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On Outage Probabilities with Correlated Locations, Channel, and Traffic in Wireless Networks

Udo Schilcher and Stavros Toumpis

***Abstract*—In this letter we derive novel expressions for the joint outage probability at two time slots of a wireless network under correlated interferer locations, channel gains, and traffic. The result is given as a function of the lag between the two slots, the channel coherence time, and the traffic burst length. Furthermore, we analyze the success probability of a transmission following a single outage, an important metric when, e.g., analyzing retransmission or cooperative relaying protocols.**

***Index Terms*—Joint outage probability, interference dynamics, stochastic geometry, wireless networks.**

I. Introduction

When evaluating the performance of a wireless network, an important performance metric is the *outage probability* [1]. It is often used when assessing the network's performance by mathematical expressions rather than by simulations or experiments.

Outage probabilities are mostly derived using tools from stochastic geometry [1]. A particular strength of this approach is that it can capture spatiotemporal correlations of interference [2], thus drawing a comprehensive picture of the performance of a network. In such an approach, the node locations are often modeled by Poisson point processes (PPPs), which is a good compromise between the accuracy of the model and mathematical simplicity and thus has given rise to a high number of publications, including results on assessing the performance of cellular networks [3], [4], mmWave communications [5], wireless ad hoc networks [6], and the Internet-of-Things (IoT) [7], [8]. Many of these works take into account the interference correlation that is caused by the fixed interferers' locations or, in case of mobility, the correlation of the interferers' locations at different times.

However, as we have shown in the past, interference can have additional sources of correlation besides interferer locations, namely correlated channel gains (e.g., when fading has a coherence time that spans several slots) and correlated traffic [2], [9] (e.g., when it is especially likely that a given node transmits again after a transmission occurred due to bursty transmissions or the use of a retransmission protocol). These correlation sources lead to rather diverse interference dynamics. For example, when nodes are immobile, the correlation of interference levels at two different time slots that is caused by the fixed interferer locations is independent of the time lag between the two slots. In contrast, the correlation caused by correlated channel gains and traffic depends on the time lag [2] between the slots; this has different implications for protocol design and network parametrization.

Our particular contributions are as follows:

- We derive closed-form expressions for the joint outage probability in two time slots separated by an arbitrary lag τ. These expressions, for the first time, quantify jointly the effects of correlations in the locations, the channel gains, and the transmission activities of the interferers.
- We also derive closed-form expressions for the joint and the conditional success (following a lost packet) probabilities under the same conditions.
- Having these closed-form expressions available, for the first time we quantify the manner in which the outage, joint success, and conditional success probabilities across two slots depend on the time separation of these slots, when there is no node mobility. Previous models did not capture this dependence.
- We present several numerical results that quantify the impact of the aforementioned correlations on outage.
- We provide guidelines for designing and configuring networking protocols, such as retransmission and scheduling schemes, cooperative relaying protocols, etc.
- We illustrate similarities between interference correlation and joint outage probabilities, which validates using the correlation as an indicator of network performance.

II. System model

We consider a wireless network with nodes distributed according to a PPP $\Phi \subseteq \mathbb{R}^2$ with intensity λ. By applying Slivnyak's theorem, we assume, with no loss of generality, that the receiver under consideration r, the so-called typical receiver, is located at the origin o. An illustration of the setup is provided in Fig. 1.

Time is slotted and in each slot any *idle* node starts a new traffic burst of length d slots with probability $\bar{p}$. Therefore, in each slot on average a fraction p of *all* nodes starts a new transmission and overall a fraction $pd \leq 1$ of the nodes is transmitting, while a fraction $1-pd$ is idle, where p and $\bar{p}$ are connected by the equation $\bar{p} = \frac{p}{1-p(d-1)}$ with $0 \leq p \leq 1/d$.

All nodes transmit with unit power. The wireless channel is subject to distance-dependent path loss and Rayleigh fading. Hence, the reception power at time t from a node $x \in \Phi$ is proportional to $\ell_x \, h_x(t)$, where $\ell_x = \|x\|^{-\alpha}$ and $\alpha > 2$ is the path gain; the fading gain $h_x(t)$ is exponentially distributed

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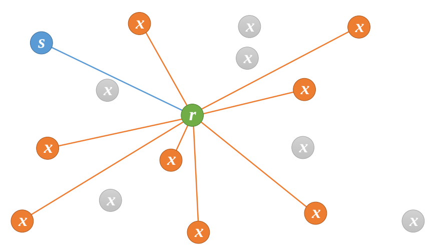

Fig. 1. Illustration of the network setup: The receiver r located at the center is aiming to receive data from a sender s. This reception is disturbed by several orange interferers $x \in \Phi$. The gray nodes are idle.

with mean $\mathbb{E}\left[h_x(t)\right] = 1$. We assume block fading, in which the channel state stays constant over c slots and then changes to an independent value. All concurrent transmissions except the desired one cause interference. The interference power at time t is the sum of all powers of interfering transmissions arriving at r, i.e., $\sum_{x\in\Phi} \ell_x h_x(t)\gamma_x(t)$. Here, $\gamma_x(t) = 1$ if x transmits at time t, and $\gamma_x(t) = 0$ otherwise. Thus, $\gamma_x(t)$ is a Bernoulli random variable with mean $\mathbb{E}\left[\gamma_x(t)\right] = pd$, which is the probability that x transmits at time t. The typical receiver r is aiming to receive transmissions from a sender $s \notin \Phi$ at distance $\|s\|$. For simpler notation, r, s, and any $x \in \Phi$ denote both the node and its location. The reception is successful if the signal-to-interference ratio SIR_t at time t is above a threshold θ, i.e.,

$$\mathrm{SIR}_t = \frac{\ell_s h_s(t)}{\sum_{x\in\Phi} \ell_x h_x(t)\gamma_x(t)} > \theta\,; \tag{1}$$

otherwise r is in outage. We define the events $\mathcal{S}_i = \{\mathrm{SIR}_{t_i} > \theta\}$ and $\mathcal{O}_i = \{\mathrm{SIR}_{t_i} \leq \theta\}$. Finally, we define $\delta = \frac{2}{\alpha}$, $\theta_{\mathrm{s}} = \frac{\theta}{\ell_s}$, $p_{11} = \mathbb{E}\left[\gamma_x(t_1)\gamma_x(t_2)\right]$, $p_{10} = \mathbb{E}\left[\gamma_x(t_1)(1-\gamma_x(t_2))\right]$, $p_{01} = \mathbb{E}\left[(1-\gamma_x(t_1))\gamma_x(t_2)\right]$, and $p_{00} = \mathbb{E}\left[(1-\gamma_x(t_1))(1-\gamma_x(t_2))\right]$.

III. Joint Outage Probabilities

A. Sending probabilities

We start by deriving expressions for the probability that a given interferer is transmitting in the considered slots.

Lemma 1: The probability that an interferer x transmits in both slots t_1 and t_2 with $t_2 - t_1 = \tau > 0$ is (see [2], Lemma 1)

$$\mathbb{E}\left[\gamma_x(t_1)\gamma_x(t_2)\right] = \max\big(0, p(d-\tau)\big) + \\ p \sum_{i=0}^{\min(\tau-1,d-1)} \sum_{j=1}^{\min(\tau-i,d)} \sum_{k=0}^{\lfloor \frac{g}{d} \rfloor} \binom{g-kd+k}{k} \bar{p}^{k+1}(1-\bar{p})^{g-kd}\,, \tag{2}$$

where $g = \tau - i - j$.

Proof: An interferer x transmits in both t_1 and t_2 in two different events: Either a single message spans both slots, or two separate messages overlap with the slots. Let $\mathbb{P}\left[\gamma_x^{\mathrm{I}}(t_1,t_2)\right]$ denote the probability of the first event. This probability vanishes for $\tau \geq d$. For $\tau < d$, we have $\mathbb{P}\left[\gamma_x^{\mathrm{I}}(t_1,t_2)\right] = p(d-\tau)$, since there are $d-\tau$ slots before t_1 in which a message can start that lasts at least until t_2, each with probability p.

Next, let $\mathbb{P}\left[\gamma_x^{\mathrm{II}}(t_1,t_2)\right]$ denote the probability of the second event. The message overlapping with t_1 can start the earliest at $d-1$ slots before t_1 and the latest at t_1 or d slots before t_2, whichever comes first, as otherwise it would overlap with t_2. Let i denote the number of slots this message extends after t_1. Then, the message overlapping with t_2 can start the earliest at t_1+i+1, but not before t_2-d+1. Let $j-1$ denote the number of slots that this message extends before t_2. Then, we have $i = 0,\ldots,\min(\tau-1,d-1)$ and $j = 1,\ldots,\min(\tau-i,d)$.

The number of slots between the two messages in t_1 and t_2 is $g = \tau - i - j$. If $g < d$, these slots are idle. Otherwise, there could be $k \leq \lfloor \frac{g}{d} \rfloor$ messages in-between, where $\lfloor\cdot\rfloor$ is the floor operator. The number of idle slots is then $e = g - dk$. The probability for given i, j, k is $\binom{g-kd+k}{k}\bar{p}^{k+1}(1-\bar{p})^{g-kd}$. Note that the power $k+1$ is because of the probability that k messages start in-between, times the probability that the message of t_2 starts. Summing over these indices gives

$$\mathbb{P}\left[\gamma_x^{\mathrm{II}}(t_1,t_2)\right] = p \sum_{i=0}^{\min(\tau-1,d-1)} \sum_{j=1}^{\min(\tau-i,d)} \sum_{k=0}^{\lfloor \frac{g}{d} \rfloor} \binom{g-kd+k}{k} \\ \cdot\ \bar{p}^{k+1}(1-\bar{p})^{g-kd}\,. \tag{3}$$

The binomial coefficient in the previous expression accounts for the number of sequences of messages and idle slots between the two messages in t_1 and t_2. Adding $\mathbb{P}\left[\gamma_x^{\mathrm{I}}(t_1,t_2)\right] + \mathbb{P}\left[\gamma_x^{\mathrm{II}}(t_1,t_2)\right]$ yields the result. ∎

Lemma 2: The probability that an interferer x transmits in exactly one of the slots t_1 and t_2 with $t_2 - t_1 = \tau$ is

$$\mathbb{E}\left[\gamma_x(t_1)(1-\gamma_x(t_2))\right] = \mathbb{E}\left[(1-\gamma_x(t_1))\gamma_x(t_2)\right] = \\ p \sum_{i=0}^{\min(\tau-1,d-1)} \sum_{k=0}^{\lfloor \frac{g}{d} \rfloor} \binom{g-kd+k}{k} \bar{p}^{k}\,(1-\bar{p})^{g-kd+1}\,, \tag{4}$$

where $g = \tau - i - 1$.

Proof: The proof is similar to the one of Lemma 1, except that x must not send in t_2. ∎

The probability that an interferer x is not transmitting in any of the two slots is then $\mathbb{E}\left[(1-\gamma_x(t_1))(1-\gamma_x(t_2))\right] = 1 - 2\,\mathbb{E}\left[\gamma_x(t_1)(1-\gamma_x(t_2))\right] - \mathbb{E}\left[\gamma_x(t_1)\gamma_x(t_2)\right]$.

B. Success probabilities

The well known success probability in a single slot is [1]:

$$\mathbb{P}\left[\mathcal{S}_1\right] = \exp\left(-\lambda p d \delta \pi^2 \theta_{\mathrm{s}}^{\delta} \csc(\pi\delta)\right)\,. \tag{5}$$

Due to block fading, for the sender and each of the interferers there are two options: either the channel states $h_x(t_1)$ and $h_x(t_2)$ are equal, or they are independent. These cases lead to different joint success probabilities.

1) Channel of sender is independent: We assume that the channel of the sender, i.e., $h_s(t_1)$, $h_s(t_2)$ are independent and identically distributed (i.i.d.).

Lemma 3 (Success for $h_x(t_1) = h_x(t_2)$): The joint success probability if $h_x(t_1) = h_x(t_2)$ for all x, but the coefficients $h_s(t_1)$, $h_s(t_2)$ of the sender are i.i.d., is

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right] = \exp\left(-\lambda\delta\pi^2\theta_{\mathrm{s}}^{\delta}\csc(\pi\delta)\big(2\,p_{10} + 2^{\delta}\,p_{11}\big)\right)\,. \tag{6}$$

Proof: The joint success probability is

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right] = \tag{7}$$

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$$\mathbb{P}\left[\frac{\ell_s h_s(t_1)}{\sum_{x\in\Phi}\ell_x h_x(t_1)\gamma_x(t_1)}>\theta, \frac{\ell_s h_s(t_2)}{\sum_{x\in\Phi}\ell_x h_x(t_2)\gamma_x(t_2)}>\theta\right]$$
$$\overset{(a)}{=} \mathbb{E}_{\Phi,h_x,\gamma_x}\left[\exp\left(-\theta_\mathrm{s}\sum_{x\in\Phi}\ell_x h_x\big(\gamma_x(t_1)+\gamma_x(t_2)\big)\right)\right]$$
$$= \mathbb{E}_\Phi\left[\prod_{x\in\Phi}\mathbb{E}_{h_x,\gamma_x}\big[\exp\big(-\theta_\mathrm{s}\ell_x h_x\big(\gamma_x(t_1)+\gamma_x(t_2)\big)\big)\big]\right]$$
$$= \mathbb{E}_\Phi\left[\prod_{x\in\Phi}\left(p_{00}+\frac{2p_{10}}{1+u}+\frac{p_{11}}{1+2u}\right)\right]$$
$$\overset{(b)}{=} \exp\left(-\lambda\int_{\mathbb{R}^2}1-\left(p_{00}+\frac{2p_{10}}{1+u}+\frac{p_{11}}{1+2u}\right)\mathrm{d}x\right),$$

where $u=\theta_\mathrm{s}\ell_x$. In (a) we define $h_x=h_x(t_1)=h_x(t_2)$ and apply the complementary cumulative distribution function of the exponentially distributed $h_s(\cdot)$. In (b) we apply the probability generating functional of Φ. Substituting $p_{00}=1-2\,p_{10}+p_{11}$ and solving the integral yields the result. ∎

Lemma 4 (Success for independent $h_x(t_1)$, $h_x(t_2)$): The joint outage probability if $h_x(t_1)$, $h_x(t_2)$ are i.i.d. for all x is

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]=\exp\left(-\lambda\delta\pi^2\theta_\mathrm{s}^\delta\csc(\pi\delta)\big(2\,p_{10}+(1+\delta)\,p_{11}\big)\right). \tag{8}$$

Proof: Along the lines of the proof to Lemma 3, we have

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]= \tag{9}$$
$$=\exp\left(-\lambda\int_{\mathbb{R}^2}1-\left(p_{00}+\frac{2p_{10}}{1+u}+\frac{p_{11}}{(1+u)^2}\right)\mathrm{d}x\right),$$

where $u=\theta_\mathrm{s}\ell_x$. The difference to Lemma 3 is in the denominator of the last fraction. Solving yields the result. ∎

Corollary 1: We assume a block fading channel with block length c for all interferers, and $t_2-t_1=\tau$. Furthermore, we assume that the channel states of the sender $h_s(t_1)$, $h_s(t_2)$ are i.i.d. In the case $\tau<c$, we have

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]=\exp\Bigg(-\lambda\delta\pi^2\theta_\mathrm{s}^\delta\csc(\pi\delta) \tag{10}$$
$$\left(\frac{c-\tau}{c}\big(2\,p_{10}+2^\delta\,p_{11}\big)+\frac{\tau}{c}\big(2\,p_{10}+(1+\delta)\,p_{11}\big)\right)\Bigg).$$

For $\tau\ge c$ the channels of all interferers become independent and the result of Lemma 4 applies.

Proof: We consider slots t_1 and t_2 with $t_2-t_1=\tau$. For block fading with length c, the expected fraction of interferers having an independent channel in the two slots is $\frac{\tau}{c}$ for $\tau<c$ and 1 otherwise. The other interferers keep the same channel. Hence, the set of interferers having a dependent / independent channel form two independent PPPs [10]. We combine them using a weighted sum of the expressions within the exponential function in Lemmata 3 and 4 yielding the result. ∎

2) Channel of sender is constant:

Lemma 5 (Success for $h_x(t_1)=h_x(t_2)$): The joint success probability if $h_x(t_1)=h_x(t_2)$ for all x and $h_s(t_1)=h_s(t_2)$ is lower bounded by

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]\ge\exp\left(-\lambda\delta\pi^2\theta_\mathrm{s}^\delta\csc(\pi\delta)\big(2\,p_{10}+p_{11}\big)\right). \tag{11}$$

Proof: Along the lines of the proof to Lemma 3, we have

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]= \tag{12}$$
$$\mathbb{P}\left[\frac{\ell_s h_s(t_1)}{\sum_{x\in\Phi}\ell_x h_x(t_1)\gamma_x(t_1)}>\theta, \frac{\ell_s h_s(t_2)}{\sum_{x\in\Phi}\ell_x h_x(t_2)\gamma_x(t_2)}>\theta\right]$$
$$=\mathbb{E}\left[\exp\left(-\theta_\mathrm{s}\max\left(\sum_{x\in\Phi}\ell_x h_x\gamma_x(t_1),\sum_{x\in\Phi}\ell_x h_x\gamma_x(t_2)\right)\right)\right]$$
$$\ge\mathbb{E}\left[\exp\left(-\theta_\mathrm{s}\sum_{x\in\Phi}\ell_x h_x\max\big(\gamma_x(t_1),\gamma_x(t_2)\big)\right)\right]$$
$$=\exp\left(-\lambda\int_{\mathbb{R}^2}1-\left(p_{00}+\frac{2p_{10}+p_{11}}{1+u}\right)\mathrm{d}x\right),$$

where $u=\theta_\mathrm{s}\ell_x$. Solving the integral yields the result. ∎

Lemma 6 (Success for independent h_1, h_2): The joint success probability in time slots t_1 and t_2 if $h_x(t_1)$, $h_x(t_2)$ independently exponentially distributed for all interferers and $h_s(t_1)=h_s(t_2)$ is lower bounded by

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]\ge\exp\left(-\lambda\delta\pi^2\theta_\mathrm{s}^\delta\csc(\pi\delta)\big(2\,p_{10}+(2-2^{-\delta})\,p_{11}\big)\right). \tag{13}$$

Proof: Along the lines of the proof to Lemma 5, we have

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]\ge \tag{14}$$
$$\ge\mathbb{E}\left[\exp\left(-\theta_\mathrm{s}\sum_{x\in\Phi}\ell_x\max\big(h_x(t_1)\gamma_x(t_1),h_x(t_2)\gamma_x(t_2)\big)\right)\right]$$
$$=\exp\left(-\lambda\int_{\mathbb{R}^2}1-\left(p_{00}+\frac{2p_{10}}{1+u}+\frac{2p_{11}}{2+3u+u^2}\right)\mathrm{d}x\right),$$

where $u=\theta_\mathrm{s}\ell_x$. Solving the integral yields the result. ∎

Corollary 2: We assume a block fading channel with block length c for all interferers, and $t_2-t_1=\tau$. Furthermore, we assume that the channel of the sender is constant, i.e., $h_s(t_1)=h_s(t_2)$. In the case $\tau<c$, we have the lower bound

$$\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]\ge\mathbb{P}_\mathrm{bound}\left[\mathcal{S}_1\mathcal{S}_2\right]=\exp\Bigg(-\pi\lambda\,\delta\pi\theta_\mathrm{s}^\delta\csc(\pi\delta) \tag{15}$$
$$\left(\frac{c-\tau}{c}\big(2\,p_{10}+p_{11}\big)+\frac{\tau}{c}\big(2\,p_{10}+(2-2^{-\delta})\,p_{11}\big)\right)\Bigg).$$

For $\tau\ge c$ the channels of all interferers become independent and the result of Lemma 6 applies.

Proof: The proof goes along the lines of the proof of Corollary 1, but applying Lemmata 5 and 6. ∎

C. Conditional and outage probabilities

The outage probability for both slots t_1 and t_2 is

$$\mathbb{P}\left[\mathcal{O}_1\mathcal{O}_2\right]=1-\mathbb{P}\left[\mathcal{S}_1\right]-\mathbb{P}\left[\mathcal{S}_2\right]+\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]. \tag{16}$$

The probability that the packet is successfully received at t_2 given that it is lost at t_1, is

$$\mathbb{P}\left[\mathcal{S}_2\,|\,\mathcal{O}_1\right]=\frac{\mathbb{P}\left[\mathcal{O}_1\mathcal{S}_2\right]}{1-\mathbb{P}\left[\mathcal{S}_1\right]}, \tag{17}$$

where $\mathbb{P}\left[\mathcal{S}_1\right]$ is given in (5). The numerator is calculated by

$$\mathbb{P}\left[\mathcal{O}_1\mathcal{S}_2\right]=\mathbb{P}\left[\mathcal{S}_2\right]-\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right], \tag{18}$$

where $\mathbb{P}\left[\mathcal{S}_1\mathcal{S}_2\right]$ is given in Corollary 1 in case of independent channels $h_s(t_1)$, $h_s(t_2)$ of the sender s.

In the case of $h_s(t_1)=h_s(t_2)$, we can bound the conditional probability by applying Corollary 2, yielding

$$\mathbb{P}\left[\mathcal{O}_1\mathcal{S}_2\right]\le\mathbb{P}\left[\mathcal{S}_2\right]-\mathbb{P}_\mathrm{bound}\left[\mathcal{S}_1\mathcal{S}_2\right]. \tag{19}$$

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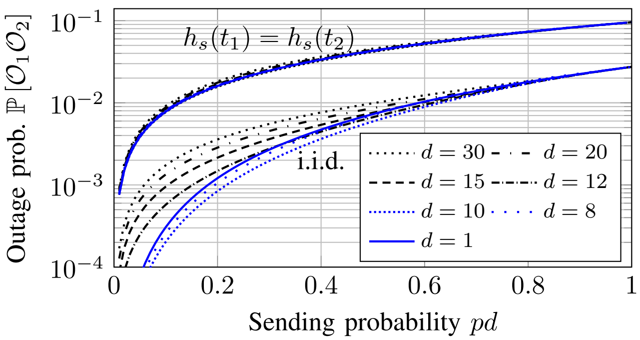

Fig. 2. The joint outage probability of slots t_1, t_2 (16) for different d is shown versus the sending probability pd. The upper group of lines is a lower bound for equal channel of the sender ($h_s(t_1) = h_s(t_2)$) as in Corollary 2, while the lower group of lines is the exact probability for i.i.d. channel of the sender as in Corollary 1. Parameters are $\tau = 10$, $\lambda = 0.01$, $c = 10$, $\alpha = 3$, and $\theta_s = 1$.

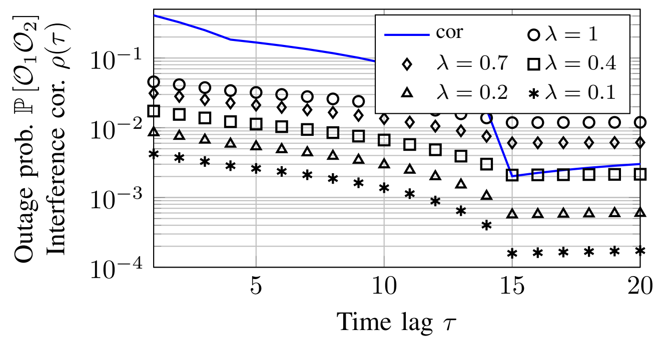

Fig. 3. $\mathbb{P}[\mathcal{O}_1\mathcal{O}_2]$ (Corollary 1) for different interferer densities λ, and the corresponding interference correlation [2] are shown versus τ. Parameters are $p = 0.001$, $c = 4$, $d = 15$, $\alpha = 3$, and $\theta_s = 1$.

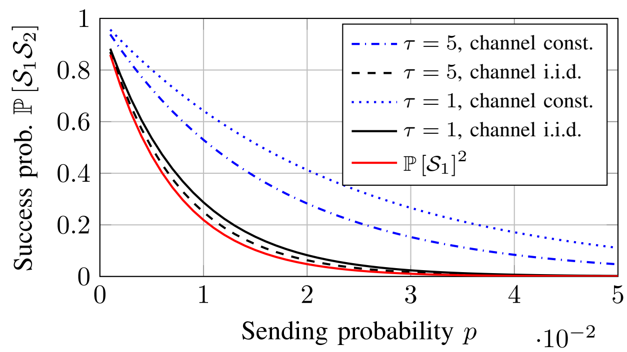

Fig. 4. $\mathbb{P}[\mathcal{S}_1\mathcal{S}_2]$ (Corollaries 1 and 2) for constant (blue lines) and i.i.d. (black lines) channel of the sender and $\tau = 1, 5$ versus different sending probabilities p. The red line indicates the baseline scenario of independent interference. Parameters are $\lambda = 1$, $c = 5$, $d = 10$, $\alpha = 3$, and $\theta_s = 1$.

IV. Numerical results

Fig. 2 shows $\mathbb{P}[\mathcal{O}_1\mathcal{O}_2]$ using (16). The sending probability is normalized to pd, which is proportional to the expected interference power as on average a fraction pd of the nodes are transmitting. As expected, a higher pd implies a higher outage probability. In the domain $d \leq \tau$ (the blue curves $d = 1, 8, 10$), we have a decrease of the outage probability with increasing d, while in the domain $d > \tau$ this trend is inverted, such that the lowest outage probability occurs for $d = \tau + 1$. Moreover, the blue traces ($d \leq \tau$) are below the black traces ($d > \tau$) for small pd, while for $pd \to 1$ the blue traces increase and partly cross the black traces. At $pd = 1$, the trace $d = 1$ reaches a higher outage probability than all $d > \tau$. This effect can be explained by a qualitative transition of interference correlation on p when d exceeds τ [2]. Overall, $d = \tau$ shows the smallest outage probability over the whole plot. Furthermore, we compare the cases of independent and identically distributed (i.i.d.) and equal ($h_x(t_1) = h_x(t_2)$) channel states of the sender. The outage probability is lower for i.i.d. channel states since when assuming there was an outage at t_1, with an independent channel there is a higher chance that at t_2 there is success. Note that the traces of $h_x(t_1) = h_x(t_2)$ are closer together than in the i.i.d. case due to the log-scale.

Fig. 3 further investigates the relationship between the lag τ, the interference correlation $\rho(\tau)$ (from [2], Case $(2, 2, 2)$) and the outage probability $\mathbb{P}[\mathcal{O}_1\mathcal{O}_2]$. This figure reveals several interesting insights: Firstly, the observation from Fig. 2 that outage is minimized for $\tau = d$ is confirmed. Secondly, the overall trend of the outage probability reflects the trend of the interference correlation. In particular, the correlation also has a minimum at $\tau = d$. Furthermore, the bend of the correlation at $\tau = c$ is also seen in the outage traces. This emphasizes the importance of the knowledge about interference correlation, such as results in [2], [9], [11], [12]. Thirdly, from a system design perspective, it is optimal when τ is chosen to be larger than the (expected) traffic burst length. In that way, the probability that both transmissions are in outage is improved.

In Fig. 4 we compare the joint success probability $\mathbb{P}[\mathcal{S}_1\mathcal{S}_2]$ for constant and i.i.d. sender channel gains. The traces of the constant channel (blue curves) are lower bounds, while the i.i.d. traces (black curves) are exact. There is a significant difference between the two cases for a given τ, where the constant channel gives a higher success probability. The real gap might be even higher as we apply a lower bound for the constant channel. The difference reduces for higher τ, which seems natural. For analyzing a real network, it makes sense to apply the result of a constant channel for $\tau \leq c$, and the result of an i.i.d. channel for $\tau > c$, based on the assumption that the channel of the sender and of the interferers c have the same coherence time. Considering these results from a network performance perspective, we can expect a jump of the success probability around $\tau = c$. Hence, it is better to time transmissions of a given sender within a delay below c slots after a successful transmission to exploit the favorable conditions before they change to independent values.

In Fig. 5 we plot the conditional probability $\mathbb{P}[\mathcal{S}_2 \,|\, \mathcal{O}_1]$ versus p and τ. The conditional probability is always below the unconditional probability $\mathbb{P}[\mathcal{S}_2]$, which indicates a positive correlation between the two slots. This correlation, and in turn the gap to the red line, varies with both p and τ. The correlation increases with p, which is a well known result [1], [9]. However, the correlation has a more intricate relation with τ: for $\tau < d$, the correlation decreases with increasing τ, leading to an increase in the conditional success probabilities with τ. However, once reaching $\tau = d$, the contribution of the traffic to interference correlation almost vanishes, and the main cause of the correlation becomes the interferer

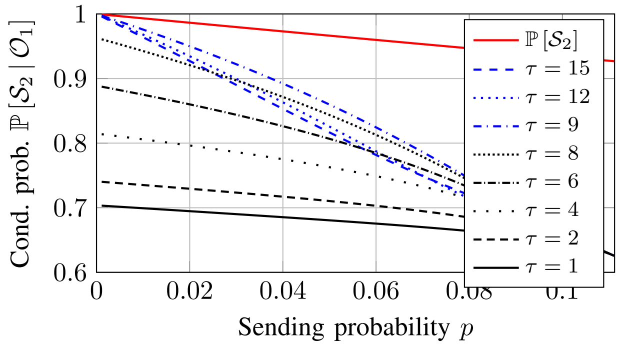

Fig. 5. The conditional success probability of slot t_2 given an outage in slot t_1 of (17) for i.i.d. channel of the sender and different values of the lag τ versus the sending probability p. Black lines are for $\tau < d$, while the blue lines are for $\tau \geq d$. The red line indicates the unconditional probability $\mathbb{P}[\mathcal{S}_2]$ for reference. Parameters are $\lambda = 0.01$, $c = 1$, $d = 9$, $\alpha = 3$, and $\theta_s = 1$.

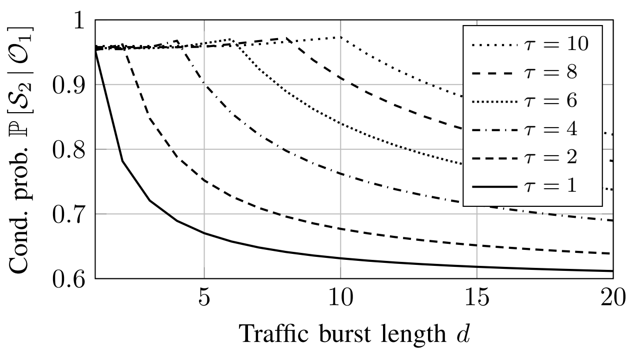

Fig. 6. The conditional success probability of slots t_1, t_2 (17) for different time lags τ is shown versus the traffic burst length d. Parameters are $\lambda = 0.01$, $pd = 0.1$, $\alpha = 3$, and $\theta_s = 1$.

locations. Hence, there is an almost linear relationship between $\mathbb{P}[\mathcal{S}_2 \mid \mathcal{O}_1]$ and τ for $\tau > d$. Further increasing τ leads to decreasing $\mathbb{P}[\mathcal{S}_2 \mid \mathcal{O}_1]$. From a systems perspective we can conclude that after an outage it is best to send the next packet after $\tau = d$. Furthermore, it can be observed that for rather high p the impact of τ decreases and vanishes close to $pd = 1$, where all nodes send all the time. This is also the point of the highest interference correlation.

In Fig. 6, we plot the conditional success probability $\mathbb{P}[\mathcal{S}_2 \mid \mathcal{O}_1]$ versus d and τ. The success probability is almost constant at a high value for $d \leq \tau$. For higher d the probability sharply drops to unfavorable values. This can be explained by interference correlation: From Fig. 3 we see that for $\tau < d$ the correlation is much higher than for $\tau > d$, where the traffic no longer contributes much. Hence, if $d > \tau$, the high correlation makes a success following an outage less likely. For increasing τ, the dropping point is at higher d, which renders a higher τ better in terms of outage. However, it has to be considered that the higher τ increases the delay for message delivery and reduces throughput due to the longer gaps between consecutive transmissions, which forms a tradeoff. In practical applications, these gaps could still be used to transmit packets to different receivers, ideally located far away from the current destination. Again the success probability is maximized for the case $d = \tau$.

V. Conclusions and Future Work

In this letter, we derived closed-form expressions of the joint success, conditional success, and outage probabilities of two transmissions separated by a given time lag. We considered, for the first time, all three sources of interference correlation, i.e., interferer locations, correlated interferer channels, and correlated interferer traffic. We provided expressions for both correlated and uncorrelated channels of the intended sender. Furthermore, we presented numerical studies of the equations to highlight the difference with respect to the uncorrelated case as well as to the case when only interferer locations are a source of correlation. Finally, we drew conclusions that allow network designers to configure the network in a manner that takes into account the underlying conditions, such as how long is the optimal time lag between two consecutive transmissions.

Future work should explore two directions: Firstly, our results should be extended to an n-dimensional outage probability. Such a result is required, e.g., to analyze a scenario in which bursty traffic occurs, or where many retransmissions are required for a successful data delivery. Secondly, the results in this letter should be exploited to analyze and improve various protocols such as cooperative relaying. Thirdly, we could investigate how the senders can exploit channel state information (CSI) or an interference predictor to reduce outage by adjusting the timing / scheduling of their transmissions.

References

[1] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2013.
[2] U. Schilcher, J. F. Schmidt, M. K. Atiq, and C. Bettstetter, "Autocorrelation and coherence time of interference in poisson networks," *IEEE Trans. Mobile Comput.*, vol. 19, no. 7, pp. 1506–1518, 2020.
[3] B. Błaszczyszyn, M. Haenggi, P. Keeler, and S. Mukherjee, *Stochastic geometry analysis of cellular networks*. Cambridge University Press, 2018.
[4] H. ElSawy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey," *IEEE Commun. Surveys & Tutorials*, vol. 15, no. 3, pp. 996–1019, 2013.
[5] J. G. Andrews, T. Bai, M. N. Kulkarni, A. Alkhateeb, A. K. Gupta, and R. W. Heath, "Modeling and analyzing millimeter wave cellular systems," *IEEE Trans. Commun.*, vol. 65, no. 1, pp. 403–430, 2016.
[6] A. Crismani, S. Toumpis, U. Schilcher, G. Brandner, and C. Bettstetter, "Cooperative relaying under spatially and temporally correlated interference," *IEEE Trans. Veh. Technol.*, vol. 64, no. 10, pp. 4655–4669, 2014.
[7] J. F. Schmidt, U. Schilcher, S. S. Borkotoky, and C. A. Schmidt, "Energy consumption in LoRa IoT: Benefits of adding relays to dense networks," in *IEEE Symp. on Computers and Commun. (ISCC)*. IEEE, 2022, pp. 1–6.
[8] R. Chevillon, G. Andrieux, L. Clavier, and J.-F. Diouris, "Stochastic geometry-based analysis of the impact of underlying uncorrelated IoT networks on LoRa coverage," *IEEE Access*, vol. 10, pp. 8790–8803, 2022.
[9] U. Schilcher, C. Bettstetter, and G. Brandner, "Temporal correlation of interference in wireless networks with Rayleigh fading," *IEEE Trans. Mobile Comput.*, vol. 11, no. 12, pp. 2109–2120, Dec. 2012.
[10] D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and Its Applications*. John Wiley & Sons Ltd, 1995, ch. 5.1.
[11] R. Ganti and M. Haenggi, "Spatial and temporal correlation of the interference in ALOHA ad hoc networks," *IEEE Commun. Lett.*, vol. 13, no. 9, pp. 631–633, Sep. 2009.
[12] U. Schilcher, J. F. Schmidt, and C. Bettstetter, "On interference dynamics in Matérn networks," *IEEE Trans. Mobile Comput.*, vol. 19, no. 7, pp. 1677–1688, 2020.