1];

$$M = d(l-1) - \sum_{i}^{n} c_i \tag{1}$$

where d represents the dimension of the feasible motion space of the entire joints of the mechanism, and l and c_i denotes the number of links including the ground and the number of constraints of the i^{th} joint, respectively. The value of d can be identified as the number of independent joint screws of the mechanism (refer to Appendix). According to the value of d, Hunt[1] categorized mechanisms into one-system through six-system.

For parallel mechanisms, due to their closed-chain structures that impose constraints on the motion of some of joints, the value of d can not be arbitrarily selected like serial mechanisms. The following is a modified form of Grubler's Formula, which is useful for parallel mechanisms [1]:

$$M = \sum_{i=1}^{n} f_i - \sum_{j=1}^{m} d_{L_j}, \tag{2}$$

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where f_i and d_{L_j} represents the degree of freedom of the i^{th} joint and the dimension of the feasible joint motion space of the independent loop L_j, respectively. m and n denotes the number of independent loops and the number of joints, respectively. For a mechanism having one closed-loop, d_{L_j} can be easily identified as the number of independent joint screws in the loop L_j.

However, in most of over-constrained mechanisms consisting of several closed-loops, the feasible motion space of all joints belonging to one loop L_j is constrained by other loops, and thus it makes difficult to identify the correct value of d_{L_j}. In light of this fact, a way to identify the dimension of the feasible motion space for each loop via the concept of "representative screw" will be summarized in the following session[9].

III. Representative Screws

Consider the parallelogrammic mechanism of Fig. 1. This mechanism is a typical example of over-constrained systems and has mobility of one. However, unless the over-constrained condition is incorporated into Eq. (1), the mobility of the mechanism turns out to be zero from Eq. (1), which is in fact incorrect. To cope with this problem, the concept of representative screw is introduced by using the same example. As shown in Fig. 1, the mechanism has two independent loops A and B. Note that joint 3 and 4 are shared by the two loops. We start the analysis from the loop A, and then check if the feasible motion space of the loop B is affected by the loop A. The joint screw for each joint of the mechanism with respect to an arbitrary origin O can be written as

$$\$_i = (0,0,1; y_1 + y_i, -x_1 - x_i, 0) \quad , \quad i = 1,2,\cdots,6 \cdot$$

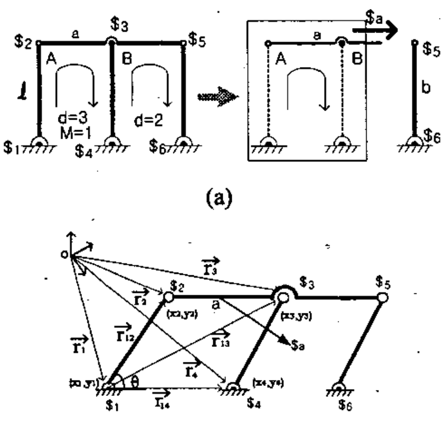


Fig. 1. A parallelogrammic planar mechanism

Using these joint screws, the number of independent joint screws for the loop A is computed as d_{L_1} =3 and its mobility is computed as 1 from either Eq. (1) or Eq. (2).

However, the feasible motion space of joints belonging to both the loop A and B (joint 3 and 4) would be constrained by the motion of loop A. Actually, the feasible motion space of joints commonly belonging to both loops A and B is reflected through the motion of the link a, which connects both loops. In fact, the motion screw of the link a in Fig. 1($\$_a$) represents the constrained motion screw of two joints 3 and 4. Thus, it is defined as "representative screw". And for this particular example it can be easily identified as

$$\$ = (0,0,0; l\sin\theta, -l\cos\theta, 0),$$

where the length of the link connecting joint l and 2 is denoted as l. From this equation it can be seen that the motion of the link a is always translational and the direction is perpendicular to the link b. Also, it can be easily confirmed that $\$_a$ is always dependent of the joint screws of the loop B (i.e., $\$_5$ and $\$_6$). Thus, the number of independent joint screws of the loop B becomes $2 (= d_{L_1})$ and the mobility of the mechanism in Fig. 1 is obtained as $M = 6 - (3+2) = 1$ from Eq. (2). Likewise, the mobility analysis could be performed in a reverse way in which the loop A is assumed constrained by the loop B. With the same procedure, we will obtain the same mobility for the mechanism.

The following summarizes the procedure of finding mobility of closed-chain mechanisms via "representative screws."

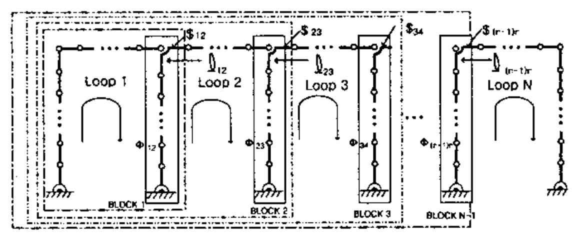


Fig. 2. Schematic model of closed-chain mechanism

Step I) Assign the sequential order to the constrained loops of the mechanism. For convenience, it is assumed that loops are constrained sequentially from 1 to n as in Fig. 2.

Step II) Compute the number of independent screws of all joints in the loop 1 and the mobility of block 1.

Step III) Identify the representative screw set $\{\$_{(k-1)k}\}$ reflecting the motion space of the link $l_{(k-1)k}$ which connects the block $k-1$ and the loop $k(k = 1,\cdots,n)$. Then, compute the number of independent joint screws for the loop k from the union set of representative screw set $\{\$_{(k-1)k}\}$

and the partial joint screw set of loop which do not belong to the block $k-1$. Then compute the mobility of the block k.

Step IV) Repeat step III until $k = n$.

In step I), it is assumed that loops are constrained sequentially from loop 1 to loop N, for convenience. However, depending on the structure of the mechanism, the order could be varied. Suppose that two different blocks (P and Q) are connected through a loop R. Then, the number of independent joint screws of the loop R can be found from the union set of the representative screws for the two blocks.

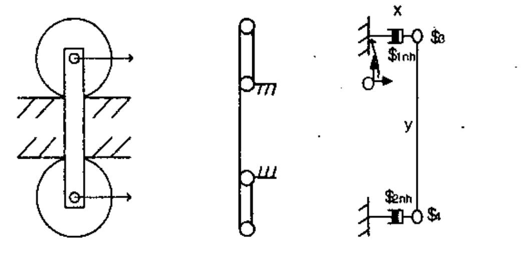
IV. Mobility Analysis for Over-constrained Mechanisms

In the followings, mobility analysis for several planar mobile robots is described. In fact, some of these examples represent over-constrained systems. It can be noted that the number of over-constraints can be easily identified by checking the number of constraints of all the independent loops in the proposed mobility analysis. Particularly in the parallelogrammic system in Fig. 1, when it is assumed that the loop A has three independent joint screws, the loop B turns out to have 2 independent joint screws. Thus, the system is indeed an over-constrained system by one over-constraint (note that if the loop B is not constrained, the number of independent joint screws of the loop must be 3).

A. Pseudo-joint representing a rolling contact

The interface of a rolling wheel with the ground has been often modeled as a revolute joint at the contact point as shown in Fig. 3(b) and Rimon and Burdick[5] showed that both the zeroth-order and first-order mobility analysis do not take the curvature characteristics of the contact surfaces into consideration, thus sometimes resulting inaccurate mobility. In spite of this fact, mobility analysis on wheeled mobile systems has not been studied much in literature.

Consider the motion of the rolling wheel of Fig. 4. The motion of the point P induced by the revolute joint model at the contact point follows a circular trajectory $\overline{PA}$. However, when observing a finite motion of this disk, the points O, C, and P follow the trajectories along the arcs $\overline{CD}$, $\overline{CC}$ and $\overline{PB}$, respectively. Thus, if a rolling interface is modeled as a revolute joint at O, it should be treated differently from the normal revolute joint in that its finite motion trajectory is different from one of a normal revolute joint shown in Fig. 3(b). Thus, in order to compensate for the missing information due to solely relying on infinitesimal motion analysis, the second-order kinematic characteristic should be taken into account. To represent the finite motion of each of the three points O, C and P, separately, the concept of pseudo-joint is introduced. Suppose in Fig. 3(a) that $\$_{0nh}, \$_{Cnh}$ and $\$_{Pnh}$ denotes the screws of the pseudo joints corresponding to the finite motion trajectories $\overline{OD}$, $\overline{CC}$ and $\overline{PB}$, respectively. Note that screws of pseudo joints $\$_{0nh}$ and $\$_{Pnh}$ are different from normal joint screw of the revolute joint, but $\$_{Cnh}$ is equal to the one of the prismatic joint. Then, an equivalent linkage model of the pseudo-joints of the mobile system of Fig. 3 can be depicted as Fig. 3(c), where the two prismatic joints denote pseudo joints representing the linear motion of the center of each wheels, and the revolute joints represent free joints connecting the wheel center to the link.



(a) (b) (c)

Fig. 3. Amobile system moving on a flat surface and its joint model. : (a) mobile system, (b) revolute joint model, (c) prismatic joint model

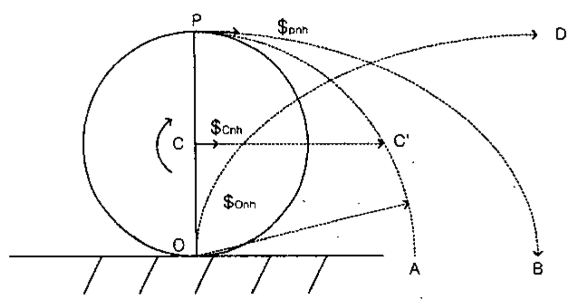


Fig. 4. Motion trajectories of points on the rolling wheel

B. Joint screw model of two different types of wheels

Table 1 shows two different types of wheels popularly employed in most of the planar mobile robots. A conventional fixed wheel is modeled as having three joints. The axis of the revolute joint (η) is along the direction normal to the ground and it passes through the contact point between the ground and the wheel, a revolute joint (θ_ω) represents the rotation about the wheel axle, and a prismatic joint ($P_\omega = r\theta$) is parallel to the contact surface on the ground. When frictional motions such as sliding and skidding occur, two prismatic joints (υ_{sl}:sliding velocity, υ_{sk}:skidding velocity) should be added to the current joint model. Likewise, the other type of wheel could be modeled similarly and summarized in Table 1.

C. Six-wheeled planar mobile robot

Table 2 shows the a little more detailed process in computing mobility of a mobile robot with two steering front wheels and four conventional rear wheels via the concepts of pseudo-joint and representative screw. Note that the motion space and mobility of the loops may depend on the order Of loops being analyzed. Starting from the loop A, its motion

Table 1. Joint Model for three Different Types of Wheels

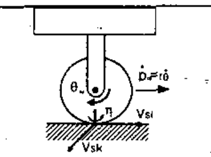
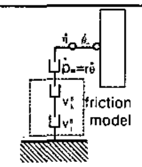
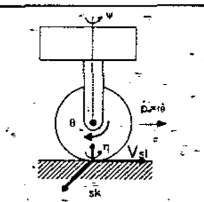
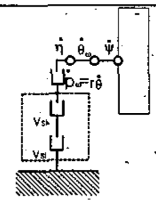
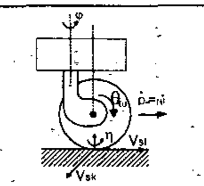
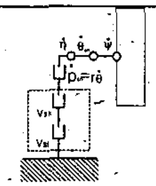
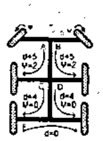
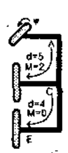
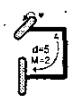
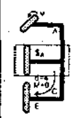
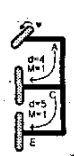
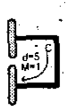
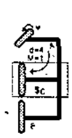
Type	Schematics	Joint Model
Conventional Wheel		
Centered Orientable Wheel		
Off-Centered Orientable Wheel		

Table 2. Mobility Analysis for Mobile Robots with Two Steering Wheels and Four Conventional Wheels (Appendix B)

Example	Half figure	Order of Constraining Loops		D			Mobility M
		A	C	No frictional motion			
				A	C	E	2
				5	4	0	
				With frictional motion			
				A	C	E	11
				5	4	3	
		C	A	No frictional motion			
				C	A	E	2
				5	4	0	
				With frictional motion			
				C	A	E	11
				5	4	3	

space and its mobility turns out to be 5 and 2, respectively. Similarly, the motion of three joints belonging to both loops A and C could be described as a representative screw, $\$_A$. Using this representative screw, the motion space of the other loop C could be obtained as 4. And the mobility of the subsystem consisting of both loops A and C is computed as 1 from Eq. (2), which represents a steering rotational motion

On the contrary, starting with the loop C, the motion space and the mobility of the loop is 5 and 1, respectively. Thus, the motion of the three joints belonging to both loops A and C can be described as a representative screw $\$_C$, which represents a forward translational motion. Using this representative screw, the motion space of the other loop A is computed as 4, differently from the previous step. Lastly, the mobility of the subsystem consisting of loops A and C turns out to be 1. It can be noted from this example that the order of analyzing loops does not make difference in computing the mobility of the mobile robot of interest after all.

Now, the other half of the mobile robot could be analyzed similarly. The mobility of each of two different subsystems of the mobile robot turns out to be 1. These two motions represent the steering motions of the two front steering wheels and do not affect the motion of joints in the Loop E. Thus the motion space of the loop E becomes O. Finally, mobility of the whole system can be computed as 2 from Eq. (2). The concept of representative screw can be applied to other various types of over-constrained mechanisms and recently, the concept was effectively used to identify the over-constrained parallel mechanism[10].

V. Mobility Analysis via Pseudo-Screws

Up to now, mobility analyses of over-constrained systems have been investigated, which can be identified by investigating the first-order kinematic characteristics (joint screw based analysis). Some of mobile robots may require the second-order mobility analysis in their singular configurations, which may be in general analytically tedious and computationally massive. Here, in replace of the second-order kinematic characteristics, the concept of "pseudo-screw" is introduced to effectively represent the real finite motion trajectory due to the rolling contact of wheels. To show the validity and effectiveness of the method, a couple of exemplary systems are examined.

A. Pseudo-screw representing the second-order kinematics

Define the pseudo-screw for a revolute joint as

$$\$^{nl} = (\vec{0}; \omega\hat{s} \times (\omega\hat{s} \times \vec{r})), \tag{3}$$

where this pseudo-screw represents the nonlinear centrifugal acceleration generated by the motion of the joint screw and in fact, is orthogonal to the direction of the linear velocity due to the joint screw, and it is related to the second order kinematic characteristics of the joint motion. This nonlinear term comes into play in analyzing mobility of mechanisms when they are at singularity configurations. Particularly, note that since the prismatic joint does not have any nonlinear acceleration orthogonal to the linear velocity term, the corresponding pseudo-screw becomes a null screw.

$$\$^{nl} = (\vec{0};\vec{0}) \tag{4}$$

For convenience, only planar cases will be considered in the followings. Suppose that there are k joints in a simple closed chain mechanism. Among these joints, l joints have distinctive d_n pseudo-screws that are orthogonal to all linear velocities from all k joints. Then the number of independent joint screws d_{Lj} of the loop L_j can be computed as follows, depending on the configuration of the mechanism. When the mechanism is not at singularity configuration, the number of independent joint screws can be computed as

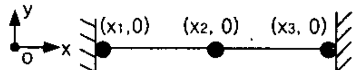
$$d_{Lj} = d_l = rank(\begin{bmatrix} \$_1 \\ \$_2 \\ \vdots \\ \$_k \end{bmatrix}). \tag{5}$$

When the mechanism is either over-constrained or at singularity configuration so that its mobility from the zeroth-order or the first-order analysis are not coincident(or inaccurate), the number of independent joint screws can be computed as

$$d_{Lj} = d_n = N(\begin{bmatrix} \$_1^{nl} \\ \$_2^{nl} \\ \vdots \\ \$_k^{nl} \end{bmatrix}), \tag{6}$$

where N denotes the number of distinctive pseudo-screws of all joints including ones with zero nonlinear acceleration components representing prismatic joint screws. Noting that pseudo screws represent nonlinear characteristics of the curvature, we define the distinctive pseudo screws as those that have different magnitudes even though they are linearly dependent.

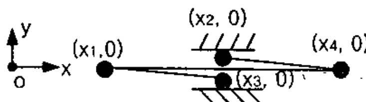
Through the following examples, the proposed method will be described in detail. Consider a mechanism in which three joints are aligned along the same line as in Fig. 5. The number of independent joint screws is 2 and there are three nonlinear joint pseudo-screws orthogonal to all the linear joint screws. Therefore, the mobility of the mechanism is computed as $M = 3 - 3 = 0$. Likewise, mobility of a mechanism in which all n joints are located at distinctive locations along the same line can be computed as $M = n - n = 0$. However, for the mechanism in which l joints out of all n joints are not located at distinct locations but distributed at other k locations, the mobility is calculated as $M = n - (n - l + k)) = l - k$. Fig. 6 shows one example in which only three joints out of four joints are located at distinct locations, but two joints are located at the same location. Thus, its mobility can be computed as $M = 4 - (4 - 2 + 1) = 1$.



$\$_1 = (0\ 0\ 1;0\ -x1\ 0)$, $\$_1^{nl} = (0\ 0\ 0;a_{x1}\ 0\ 0)$
$\$_2 = (0\ 0\ 1;0\ -x2\ 0)$, $\$_2^{nl} = (0\ 0\ 0;a_{x2}\ 0\ 0)$
$\$_3 = (0\ 0\ 1;0\ -x3\ 0)$, $\$_3^{nl} = (0\ 0\ 0;a_{x3}\ 0\ 0)$

$a_{xi} = \frac{v_i^2}{x_i}$, $v_i = \omega x_i$, $i = 1,2,3$

Fig. 5 Joint screws and its pseudo-screws for a mechanism with aligned three joints



$\$_1 = (0\ 0\ 1;0\ -x1\ 0)$, $\$_1^{nl} = (0\ 0\ 0;a_{x1}\ 0\ 0)$
$\$_2 = \$_3 = (0\ 0\ 1;0\ -x2\ 0)$, $\$_2^{nl} = \$_3^{nl} = (0\ 0\ 0;a_{x2}\ 0\ 0)$
$\$_4 = (0\ 0\ 1;0\ -x3\ 0)$, $\$_4^{nl} = (0\ 0\ 0;a_{x3}\ 0\ 0)$

$a_{xi} = \frac{v_i^2}{x_i}$, $v_i = \omega x_i$, $i = 1,2,3,4$

Fig. 6 Joint screws and its pseudo-screws for a mechanism with aligned four joints but with two joints at the same location

Now, consider another two examples in Fig. 7. The mobility of the mechanism in Fig. 7(a) can be computed as $M = 3(4-1) - 2 \times 4 = 1$ from Eq. (1). However, the mobility of the same mechanism can be computed as $M = 4 - 2 = 2$ from (2) via. the screw analysis. On the contrary, the mechanisms in Fig. 7(b), mobility can be identified as 1 from those two methods. The conflict in mobility analysis of the mechanism in Fig. 7(a) can be cleared by the proposed method in this paper. The following describes its detailed mobility analysis. The joint screws and pseudo-joint screws for the mechanism in Fig. 7(a) with respect to the referred origin can be expressed as

$$\$_1 = \$_2 = (000;100), \$_1^{nl} = \$_2^{nl} = (000;000)$$

$$\$_3 = (001;-y_1 00), \$_3^{nl} = (000;0\upsilon_1^2 / y_1 0)$$

where $\upsilon_1 = y_1\omega$, and

$$\$_4 = (001;-y_2 00), \$_4^{nl} = (000;0\upsilon_2^2 / y_2 0)$$

where $\upsilon_2 = y_2\omega$.

The number of independent joint screws can be computed as 2 and it can be seen that nonlinear screws are orthogonal to all the joint screws. Thus, the number of distinctive pseudo-screws including the ones representing prismatic joint screws will be the number of the independent pseudo-joint screws and can be obtained directly as 3. Thus, mobility of this mechanism can be computed from (2) as $M = 4-3=1$.

Similarly, the joint screws and pseudo-joint screws for the mechanism in Fig. 7(b) with respect to the referred origin can be expressed as

$$\$_1 = \$_2 = (000;100), \$_1^{nl} = \$_2^{nl} = (000;000),$$

$$\$_3 = (001;-y_1 00), \$_3^{nl} = (000;0\upsilon_1^2/y_1 0),$$

where $\upsilon_1 = y_1\omega$, and

$$\$_4 = (001;-y_2 - x_2 0), \$_4^{nl} = (000;a_x a_\upsilon 0)$$

where $\upsilon_2 = (x_2^2+y_2^2)^{-1/2}\omega_2$

$$a_x = \upsilon_2^2 x_2/(x_2^2+y_2^2), \quad a_\upsilon = \upsilon_2^2 y_2/(x_2^2+y_2^2).$$

In this case, the number of the independent joint screws can be computed as 3 and all nonlinear screws spans spaces of all linear joint screws. Thus, mobility of this mechanism can directly be computed from (2) as $M = 4-3=1$.

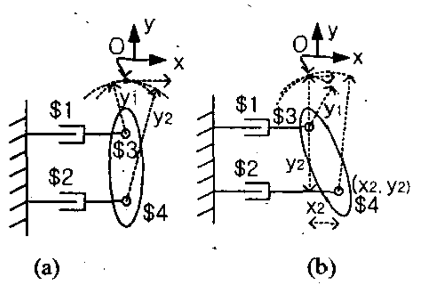


Fig. 7 Mobility analyses for (a) a one-system Mechanism and (b) a two-system mechanism

B. Mechanisms with rolling contact

Now, consider a mobile system consisting of two wheels contacted a flat surface in Fig. 3(a). Fig. 3(b) represents the joint model in which the rolling is modeled as a revolute joint at contact point. The joint model is in a singular configuration. However, in this model, the results of the zero- and first-order mobility analysis become 1 and 2, respectively. That is, the zero-order mobility using Eq. (1) becomes 1 and the first-order mobility turns out as 2 by counting the number of independent joint screws. Thus, the second-order mobility analysis is necessary to identify the true mobility.

Since the center of each wheel actually moves along the line parallel to the contact surface, the rolling interface of each wheel is modeled as a prismatic joint in Fig. 3(c). In this particular case, the screw corresponding to this prismatic joint perfectly represents the motion trajectory of the center of the wheel. That is, it also describes the finite motion of the center of the wheel correctly. This model is equivalent to one in Fig. 7 (a) and its mobility is computed as 1.

Fig. 8(a) shows a mobile robot moving on a flat surface. The feature different from Fig. 3 is the off-centered allocation of the connecting link with respect to the center of the bottom wheel. Since the trajectories of the two connecting points are different, the joint model should be different from Fig. 3(c). In Fig. 8(b), a point on the bottom wheel can no longer be modeled as a prismatic joint, but it is modeled as a pseudo-revolute joint ($\$_{2nk}$), which has different motion characteristics as compared to the normal revolute joint of Fig. 3(b), and thus it is marked as a black circle in Fig. 8(b). In this model, three joint screws $\$_1, \$_3, \$_4$ are independent one another, implying that its feasible joint motion space spans the whole 3-DOF planar space. It can be seen easily that the joint screw of the pseudo-revolute joint is dependent of those three joint screws since the motion space of the pseudo-revolute joint is in planar space. Thus, the dimension of independent joint screws is 3 and the mobility of this mechanism can be identified as $M = 4-3=1$.

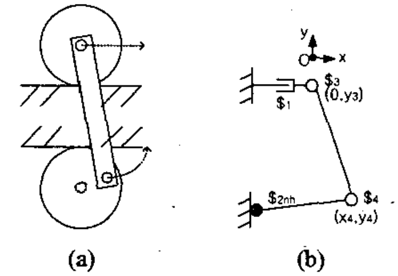


Fig. 8. (a) A mobile system and (b) its joint model

Fig. 9(a) represents a mobile system moving on two different circular surfaces. Its contact interface can be modeled as a revolute joint, the origin of which is located at the center of the curvature of the surface, as shown in Fig. 9(b). In fact, Fig. 9(a) *shows the finite motion trajectories for each of two pseudo* joints. Similarly to Fig. 3 its mobility can be computed from Eq. (2) as $M = 4-4=0$. Also, for the similar system of Fig. 9(c), Fig. 9(d) denotes its joint model. The number of joint is 4 but the number of independent joint screws is easily seen to be 3. Thus, the mobility of the system can also be identified as $M = 4-3=1$.

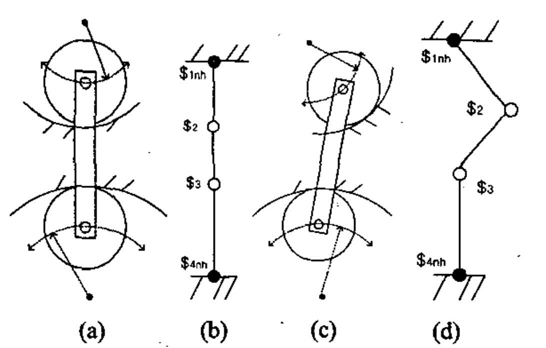


Fig. 9. A mobile robot on two circular surfaces and its joint model

VI. Conclusions

In this paper, a method of finding mobility for various types of closed-chain mechanisms with over-constraints or with rolling contacts is proposed. The method primarily utilizes joint screws of mechanisms, based on modified Grubler's Mobility formula. Importantly, the concepts of "pseudo-joint" and "pseudo-screw" are introduced to analyze the real finite motion characteristic due to the rolling contact of wheels as well as the motion of the mechanism at singular configuration. To show the validity and effectiveness of the method, a variety of planar exemplary systems are examined.

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Appendix A

Motion of a body due to the joint motion could be represented by joint screw

$$\omega \$ = \omega(\hat{s};\hat{s}_o) = \omega(\hat{s};\vec{r}\times\hat{s} + h\hat{s}), \quad \text{(A-1)}$$

where ω and $\vec{r}$ represents the angular speed about joint axis $\hat{s}$ and the position vector from an arbitrary origin, respectively. And $h = (\hat{s}\cdot\hat{s}_o)/(\hat{s}\cdot\hat{s})$ represents the screw pitch. A revolute joint with zero screw pitch and a prismatic joint with infinite screw pitch can be written as, respectively.

$$\omega \$ = \omega(\hat{s};\hat{s}_o) = \omega(\hat{s};\vec{r}\times\hat{s}), \quad \text{(A-2)}$$

$$\upsilon \$ = \upsilon(0;\hat{s}). \quad \text{(A-3)}$$

When there are k joint screws, the number of its independent screws can be computed by analyzing the rank of

$$R = rank\left(\begin{bmatrix} \$_1 \\ \$_2 \\ \vdots \\ \$_k \end{bmatrix}\right). \quad \text{(A-4)}$$

Appendix B

Joint screws and mobility analyses of three different types wheeled mobile robots are given here. These results are referred in calculating the number of independent joint screws for six-wheeled planar mobile robot in Table 2.

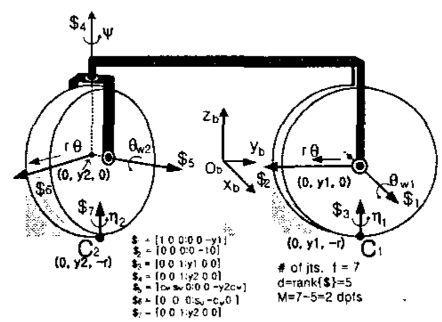

Fig. B1. Screws of a Bicycle Type Wheels

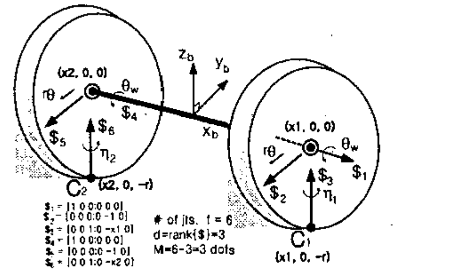

Fig. B2. Screws of Two Conventional Wheels in Parallel

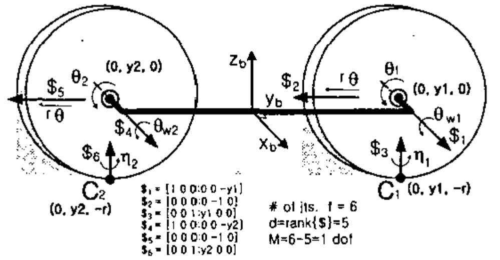

Fig. B3. Screws of Two Conventional Wheels in Series