Convergence Analysis and Optimization of SWIPT-Based Over-the-Air Federated Learning

Shaoshuai Fan, Shilin Tao, Wanli Ni, and Hui Tian

***Abstract*—Federated learning (FL) are challenging for low-end Internet of Things (IoT) devices with limited energy storage. In this letter, to solve this difficulty, Base Station (BS) uses simultaneous wireless information and power transfer (SWIPT) to spread the global model and charge every device during each FL round. The convergence gap of SWIPT-based FL is derived to capture the effect of wireless communications on the learning performance. To speed FL convergence, a non-convex problem is formulated by jointly optimizing the transceiver beamforming and power-splitting ratio. Then, an alternating optimization algorithm is designed to obtain a sub-optimal solution. Simulation results show that our proposed scheme outperforms benchmarks in terms of prediction accuracy and convergence.**

***Index Terms*—Beamforming design, convergence analysis, federated learning, over-the-air computation, SWIPT.**

I. Introduction

Federated learning (FL), a distributed learning architecture, is based on the principle of model sharing rather than raw data uploading, which dramatically decreases communication overhead and latency while also protecting user privacy [1]. With the aforementioned benefits, FL may be used to various IoT situations (e.g. smart factories and smart cities), where a large number of low-end IoT devices with sufficient computing capability but limited energy are placed [2]. Nevertheless, there are certain difficulties in integrating FL in IoT networks. First off, low-end IoT devices' small battery packs are unable to supply the energy required for prolonged model training. Second, there is a shortage of communication capacity as a result of the quick increase in IoT devices.

Recently, to conquer the energy shortage in FL, the authors of [3]–[5] considered a simultaneous wireless information and power transfer (SWIPT)-based FL system to charge local devices while broadcasting the global model. Specifically, the authors of [3] have offered potential IoT and unmanned aerial network applications of the SWIPT-based FL model. Meanwhile, research is done on the difficulties and related solutions with applying SWIPT to FL. The authors of [4] researched the SWIPT-based FL model in IoT and proposed an enhanced block coordinate descent algorithm to lower FL's overall latency. Similarly, the authors of [5] proposed a dynamic optimization of device scheduling, transmit power, and power-splitting ratio, to reduce the long-term energy usage.

The work was supported by the Natural Science Foundation of Beijing under Grant L232052.(*Corresponding author: Shaoshuai Fan.*)

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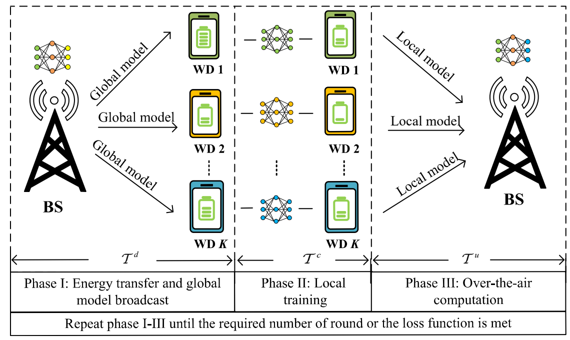

Fig. 1. An illustration of the considered SWIPT-based AirFL network.

To increase the communication efficiency in FL, the authors of [5], [6] used non-orthogonal multiple access (NOMA) to transmit local models to the base station (BS). Nevertheless, utilizing the restricted bandwidth to support massive IoT devices is challenging due to the intricacy of NOMA's decoding. Alternatively, with the waveform superposition property of multiple access channels, over-the-air computation (AirComp) integrates the data transmission and computation, which is ideal for FL's model aggregation requirements. For instance, the authors of [7] optimized the local learning rates to reduce aggregation distortion in over-the-air FL (AirFL). The authors of [8] proposed a hierarchical AirFL scheme to mitigate the negative effects of these IoT devices that are far away from the BS.

Although the authors of [9], [10] have researched SWIPT-based AirFL, they focus on improving the long-term energy efficiency of the system whose rationality is worth considering. It still remains an open question about how to properly design the transceiver beamforming and power-splitting ratio such that an optimal learning performance can be reached under the latency and energy constraints. The following is a summary of this letter's primary contributions: 1) We propose a SWIPT-based AirFL scheme to enable FL on low-end IoT devices. We construct a non-convex problem to reduce the convergence gap by jointly optimizing the transceiver beamforming and power-splitting ratio. 2) By decoupling closely-coupled variables and splitting the difficult subproblems into multiple convex problems, we design an alternating optimization algorithm to achieve a high-performance sub-optimal solution. 3) Numerical results demonstrate the effectiveness of the proposed scheme.

II. System Model

We consider a SWIPT-based AirFL network, as shown in Fig. 1, which contains one BS with M antennas and K wireless devices (WDs) with N antennas each. Let $\mathcal{D}_k = \{\mathbf{X}_k, \mathbf{y}_k\}$ and D_k represent the local dataset and the number of samples collected by WD $k \in \mathcal{K} \triangleq \{1, 2, \dots, K\}$, respectively, where

This article has been accepted for publication in IEEE Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LCOMM.2024.3371216

$\mathbf{X}_k = [\mathbf{x}_{k,1}, \mathbf{x}_{k,2}, \ldots, \mathbf{x}_{k,D_k}]$ and $\mathbf{y}_k = [y_{k,1}, y_{k,2}, \ldots, y_{k,D_k}]$ denote the input and output. Let $D = \sum_{k=1}^{K} D_k$ be the total number of samples of all WDs, and $\mathbf{w}_t \in \mathbb{R}^{\Psi}$ be the parameter vector of global model with dimension Ψ in round t. One FL round in SWIPT-based AirFL network is divided into three phases. In phase I, the BS charges all WDs and send the global model to them via SWIPT. In phase II, the gradient descent method is used to update the local model as

$$\mathbf{w}_{k,t+1} = \mathbf{w}_t - \frac{\eta}{D_k} \sum_{i=1}^{D_k} \nabla f(\mathbf{w}_t, \mathbf{x}_{k,i}, y_{k,i}) \tag{1}$$

where η is the learning rate and $\nabla f(\mathbf{w}_t, \mathbf{x}_{k,i}, y_{k,i})$ is the gradient of loss function $f(\mathbf{w}_t, \mathbf{x}_{k,i}, y_{k,i})$ with respect to $\mathbf{w}_t$. In phase III, all WDs concurrently send local models to BS by AirComp and the desired aggregated model is calculated by $\mathbf{w}_{t+1} = \frac{1}{D} \sum_{k=1}^{K} D_k \mathbf{w}_{k,t+1}$. Through multiple iterations, FL aims to minimize the global loss function

$$F(\mathbf{w}) = \frac{1}{D} \sum_{k=1}^{K} \sum_{i=1}^{D_k} f(\mathbf{w}, \mathbf{x}_{k,i}, y_{k,i}) \tag{2}$$

with dataset from all WDs, and the desired global model satisfies $\mathbf{w}^* = \mathrm{argmin} F(\mathbf{w})$.

A. SWIPT Model

In phase I, BS broadcasts updated global model to all WDs and charges them concurrently. Without loss of generality, the global model $\mathbf{w}_t$ is normalized to $\mathbf{s}_{k,t}$ with zero mean and unit variance. Note that we use various $\mathbf{s}_{k,t}$ to customize a unique energy beam for every WD, which can enhance energy efficiency and allocation flexibility. The receiving signal of WD k in round t is

$$\mathbf{y}_{k,t} = \mathbf{H}_{k,t}^{\mathrm{H}} \sum_{i=1}^{K} \mathbf{W}_{i,t} \mathbf{s}_{i,t} + \mathbf{n}_0 \tag{3}$$

where $\mathbf{H}_{k,t} \in \mathbb{C}^{M \times N}$ is the channel matrix from WD k to BS which is composed of both path loss and small-scale fading, $\mathbf{W}_{i,t} \in \mathbb{C}^{M \times \Psi}$ is the transmit beamforming, and $\mathbf{n}_0 \in \mathbb{C}^N$ is the additive white Gaussian noise with distribution $\mathcal{CN}\left(0, \sigma^2 \mathbf{I}_N\right)$. Then, the power splitting is used to allocate RF energy for information decoding and energy harvesting. Let $\rho_{k,t} \in (0,1)$ denotes the power-splitting ratio, namely $\sqrt{\rho_{k,t}} \mathbf{y}_{k,t} + \mathbf{n}_1$ for information decoding where $\mathbf{n}_1 \in \mathbb{C}^{N \times 1}$ is the decoding noise which is modelled as an additive white Gaussian noise with zero mean and variance σ_1^2, and the power of energy harvesting of WD k in round t is

$$Q_{k,t} = (1 - \rho_{k,t}) \xi_k (\textstyle\sum_{i=1}^{K} \|\mathbf{H}_{k,t}^{\mathrm{H}} \mathbf{W}_{i,t}\|_F^2 + \sigma^2) \tag{4}$$

where ξ_k is the energy conversion efficiency, and the signal-to-interference-plus-noise ratio (SINR) is

$$\mathrm{SINR}_{k,t} = \frac{\rho_{k,t} \|\mathbf{H}_{k,t}^{\mathrm{H}} \mathbf{W}_{k,t}\|_F^2}{\rho_{k,t} (\sum_{i=1, i \neq k}^{K} \|\mathbf{H}_{k,t}^{\mathrm{H}} \mathbf{W}_{i,t}\|_F^2 + \sigma^2) + \sigma_1^2}. \tag{5}$$

Suppose that the channel bandwidth is B_c and one FL model requires b bits to be transmitted successfully, the global model transmission rate is given by $r_{k,t}^d = B_c \log_2 (1 + \mathrm{SINR}_{k,t})$. Then, the latency is $\mathcal{T}_t^d = \max_{k \in \mathcal{K}} \{b / r_{k,t}^d\}$ to ensure that the global model can be successfully received by all WDs, and the harvesting energy is given by

$$E_{k,t} = (1 - \rho_{k,t}) \xi_k \mathcal{T}_t^d (\textstyle\sum_{i=1}^{K} \|\mathbf{H}_{k,t}^{\mathrm{H}} \mathbf{W}_{i,t}\|_F^2 + \sigma^2). \tag{6}$$

B. AirFL Model

In phase III, AirComp is used to aggregate local models of all WDs. Without loss of generality, local model $\mathbf{w}_{k,t+1}, \forall k \in \mathcal{K}$ is normalized as $s_{k,t+1,i} = (w_{k,t+1,i} - \bar{w}_{t+1}) / \delta_{t+1}, \forall i = 1, 2, \ldots, \Psi$, where $\bar{w}_{t+1} = \frac{1}{K\Psi} \sum_{k=1}^{K} \sum_{i=1}^{\Psi} w_{k,t+1,i}$ and $\delta_{t+1}^2 = \frac{1}{K\Psi} \sum_{k=1}^{K} \sum_{i=1}^{\Psi} (w_{k,t+1,i} - \frac{1}{\Psi} \sum_{i=1}^{\Psi} w_{k,t+1,i})^2$ are the estimate of mean and variance of model parameters, respectively. $s_{k,t+1,i}$ is i-th entry of $\mathbf{s}_{k,t+1}$ with zero mean and unit variance, namely $\mathrm{E}\{\mathbf{s}_{k,t+1} \mathbf{s}_{k,t+1}^{\mathrm{H}}\} = \mathbf{I}$. Let $\mathbf{F}_{k,t} \in \mathbb{C}^{N \times \Psi}$ denote the transmit beamforming of WD k in round t, and the superposition signal received at BS is

$$\hat{\mathbf{s}}_{t+1} = \mathbf{Z}_t^{\mathrm{H}} \textstyle\sum_{i=1}^{K} \mathbf{H}_{i,t} \mathbf{F}_{i,t} \mathbf{s}_{i,t+1} + \mathbf{Z}_t^{\mathrm{H}} \mathbf{n}_2 \tag{7}$$

where $\mathbf{Z}_t \in \mathbb{C}^{M \times \Psi}$ is the receive beamforming of BS and $\mathbf{n}_2 \in \mathbb{C}^{M \times 1}$ is the additive white Gaussian noise.

Given the ideal summation $\mathbf{s}_{t+1} = \frac{1}{D} \sum_{k=1}^{K} D_k \mathbf{s}_{k,t+1}$, the aggregation distortion between $\hat{\mathbf{s}}_{t+1}$ and $\mathbf{s}_{t+1}$ is measured by $\mathrm{MSE}_t = \mathbb{E}[\mathrm{tr}((\hat{\mathbf{s}}_{t+1} - \mathbf{s}_{t+1})(\hat{\mathbf{s}}_{t+1} - \mathbf{s}_{t+1})^{\mathrm{H}})]$, which is expanded as

$$\mathrm{MSE}_t = \textstyle\sum_{k=1}^{K} \|\mathbf{Z}_t^{\mathrm{H}} \mathbf{H}_{k,t} \mathbf{F}_{k,t} - D_k / D \mathbf{I}\|_F^2 + \sigma^2 \|\mathbf{Z}_t\|_F^2. \tag{8}$$

Then, the transmission rate of WD k is given by

$$r_{k,t}^u = B_c \log_2 (1 + \|\mathbf{H}_{k,t} \mathbf{F}_{k,t}\|_F^2 / \sigma^2) \tag{9}$$

and the latency is $\mathcal{T}_t^u = \max_{k \in \mathcal{K}} \{b / r_{k,t}^u\}$ due to the synchronization requirements in AirComp. Note that the reason for (9) is that the transmitted signals of other WDs will not interfere with WD k in AirComp. The transmission energy consumption of WD k is $E_{k,t}^u = \mathcal{T}_t^u \|\mathbf{F}_{k,t}\|_F^2$. After aggregation, the BS de-normalizes $\hat{\mathbf{s}}_{t+1}$ to obtain the global model $\hat{\mathbf{w}}_{t+1}$ as

$$\hat{w}_{t+1,i} = \delta_{t+1} \hat{s}_{t+1,i} + \bar{w}_{t+1}, \forall i = 1, 2, \ldots, \Psi. \tag{10}$$

C. Latency and Energy Consumption

Let C_k and $f_{k,t}$ denote the number of CPU cycles to process one data sample and CPU frequency of WD k, respectively. Considering that only one iteration is run locally, the computation latency of WD k is $\mathcal{T}_{k,t}^c = C_k D_k / f_{k,t}$. The energy consumption for processing one CPU cycle is calculated as $\gamma f_{k,t}^2$, where γ is a constant determined by the switched capacitance, and the energy consumption of WD k for computation is $E_{k,t}^c = C_k D_k \gamma f_{k,t}^2$. Overall, the total latency of one round consists of the latency for transmitting global model, computation and uploading local model, given by

$$\mathcal{T}_t^r = \mathcal{T}_t^d + \mathcal{T}_t^c + \mathcal{T}_t^u \tag{11}$$

where $\mathcal{T}_t^c = \max_{k \in \mathcal{K}} \{\mathcal{T}_{k,t}^c\}$ is the computation latency. The total energy consumption for WD k consisting of one part for training local model and the other part for uploading local model is given by

$$E_{k,t}^r = E_{k,t}^c + E_{k,t}^u. \tag{12}$$

III. Analysis and Problem Formulation

Assumption 1 (*L-smooth*). *For any model parameters* $\mathbf{w}$ *and* $\mathbf{v}$, *there always exists a non-negative constant L, we have*

$$F(\mathbf{w}) - F(\mathbf{v}) \leq \nabla F(\mathbf{v})^T (\mathbf{w} - \mathbf{v}) + L/2 \|\mathbf{w} - \mathbf{v}\|_2^2. \tag{13}$$

Assumption 2 (*PL Inequality*). *For any global loss function* $F(\mathbf{w})$, *there always exists a non-negative constant* μ, *we have*

$$\|\nabla F(\mathbf{w})\|_2^2 \geq 2\mu (F(\mathbf{w}) - F(\mathbf{w}^*)). \tag{14}$$

Based on the above assumptions, the convergence upper bound of SWIPT-based AirFL is provided in Theorem 1.

Theorem 1 (*Convergence upper bound of SWIPT-based AirFL*). *When Assumptions 1 and 2 are satisfied and the learning rate is $\eta = 1/L$, the convergence upper bound of SWIPT-based AirFL after T rounds is given by*

$$\mathbb{E}[F(\mathbf{w}_{T+1}) - F(\mathbf{w}^*)] \leq \mathbb{E}[F(\mathbf{w}_1) - F(\mathbf{w}^*)](1-\mu/L)^T + \textstyle\sum_{t=1}^{T}(1-\mu/L)^{T-t} \cdot L\delta_{t+1}^2/2 \cdot \mathrm{MSE}_t. \quad (15)$$

Proof: Please refer to Appendix A. □

The first term on the right-hand-side of (15) shows that as T increases, the effect of initial optimality gap disappears. The second term characterizes the effect of communication-related factors, such as transmit beamforming $\mathbf{F}_{k,t}$ and receive beamforming $\mathbf{Z}_t$. To accelerate the convergence of SWIPT-based AirFL, we minimize the convergence upper bound by jointly optimizing these factors, while guaranteeing the communication latency and energy requirements, and $\tau \triangleq \{1, 2, \ldots, T\}$. The minimization problem is given by

$$\begin{aligned}
\underset{\mathbf{W}_{k,t},\rho_{k,t},\mathbf{F}_{k,t},\mathbf{Z}_t}{\text{minimize}} \ & \textstyle\sum_{t=1}^{T}(1-\mu/L)^{T-t} \cdot L\delta_{t+1}^2/2 \cdot \mathrm{MSE}_t && (16a)\\
\text{s.t.} \quad & \mathcal{T}_t^{r} \leq \mathcal{T}_{\max}, \quad \forall t \in \tau && (16b)\\
& E_{k,t}^{r} \leq E_{k,t}, \quad \forall t \in \tau, \forall k \in \mathcal{K} && (16c)\\
& 0 < \rho_{k,t} \leq 1, \quad \forall t \in \tau, \forall k \in \mathcal{K} && (16d)\\
& \textstyle\sum_{k=1}^{K} \|\mathbf{W}_{k,t}\|_F^2 \leq P_{\max}^{\mathrm{BS}}, \quad \forall t \in \tau && (16e)\\
& \|\mathbf{F}_{k,t}\|_F^2 \leq P_k^{\max}, \quad \forall t \in \tau, \forall k \in \mathcal{K} && (16f)
\end{aligned}$$

where $\mathcal{T}_{\max}$ is the latency upper bound for one round. $P_{\max}^{\mathrm{BS}}$ and $P_k^{\max}$ are the transmission power upper bound of BS and WD k, respectively. (16c) indicates that each WD should consume less energy than its harvesting energy. Since the objective function is the weighted sum of MSE and wireless-related factors are independent of each other between T rounds, problem (16) is decomposed into following T subproblems each for one specific outer iteration $t \in \tau$.

$$\begin{aligned}
\underset{\mathbf{W}_{k,t},\rho_{k,t},\mathbf{F}_{k,t},\mathbf{Z}_t}{\text{minimize}} \ & \mathrm{MSE}_t && (17a)\\
\text{s.t.} \quad & (16b) - (16f). && (17b)
\end{aligned}$$

Due to the strong coupling of $\mathbf{F}_{k,t}$ and $\mathbf{Z}_t$ in objective function and constraints, Problem (17) is non-convex and can be solved via alternating optimization (AO). By continuously iterating and gradually decreasing the degree of coupling between variables, AO can achieve global or approximate optimal solutions [7].

IV. Proposed Solution

A. Joint Transmit and Receive Beamforming for AirComp

Given $\mathbf{W}_{k,t}$, $\rho_{k,t}$ and $\mathbf{F}_{k,t}$, minimum mean square error (MMSE) receivers is used to balance system performance and design complexity, which is given by

$$\mathbf{Z}_t = (\sigma^2\mathbf{I} + \textstyle\sum_k \mathbf{H}_{k,t}\mathbf{F}_{k,t}\mathbf{F}_{k,t}^{\mathrm{H}}\mathbf{H}_{k,t}^{\mathrm{H}})^{-1}\sum_k \frac{D_k}{D}\mathbf{H}_{k,t}\mathbf{F}_{k,t}. \quad (18)$$

Given $\mathbf{W}_{k,t}$, $\rho_{k,t}$ and $\mathbf{Z}_t$, to simplify expression, we introduce an auxiliary variable $v_t = \mathcal{T}_t^{u} = \max_{k\in\mathcal{K}}\{b/r_{k,t}^{u}\} > 0$. Then, problem (17) is reduced to

$$\begin{aligned}
\underset{\mathbf{F}_{k,t},v_t}{\text{minimize}} & \textstyle\sum_{k=1}^{K} \|\mathbf{Z}_t^{\mathrm{H}}\mathbf{H}_{k,t}\mathbf{F}_{k,t} - D_k/D\mathbf{I}\|_F^2 && (19a)\\
\text{s.t.} \quad & v_t + \mathcal{T}_t^{c} + \mathcal{T}_t^{d} \leq \mathcal{T}_{\max}, \text{ and (16f)} && (19b)\\
& \|\mathbf{F}_{k,t}\|_F^2 - (E_{k,t} - E_{k,t}^{c})1/v_t \leq 0, \forall k \in \mathcal{K} && (19c)\\
& (2^{b/(B_c v_t)} - 1)\sigma^2 - \|\mathbf{H}_{k,t}\mathbf{F}_{k,t}\|_F^2 \leq 0, \forall k \in \mathcal{K}. && (19d)
\end{aligned}$$

Algorithm 1 SCA-based WD transmit beamforming

1: **Initialize** $\mathbf{F}_{k,t}[0]$, $v_t[0]$, the value $obj_1[0]$ of objective function (21), minimum threshold ϵ and maximum iteration number N_1, and set $i = 0$.
2: **repeat**
3: Given $\mathbf{F}_{k,t}[i]$ and $v_t[i]$, we can obtain $\mathbf{F}_{k,t}^*[i]$,$v_t^*[i]$ and $obj_1^*[i]$ by solving problem (21);
4: Update $\mathbf{F}_{k,t}[i+1] = \mathbf{F}_{k,t}^*[i]$,$obj_1[i+1] = obj_1^*[i]$;
5: Update $v_t[i+1] = v_t^*[i]$, $i = i+1$;
6: **until** $i \geq N_1$ or $|obj_1[i] - obj_1[i-1]| \leq \epsilon$.

Problem (19) is non-convex due to the difference of convex (DC) function in constraints which can be solved by the Successive Convex Approximation (SCA). Through continuous iteration, SCA can achieve global or approximate optimal solutions and is widely used in engineering [11]. Meanwhile, to ensure that the solution of approximate problem is within the definition domain of the original problem, we get the convex upper bounds of (19c) and (19d) by the first-order Taylor expansion of only the second term of DC function at a point $(\mathbf{F}_{k,t}[i], v_t[i])$ obtained in the i-th iteration. The Taylor expansion is given by

$$1/v_t \geq \tilde{\mathcal{V}}(v_t) = 1/v_t[i] - 1/v_t^2[i](v_t - v_t[i]), \quad (20a)$$

$$\|\mathbf{H}_{k,t}\mathbf{F}_{k,t}\|_F^2 \geq \tilde{\Phi}_k(\mathbf{F}_{k,t}) = \|\mathbf{H}_{k,t}\mathbf{F}_{k,t}[i]\|_F^2 + 2\mathrm{tr}[(\mathbf{F}_{k,t} - \mathbf{F}_{k,t}[i])^{\mathrm{H}}\mathbf{H}_{k,t}^{\mathrm{H}}\mathbf{H}_{k,t}\mathbf{F}_{k,t}[i]], \forall k \in \mathcal{K}. \quad (20b)$$

After substituting (20a) and (20b) into problem (19), the problem is transformed as

$$\begin{aligned}
\underset{\mathbf{F}_{k,t},v_t}{\text{minimize}} & \textstyle\sum_{k=1}^{K} \|\mathbf{Z}_t^{\mathrm{H}}\mathbf{H}_{k,t}\mathbf{F}_{k,t} - D_k/D\mathbf{I}\|_F^2 && (21a)\\
\text{s.t.} \quad & \text{(16f), and } (19b) && (21b)\\
& \|\mathbf{F}_{k,t}\|_F^2 - (E_{k,t} - E_{k,t}^{c})\tilde{\mathcal{V}}(v_t) \leq 0, \forall k \in \mathcal{K} && (21c)\\
& (2^{b/(B_c v_t)} - 1)\sigma^2 - \tilde{\Phi}_k(\mathbf{F}_{k,t}) \leq 0, \forall k \in \mathcal{K}. && (21d)
\end{aligned}$$

Since problem (21) is convex, the standard optimization toolbox such as CVX can be used to solve it efficiently. The proposed SCA-based algorithm for solving problem (19) is given in Algorithm 1.

B. SWIPT Transmit Beamforming and Power Splitting

Given $\mathbf{F}_{k,t}$ and $\mathbf{Z}_t$, problem (17) is simplified as a feasibility-check problem w.r.t. $\mathbf{W}_{k,t}$ and $\rho_{k,t}$ as

$$\begin{aligned}
\text{Find} \quad & \{\mathbf{W}_{k,t}, \rho_{k,t}\} && (22a)\\
\text{s.t.} \quad & (16b) - (16e). && (22b)
\end{aligned}$$

To obtain additional performance on energy consumption, we search the feasible solution space of problem (22) to optimize total energy consumption in single round instead of arbitrary solutions on $\mathbf{W}_{k,t}$ and $\rho_{k,t}$. The minimization problem is given by

$$\begin{aligned}
\underset{\mathbf{W}_{k,t},\rho_{k,t}}{\text{minimize}} \quad & \mathcal{T}_t^{d}\textstyle\sum_{k=1}^{K} \|\mathbf{W}_{k,t}\|_F^2 && (23a)\\
\text{s.t.} \quad & (16d), (16e), && (23b)\\
& \mathcal{T}_t^{d} + \mathcal{T}_t^{c} + \mathcal{T}_t^{u} \leq \mathcal{T}_{\max}, && (23c)\\
& E_{k,t}^{c} + E_{k,t}^{u} \leq (1-\rho_{k,t})\xi_k\mathcal{T}_t^{d}(\textstyle\sum_{i=1}^{K} \left\|\mathbf{H}_{k,t}^{\mathrm{H}}\mathbf{W}_{i,t}\right\|_F^2 + \sigma^2),\\
& \forall k \in \mathcal{K}. && (23d)
\end{aligned}$$

$$f_t = \max\{\max_{k\in\mathcal{K}}\{\frac{E^c_{k,t}+E^u_{k,t}}{(1-\rho_{k,t})\xi_k(\sum_{i=1}^K \|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}\|_F^2+\sigma^2)}\}, \max_{k\in\mathcal{K}}\{\frac{b}{B_c\log_2(\frac{\|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{k,t}\|_F^2}{\sum_{i=1,i\neq k}^K\|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}\|_F^2+\sigma^2+\frac{\sigma_1^2}{\rho_{k,t}}}+1)}\}\} \quad (29)$$

Since the complex expansion of $\mathcal{T}_t^d$, we introduce an auxiliary variable $f_t = \mathcal{T}_t^d = \max_{k\in\mathcal{K}}\{b/r^d_{k,t}\} > 0$. Then, problem (23) is transformed as

$$\underset{\mathbf{W}_{k,t},\rho_{k,t},f_t}{\text{minimize}} \quad f_t\textstyle\sum_{k=1}^K\|\mathbf{W}_{k,t}\|_F^2 \quad (24a)$$

$$\text{s.t.} \quad (16d), \text{ and } (16e), \quad (24b)$$

$$f_t+\mathcal{T}_t^c+\mathcal{T}_t^u \le \mathcal{T}_{\max}, \quad (24c)$$

$$(E^c_{k,t}+E^u_{k,t})1/((1-\rho_{k,t})\xi_k f_t)-\textstyle\sum_{i=1}^K\|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}\|_F^2$$
$$-\sigma^2\le 0, \forall k\in\mathcal{K}, \quad (24d)$$

$$\textstyle\sum_{i=1,i\neq k}^K\|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}\|_F^2-\|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{k,t}\|_F^2 1/(2^{b/(B_cf_t)}-1)$$
$$+\sigma^2+\sigma_1^2/\rho_{k,t}\le 0,\forall k\in\mathcal{K}. \quad (24e)$$

Problem (24) is non-convex due to the coupling of $\mathbf{W}_{k,t}$, $\rho_{k,t}$ and f_t in objective function and constraints. We adopt the alternating optimization (AO) technique to divide the problem into two sub-problems. The first sub-problem is given by

$$\underset{\mathbf{W}_{k,t},\rho_{k,t}}{\text{minimize}} \quad \textstyle\sum_{k=1}^K\|\mathbf{W}_{k,t}\|_F^2 \quad (25a)$$

$$\text{s.t.} \quad (16d),\ (16e),\ (24d), \text{ and } (24e). \quad (25b)$$

Problem (25) is non-convex due to the DC function in constraint (24d) and (24e). We adopt the same method as problem (19) to transform it into a convex problem, and the Taylor expansion of the second term of DC function at a point $(\mathbf{W}_{k,t}[j], \rho_{k,t}[j])$ obtained in j-th iteration is given by

$$\|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}\|_F^2 \ge \tilde{\Theta}_{k,i}(\mathbf{W}_{i,t}) = \|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}[j]\|_F^2$$
$$+2\mathrm{tr}[(\mathbf{W}_{i,t}-\mathbf{W}_{i,t}[j])^{\mathrm{H}}\mathbf{H}_{k,t}\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}[j]], \forall k,i\in\mathcal{K}. \quad (26)$$

After substituting the (26) into problem (25), the problem is transformed as

$$\underset{\mathbf{W}_{k,t},\rho_{k,t}}{\text{minimize}} \quad \textstyle\sum_{k=1}^K\|\mathbf{W}_{k,t}\|_F^2 \quad (27a)$$

$$\text{s.t.} \quad (16d), \text{ and } (16e), \quad (27b)$$

$$(E^c_{k,t}+E^u_{k,t})1/((1-\rho_{k,t})\xi_k f_t)-\textstyle\sum_{i=1}^K\tilde{\Theta}_{k,i}(\mathbf{W}_{i,t})$$
$$-\sigma^2\le 0,\forall k\in\mathcal{K}, \quad (27c)$$

$$\textstyle\sum_{i=1,i\neq k}^K\|\mathbf{H}^{\mathrm{H}}_{k,t}\mathbf{W}_{i,t}\|_F^2-\tilde{\Theta}_{k,k}(\mathbf{W}_{k,t})1/(2^{b/(B_cf_t)}-1)$$
$$+\sigma^2+\sigma_1^2/\rho_{k,t}\le 0,\forall k\in\mathcal{K}. \quad (27d)$$

Problem (27) is also convex. The SCA-Based algorithm to solve problem (25) is similar to Algorithm 1 and thus is omitted here. The second sub-problem is given by

$$\text{minimize}_{f_t} \quad f_t \quad (28a)$$

$$\text{s.t.} \quad (24c),\ (24d), \text{ and } (24e). \quad (28b)$$

Problem (28) is convex, and we can obtain its solution as (29). Now, after the above alternative analysis, the algorithm to solve problem (17) is given in algorithm (2). Although the AO and SCA algorithms become more sophisticated as systems get larger, the threshold can be adjusted to balance latency and model accuracy. Meanwhile, latency can be reduced to the lowest feasible level due to server-side execution of algorithm.

Algorithm 2 AO-based algorithm for solving problem (17)

1: **Initialize** $\mathbf{W}^{(0)}_{k,t}$, $\rho^{(0)}_{k,t}$, $\mathbf{F}^{(0)}_{k,t}$, $\mathbf{Z}^{(0)}_t$, $f^{(0)}_t$, $obj^{(0)}_m$ of (17), $obj^{(0)}_a$ of (23), ϵ_1, ϵ_2, N_1, N_2, $i=j=0$.
2: **repeat**
3: update $j=j+1$;
4: Given $\mathbf{W}^{(0)}_{k,t}$, $\rho^{(0)}_{k,t}$ and $\mathbf{F}^{(j-1)}_{k,t}$, obtain $\mathbf{Z}^{*(j)}_t$ by using the MMSE receiver (18);
5: With $\mathbf{Z}^{*(j)}_t$, obtain $\mathbf{F}^{*(j)}_{k,t}$ by solving problem (21);
6: Update $\mathbf{F}^{(j)}_{k,t}=\mathbf{F}^{*(j)}_{k,t}$, $\mathbf{Z}^{(j)}_t=\mathbf{Z}^{*(j)}_t$,$obj^{(j)}_m$;
7: **until** $j\ge N_1$ or $|obj^{(j)}_m-obj^{(j-1)}_m|\le\epsilon_1$;
8: **repeat**
9: update $i=i+1$;
10: With $\mathbf{F}^*_{k,t}$, $\mathbf{Z}^*_t$ and $f^{(i-1)}_t$, obtain $\mathbf{W}^{*(i)}_{k,t}$, $\rho^{*(i)}_{k,t}$ by solving problem (27);
11: With $\mathbf{W}^{*(i)}_{k,t}$, $\rho^{*(i)}_{k,t}$, obtain $f^{*(i)}_t$ by using (29).
12: Update $\mathbf{W}^{(i)}_{k,t}=\mathbf{W}^{*(i)}_{k,t}$,$\rho^{(i)}_{k,t}=\rho^{*(i)}_{k,t}$;
13: Update $f^{(i)}_t=f^{*(i)}_t$, $obj^{(i)}_a$;
14: **until** $i\ge N_2$ or $|obj^{(i)}_a-obj^{(i-1)}_a|\le\epsilon_2$;
15: **Output** $\mathbf{F}^*_{k,t}$, $\mathbf{Z}^*_t$,$\mathbf{W}^*_{k,t}$ and $\rho^*_{k,t}$.

TABLE I
SIMULATION PARAMETERS

Name	Value	Name	Value	Name	Value
D_k	200	B_c	20 MHz	P^{BS}_{max}	50 W
Ψ	159010	C_k	10000∼30000	P^{max}_k	0.05 W
η	0.05	γ	$1e^{-28}$	T_{max}	1 s
σ^2	-80 dBm	μ	0.8	T	80
σ_1^2	-80 dBm	L	1	$f_{k,t}$	100 MHz
ξ_k	0.7	b	159010 × 32	M	2
N	8				

V. Numerical Results

We consider an SWIPT-based AirFL network with $K=10$ WDs which collaboratively train a shared multi-layer perceptron (MLP) network with one hidden layer of 200 neurons based on the MNIST dataset or FashionMNIST dataset, and the settings of simulation parameters are listed in Table I. For comparison, we consider three benchmarks: 1) Noise-free AirFL 2) Fixed beamforming, and 3) Conventional AirFL [12](WDs have limited energy and cannot replenish energy during federated learning processes).

Fig. 2 plots the accuracy on MNIST(M) dataset and FashionMNIST(FM) dataset with the increasing number of rounds, respectively. We observe that the Proposed AirFL outperforms Fixed beamforming and Conventional AirFL schemes in accuracy and convergence. Compared to Fixed beamforming, the Proposed AirFL optimizes the transceiver beamforming to greatly reduce the MSE, thereby reducing the impact of environmental noise on the aggregation model. Compared to Conventional AirFL, the Proposed AirFL adopts SWIPT to

This article has been accepted for publication in IEEE Communications Letters. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/LCOMM.2024.3371216

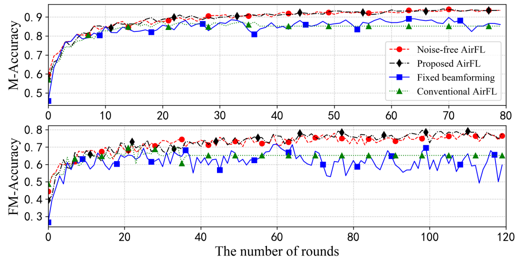

Fig. 2. Accuracy versus the number of rounds.

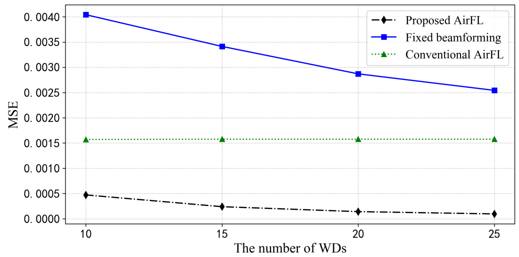

Fig. 3. MSE versus the number of WDs.

replenish energy for IoT devices, increasing the number of devices participating in FL. Note that the Proposed AirFL approaches Noise-free AirFL in accuracy.

Fig. 3 shows that the MSE decreases with the number of WDs. As the total number of WDs increases, the amount of samples trained in FL also increases, which leads to a reduced MSE value. Meanwhile, compared to Fixed beamforming and Conventional AirFL schemes, the Proposed AirFL has a lower MSE value, resulting in smaller convergence gap and better accuracy.

VI. Conclusion

In this letter, a SWIPT-based AirFL scheme was proposed to support FL in energy constrained scenarios. We considered that BS adopted SWIPT to simultaneously send global model and energy to all WDs in each round. After local training, WDs participating in FL adopted AirComp to send local models to BS, and utilized the superposition property of multiple access channels to integrate the communication and calculation of model parameters. A SCA-based algorithm was designed to obtain a sub-optimal solution of the formulated non-convex problem. Simulation results validated that the proposed SWIPT-based AirFL achieved improvements in accuracy and convergence compared to conventional FL.

Appendix A: Proof of Theorem 1

Let $\tilde{\mathbf{w}}_{t+1}$ denote the ideal aggregated model in round t, satisfying $\tilde{\mathbf{w}}_{t+1} = \frac{1}{D}\sum_{k=1}^{K} D_k \mathbf{w}_{k,t+1}$. According to (1), we have $\tilde{\mathbf{w}}_{t+1} = \mathbf{w}_t - \eta \nabla F(\mathbf{w}_t)$. Then, we denote $\mathbf{w}_{t+1}$ as the estimate of aggregated model in round t, and $\varepsilon_{t+1} = \mathbf{w}_{t+1} - \tilde{\mathbf{w}}_{t+1}$, we have

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta \nabla F(\mathbf{w}_t) + \varepsilon_{t+1}. \tag{30}$$

Through the second-order Taylor expansion, $F(\mathbf{w}_{t+1})$ is rewrote as

$$\begin{aligned} F(\mathbf{w}_{t+1}) &= F(\mathbf{w}_t) + (\mathbf{w}_{t+1} - \mathbf{w}_t)^T \nabla F(\mathbf{w}_t) \\ &\quad + 1/2(\mathbf{w}_{t+1} - \mathbf{w}_t)^T \nabla^2 F(\mathbf{w}_t)(\mathbf{w}_{t+1} - \mathbf{w}_t) \\ &\le F(\mathbf{w}_t) + (\mathbf{w}_{t+1} - \mathbf{w}_t)^T \nabla F(\mathbf{w}_t) + L/2\|\mathbf{w}_{t+1} - \mathbf{w}_t\|^2, \end{aligned} \tag{31}$$

where the inequality stems from the Assumption 1. Given the learning rate $\eta = 1/L$ and based on (30), we have

$$\begin{aligned} F(\mathbf{w}_{t+1}) &\le F(\mathbf{w}_t) + (\varepsilon_{t+1} - \eta \nabla F(\mathbf{w}_t))^T \nabla F(\mathbf{w}_t) \\ &\quad + L/2\|\varepsilon_{t+1} - \eta \nabla F(\mathbf{w}_t)\|^2 \\ &= F(\mathbf{w}_t) - 1/(2L)\|\nabla F(\mathbf{w}_t)\|^2 + L/2\|\varepsilon_{t+1}\|^2. \end{aligned} \tag{32}$$

Then taking the expectation of both sides of this inequality, and based on (14), we have

$$\begin{aligned} \mathbb{E}[F(\mathbf{w}_{t+1})] &\le \mathbb{E}[F(\mathbf{w}_t)] - \mu/L\mathbb{E}[F(\mathbf{w}_t) - F(\mathbf{w}^*)] \\ &\quad + L/2\mathbb{E}\|\varepsilon_{t+1}\|^2. \end{aligned} \tag{33}$$

Subtracting $\mathbb{E}\left[F(\mathbf{w}^*)\right]$ from both sides of the above inequality, we have one-round convergence as

$$\begin{aligned} \mathbb{E}[F(\mathbf{w}_{t+1}) - F(\mathbf{w}^*)] &\le (1 - \mu/L)\mathbb{E}[F(\mathbf{w}_t) - F(\mathbf{w}^*)] \\ &\quad + L/2\mathbb{E}\|\varepsilon_{t+1}\|^2. \end{aligned} \tag{34}$$

According to (10), we can rewrite $\tilde{\mathbf{w}}_{t+1} = \delta_{t+1}/D\sum_{k=1}^{K} D_k \mathbf{s}_{k,t+1} + \bar{w}_{t+1}\mathbf{e}_\Psi$ and $\mathbf{w}_{t+1} = \delta_{t+1}\sum_{k=1}^{K} \mathbf{Z}_t^H \mathbf{H}_{k,t}\mathbf{F}_{k,t}\mathbf{s}_{k,t+1} + \delta_{t+1}\mathbf{Z}_t^H \mathbf{n}_2 + \bar{w}_{t+1}\mathbf{e}_\Psi$, where $\mathbf{e}_\Psi = (1, 1, \ldots, 1)^T$ is a vector with dimension Ψ. Based on two above expressions, we can expand $\mathbb{E}\|\varepsilon_{t+1}\|^2$ as

$$\begin{aligned} &\mathbb{E}\|\varepsilon_{t+1}\|^2 = \mathbb{E}\|\mathbf{w}_{t+1} - \tilde{\mathbf{w}}_{t+1}\|^2 \\ &= \delta_{t+1}^2 \mathbb{E}\|\textstyle\sum_{k=1}^{K}(\mathbf{Z}_t^H \mathbf{H}_{k,t}\mathbf{F}_{k,t} - \frac{D_k}{D}\mathbf{I})\mathbf{s}_{k,t+1} + \mathbf{Z}_t^H \mathbf{n}_2\|^2. \end{aligned} \tag{35}$$

Substituting (35) into (34) and applying (34) recursively for T times, we finally reach (15).

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