

Approximating Throughput and Packet Decoding Delay in Linear Network Coded Wireless Broadcast

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Abstract—In this paper, we study a wireless packet broadcast system that uses linear network coding (LNC) to help receivers recover data packets that are missing due to packet erasures. We study two intertwined performance metrics, namely throughput and average packet decoding delay (APDD) and establish strong/weak approximation relations based on whether the approximation holds for the performance of every receiver (strong) or for the average performance across all receivers (weak). We prove an equivalence between strong throughput approximation and strong APDD approximation. We prove that throughput-optimal LNC techniques can strongly approximate APDD, and partition-based LNC techniques may weakly approximate throughput. We also prove that memoryless LNC techniques, including instantly decodable network coding techniques, are not strong throughput and APDD approximation nor weak throughput approximation techniques.

Index Terms—Wireless broadcast, network coding, throughput, decoding delay, approximation.

I. INTRODUCTION

In this paper, we consider a wireless broadcast problem where a sender wishes to broadcast a block \mathcal{P} of K data packets to a set of N receivers using linear network coding (LNC) [1], [2]. Each receiver is assumed to already possess a subset of \mathcal{P} and still wants all the remaining data packets.

For such systems, two important performance metrics are throughput and average packet decoding delay (APDD). While throughput measures how fast the broadcast can be completed, APDD measures how fast each individual data packet can be decoded by each receiver. A lower APDD implies faster data delivery to the application layer on average, and is particularly important when individual data packets are informative.

Throughput can be maximized if every LNC coded packet is innovative to every receiver who has not fully recovered \mathcal{P} . Such packets can be generated either randomly (i.e., the classic random LNC (RLNC) technique [3]) or deterministically (e.g., by solving a hitting set problem [4], or by adding extra data packets to instantly decodable coded packets [5]). However, the APDD performance of these techniques have not been well studied. Recently, Yu *et al* proved that RLNC approximates the minimum APDD average over all receivers with a ratio of 2 [6], i.e., its APDD is at most two times of the minimum.

APDD can be minimized if every LNC coded packet allows every receiver to instantly decode a wanted data packet [6]. Such coded packets, however, are NP-hard to find [6]. Instead, instantly decodable network coding (IDNC) techniques generate in each transmission a coded packet that allows a subset of receivers to instantly decode a wanted packet, and

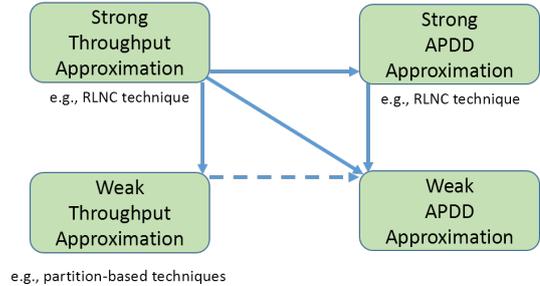


Fig. 1. Interplay between throughput and APDD approximation.

ask the remaining receivers to discard this coded packet rather than storing it in the memory for future decoding. Thus, IDNC techniques are memoryless. Due to this feature, IDNC techniques are generally not throughput optimal [7]. It has also been proved in [8] that it is intractable to maximize the throughput of general IDNC techniques. Although a large body of heuristics have been developed as a remedy, it is an open problem whether memoryless LNC techniques are able to approximate the optimal throughput and APDD.

A tradeoff between throughput and APDD can be achieved by partitioning \mathcal{P} into disjoint sub-blocks and broadcasting them separately using certain LNC techniques [9]–[12]. Although it is understood that such partitioned-based LNC techniques are generally not throughput and APDD optimal, their approximation performance has not been studied.

Due to the fact that throughput and APDD optimization could be intractable and that heuristics cannot guarantee bounded performance, throughput and APDD approximation is important in the design and evaluation of LNC techniques. However, to the best of our knowledge, LNC throughput and APDD approximation has not received much attention in the literature. Moreover, the interplay between throughput and APDD optimization has not been well studied. We also note that optimization is a special approximation with a ratio of 1.

Therefore, in this paper, we study the more general problem of throughput and APDD approximation in linear network coded wireless broadcast. Specifically, we will

- 1) introduce the concepts of strong/weak throughput and APDD approximation. Here strong (resp. weak) means that the approximation holds for the performance of every receiver (resp. averaged over all receivers);
- 2) investigate the interplay between throughput and APDD approximation;

- 3) evaluate the approximation performance of the aforementioned three classes of LNC techniques.

Some main findings of this paper are (also depicted in Fig. 1):

- A strong throughput β -approximation technique also strongly approximates APDD with a ratio of at most 2β . This relation does not necessarily hold between weak throughput and APDD approximation;
- All strong throughput-optimal LNC techniques strongly approximate APDD with a ratio between $\frac{4}{3}$ and 2;
- A technique that partitions the packet block into M disjoint sub-blocks and applies a weak throughput β -approximation technique to each sub-block weakly approximates throughput with a ratio of at most βM ;
- Memoryless LNC techniques are not strong throughput and APDD approximation nor weak throughput approximation techniques.

II. SYSTEM MODEL AND PERFORMANCE MEASUREMENTS

We consider a block-based wireless broadcast scenario, in which the sender wishes to deliver a block of K data packets, denoted by $\mathcal{P} = \{\mathbf{p}_k\}_{k=1}^K$, to a set of N receivers, denoted by $\mathcal{R} = \{r_n\}_{n=1}^N$. All data packets are vectors of the same length, with entries taken from a finite field \mathbb{F}_q . Time is slotted, and in each time slot the sender broadcasts a coded packet to all receivers. The wireless channel between the sender and each receiver r_n is independent of each other, and is subject to Bernoulli random packet erasures with a probability of $P_{e,n}$.

We assume each receiver has already received a subset of packets in \mathcal{P} and still wants all the rest. Such a packet reception state could be the consequence of previous uncoded transmissions [13], and is a common assumption in network coding and index coding literature [14]. This state can be summarized by a binary $N \times K$ state feedback matrix (SFM) \mathbf{A} : $\mathbf{A}(n, k) = 1$ means r_n has missed \mathbf{p}_k and wants it, and $\mathbf{A}(n, k) = 0$ means r_n already has \mathbf{p}_k . The set of data packets wanted by r_n is denoted by \mathcal{W}_n . Its size is denoted by w_n .

The sender then applies an LNC technique to help receivers recover their missing data packets. In each time slot, it broadcasts an LNC packet \mathbf{c} , which takes the form of:

$$\mathbf{c} = \sum_{\mathbf{p}_k \in \mathcal{P}} \alpha_k \mathbf{p}_k, \quad (1)$$

where $\{\alpha_k\}$ are coding coefficients chosen from \mathbb{F}_q . In particular, RLNC technique chooses $\{\alpha_k\}$ uniformly at random.

A. Performance Metrics

Our first performance metric is throughput. It measures how fast the broadcast of \mathcal{P} can be finished. Noting that a minimum of K uncoded transmissions is initially needed, we measure throughput by the total number of coded transmissions in the broadcast, which is denoted by U . Clearly, a smaller U indicates higher throughput. We further denote by U_n the number of coded transmissions after which receiver r_n decodes all its wanted data packets. Consequently, $U = \max(\{U_n\}_{n=1}^N)$.

Our second performance metric is average packet decoding delay (APDD). It reflects how fast each individual data packet

is decoded by each receiver on average. Given a realization of \mathbf{A} , the APDD of receiver r_n , denoted by D_n , is:

$$D_n = \frac{1}{w_n} \sum_{\forall k: \mathbf{p}_k \in \mathcal{W}_n} u_{n,k}, \quad (2)$$

where $u_{n,k}$ is the index of the coded transmission after which r_n decodes \mathbf{p}_k . The APDD across all receivers is similar:

$$D = \frac{1}{\text{sum}(\mathbf{A})} \sum_{\forall k, n: \mathbf{A}(n,k)=1} u_{n,k}, \quad (3)$$

where $\text{sum}(\mathbf{A})$ is the sum of the entries of \mathbf{A} , and is equal to the number of ones in \mathbf{A} .

B. Performance Limits and Expectations

We denote by $U_{\min,n}$ (resp. U_{\min}) the minimum possible U_n (resp. U) that any LNC techniques can offer without packet erasures. It is clear that $U_{\min,n} = w_n$ and $U_{\min} = \max(\{w_n\}_{n=1}^N)$. We further denote by $\bar{U}_{\min,n}$ (resp. \bar{U}_{\min}) the minimum expected U_n (resp. U) that any LNC techniques can offer with random packet erasures. It is clear that $\bar{U}_{\min,n} \geq U_{\min,n}$ and $\bar{U}_{\min} \geq U_{\min}$, and the equalities hold when there are no packet erasures. \bar{U}_{\min} has been studied in the literature through studying RLNC [15].

Similarly, we denote by $D_{\min,n}$ (resp. D_{\min}) the minimum D_n (resp. D) that any LNC techniques can offer without packet erasures. It holds that $D_{\min,n} \geq \frac{w_n+1}{2}$ and $D_{\min} \geq \frac{\sum w_n D_{\min,n}}{\sum w_n}$, where the equality holds when every coded packet allows every receiver to instantly decode a wanted data packet. We further denote by $\bar{D}_{\min,n}$ (resp. \bar{D}_{\min}) the minimum expected D_n (resp. D) that any LNC techniques can offer with random packet erasures. Again, $\bar{D}_{\min,n} \geq D_{\min,n}$ and $\bar{D}_{\min} \geq D_{\min}$. It is proved in [16] that \bar{D}_{\min} is NP-hard to find.

If an LNC technique called “X” is applied, we add (X) to the end of the above. For example, $\bar{D}_{\min,n}(\text{RLNC})$ denotes the minimum expected APDD D_n of r_n under RLNC.

III. DEFINING PERFORMANCE APPROXIMATION

In this section, we define strong and weak approximation of throughput and APDD.

A. Strong Approximation

We define strong throughput approximation as follows:

Definition 1. An LNC technique X is a strong throughput β -approximation technique if and only if:

$$\frac{\bar{U}_{\min,n}(\text{X})}{\bar{U}_{\min,n}} \leq \beta \quad (4)$$

for every receiver r_n under any SFM and any packet erasure probabilities $\{P_{e,n}\}_{n=1}^N$, where $\beta \geq 1$ is a constant. In particular, if $\beta = 1$, then technique-X is a strong throughput-optimal technique.

According to this definition, when a strong throughput β -approximation LNC technique is applied, every receiver r_n can expect to decode all its wanted data packets within $\beta \bar{U}_{\min,n}$

coded transmissions regardless of the packet reception state and packet erasure probability of the other receivers.

We define strong APDD approximation similarly:

Definition 2. An LNC technique X is a strong APDD β -approximation technique if and only if:

$$\frac{\bar{D}_{\min,n}(X)}{\bar{D}_{\min,n}} \leq \beta \quad (5)$$

for every receiver r_n under any SFM and any packet erasure probabilities $\{P_{e,n}\}_{n=1}^N$, where $\beta \geq 1$ is a constant. In particular, if $\beta = 1$, then technique- X is a strong APDD-optimal technique.

B. Weak Approximation

We define weak throughput approximation as follows:

Definition 3. An LNC technique X is a weak throughput β -approximation technique if and only if:

$$\frac{\bar{U}_{\min}(X)}{\bar{U}_{\min}} \leq \beta \quad (6)$$

for any SFM and any packet erasure probabilities $\{P_{e,n}\}_{n=1}^N$, where $\beta \geq 1$ is a constant. In particular, if $\beta = 1$, then technique- X is a weak throughput-optimal technique.

According to this definition, with a weak throughput β -approximation LNC technique, we can expect to complete the coded broadcast within $\beta\bar{U}_{\min}$ coded transmissions.

Similarly, we define weak APDD approximation as follows:

Definition 4. An LNC technique X is a weak APDD β -approximation technique if and only if:

$$\frac{\bar{D}_{\min}(X)}{\bar{D}_{\min}} \leq \beta \quad (7)$$

for any SFM and any packet erasure probabilities $\{P_{e,n}\}_{n=1}^N$, where $\beta \geq 1$ is a constant. In particular, if $\beta = 1$, then technique- X is a weak APDD-optimal technique.

It is clear that a strong throughput/APDD approximation technique is also a weak one, but not necessarily vice versa. Our main interest in this paper is the interplay between throughput and APDD approximation. To this end, we first establish the performance of a reference technique, namely RLNC, that strongly approximates both throughput and APDD.

IV. THE PERFORMANCE OF RLNC

In this section, we study the approximation performance of RLNC, and then extend the result to the general class of throughput-optimal LNC techniques.

Theorem 1. RLNC is a strong throughput-optimal and strong APDD 2-approximation technique. Mathematically, for every receiver r_n , it always holds that $\bar{U}_{\min,n}(\text{RLNC}) = \bar{U}_{\min,n}$ and $\bar{D}_{\min,n}(\text{RLNC}) \leq 2\bar{D}_{\min,n}$.

Proof: It is clear that

$$\bar{U}_{\min,n}(\text{RLNC}) = \bar{U}_{\min,n} = \frac{w_n}{1 - P_{e,n}}, \quad (8)$$

Then, since r_n decodes all its wanted data packets on average after $\bar{U}_{\min,n}(\text{RLNC})$ coded transmissions, we have

$$\bar{D}_{\min,n}(\text{RLNC}) = \bar{U}_{\min,n}(\text{RLNC}) = \frac{w_n}{1 - P_{e,n}}. \quad (9)$$

On the other hand, the authors of [16] has proved that

$$\bar{D}_{\min,n} \geq \frac{w_n + 1}{2(1 - P_{e,n})}, \quad (10)$$

Therefore,

$$\frac{\bar{D}_{\min,n}(\text{RLNC})}{\bar{D}_{\min,n}} \leq \frac{2w_n}{w_n + 1} \leq 2, \quad (11)$$

which completes the proof. \blacksquare

We note that in terms of APDD performance, RLNC is in fact, the worst technique in the class of strong throughput-optimal LNC techniques, as it generally does not provide early packet decodings (excluding occasional early decodings). Thus, we can state that all LNC techniques in this class strongly approximate APDD with a ratio of at most 2. This result can be further strengthened into the following:

Theorem 2. All strong throughput-optimal LNC techniques strongly approximate APDD with a ratio between $\frac{4}{3}$ and 2.

Proof: Since the approximation ratio of an LNC technique is the largest ratio it provides across any SFMs, to prove that the ratio is at least $\frac{4}{3}$ for strong throughput-optimal techniques we only need an instance of SFM where $\frac{4}{3}$ is achieved by them.

Our SFM consists of 2 data packets and 3 receivers. r_1 only wants \mathbf{p}_1 , r_2 only wants \mathbf{p}_2 , and r_3 wants both packets. For this SFM, any strong throughput-optimal LNC technique- X will send as \mathbf{c}_1 a linear combination of \mathbf{p}_1 and \mathbf{p}_2 to satisfy both r_1 and r_2 . However, \mathbf{c}_1 does not allow r_3 to decode. r_3 can only decode after the second coded transmission. Thus, $D_{\min,3}(X) = 2$. On the other hand, by sending \mathbf{p}_1 and \mathbf{p}_2 separately, $D_{\min,3} = 1.5$. Thus, $\frac{U_{\min,3}(X)}{U_{\min,3}} = \frac{4}{3}$. \blacksquare

V. INTERPLAY BETWEEN THROUGHPUT AND APDD APPROXIMATION

With the help of RLNC, we establish the following relation between throughput and APDD approximation:

Theorem 3. Strong throughput β -approximation techniques strongly approximate APDD with a ratio of at most 2β .

Proof: Consider a strong throughput β -approximation LNC technique called X . By definition,

$$\bar{U}_{\min,n}(X) \leq \beta\bar{U}_{\min,n}, \quad (12)$$

for any receiver r_n in any given SFM. Then, since

$$\begin{cases} \bar{D}_{\min,n}(X) \leq \bar{U}_{\min,n}(X), \\ \bar{U}_{\min,n} = \bar{U}_{\min,n}(\text{RLNC}) = \bar{D}_{\min,n}(\text{RLNC}), \text{ and} \\ \bar{D}_{\min,n}(\text{RLNC}) \leq 2\bar{D}_{\min,n}, \end{cases}$$

we obtain $\bar{D}_{\min,n}(X) \leq 2\beta D_{\min,n}$. ■

On the other hand, weak throughput approximation techniques do not necessarily weakly approximate APDD. To see this, we will prove in the next section that partition-based LNC techniques may weakly approximate throughput but may not weakly approximate APDD. We summarize the interplay between throughput and APDD approximation in Fig. 1.

VI. PARTITION-BASED LNC TECHNIQUES

Given an SFM, partition-based LNC techniques partition the packet block \mathcal{P} into M ($M > 1$) disjoint or overlapped sub-blocks $\{\mathcal{P}_m\}_{m=1}^M$ [9]–[12], [17]. In this paper, we only consider the disjoint case. Correspondingly, the SFM is partitioned into M sub-SFMs $\{\mathbf{A}_m\}_{m=1}^M$, where in \mathbf{A}_m , the N receivers want data packets from \mathcal{P}_m . A certain LNC technique (e.g. RLNC) is then applied to each sub-block separately in order.

Theorem 4. *If a partition-based LNC technique applies a weak throughput β -approximation LNC technique to each of the M sub-SFMs, then it is at most a weak throughput $2\beta M$ -approximation technique.*

Proof: When a weak throughput β -approximation technique called X is applied to any given SFM \mathbf{A} , by definition it holds that $\bar{U}_{\min}(X) \leq \beta \bar{U}_{\min}$. Since any sub-SFM of \mathbf{A} requires, on average, at most $\bar{U}_{\min}(X)$ coded transmissions, any M -partition of \mathbf{A} need at most $M\bar{U}_{\min}(X)$ coded transmissions. Thus, $\bar{U}_{\min}(\text{partition}, X) \leq \beta M \bar{U}_{\min}$. ■

However, weakly approximating throughput may not help these techniques weakly approximate APDD:

Theorem 5. *Partition-based throughput weak approximation techniques do not necessarily weakly approximate APDD.*

Proof: For any given SFM \mathbf{A} , without loss of generality let us assume receiver r_1 wants the largest subset \mathcal{W}_1 of data packets of \mathcal{P} . Consider a partition-based technique, called X , that partitions \mathcal{P} into two sub-blocks: $\mathcal{P}_1 = \mathcal{W}_1$ and $\mathcal{P}_2 = \mathcal{P} \setminus \mathcal{W}_1$. Due to Theorem 5, when RLNC is applied, technique- X can weakly approximate the throughput with a ratio of 2.

We now show that technique- X cannot weakly approximate APDD. Consider an SFM with N receivers. r_1 wants \mathcal{W}_1 , and all the remaining $N-1$ receivers only want one data packet not in \mathcal{W}_1 . When $N \gg w_1$, we have $D_{\min} \approx 1$. But if technique- X is applied, the remaining $N-1$ receivers can only decode after r_1 has fully decoded, indicating that $D_{\min}(X) \approx w_1 + 1$, which is not within a constant multiple of D_{\min} .¹ ■

This theorem also indicates the general relation between weak throughput and APDD approximation:

Corollary 1. *Weak throughput approximation techniques are not necessarily weak APDD approximation techniques.*

However, we are not able to identify the strong throughput and APDD approximation performance of partition-based LNC techniques without specifying the partitioning strategy, which is out of the scope of this paper.

¹ For this particular SFM, there exist better partition strategies that are able to minimize APDD. This, however, is irrelevant to the theorem and its proof.

VII. MEMORYLESS LNC TECHNIQUES

An LNC technique is memoryless if its receivers discard undecodable coded packet(s) rather than storing them for future decodings. A well-known class of memoryless LNC techniques is IDNC, which allows a subset of receivers to instantly decode a wanted data packet from each coded packet, so that APDD could be reduced. However, the cost is a degradation in the throughput of receivers who discard useful, but instantly undecodable coded packets. In this section, we prove the following two theorems:

Theorem 6. *Memoryless LNC techniques are not weak throughput approximation techniques.*

We prove this theorem in the appendix by showing that, for an SFM where every pair of two data packets is wanted by a different receiver, memoryless LNC techniques require at least $\lceil \log_2 K \rceil + 1$ coded transmissions, which is not within a constant multiple of $U_{\min} = 2$. (Here $\lceil x \rceil$ is the smallest integer greater than or equal to x .)

Then, since every strong throughput approximation technique is a weak one, the above theorem indicates that:

Corollary 2. *Memoryless LNC techniques are not strong throughput approximation techniques.*

The proof of Theorem 6 also sheds some light on the APDD approximation performance of memoryless LNC techniques.

Theorem 7. *Memoryless LNC techniques are not strong APDD approximation techniques.*

Proof: In the proof of Theorem 6, receivers who decode their second wanted data packet after the last coded transmission have an APDD of at least $\lceil \log_2 K \rceil / 2 + 1$. However, $D_{\min,n} \leq D_{\min,n}(\text{RLNC}) = 2$. Thus, $D_{\min,n}(\text{memoryless})$ is not within a constant multiple of $D_{\min,n}$. ■

We summarize our results on the approximation performance of the three classes of LNC techniques in Table I.

VIII. CONCLUSION

In this paper, we generalized the problem of throughput and APDD optimization in linear network coded wireless broadcast to their strong and weak approximations. This generalization fills the gap between optimal and heuristic LNC techniques with approximation techniques, such as RLNC (strong throughput optimal and strong APDD 2-approximation) and partition-based LNC techniques (weak throughput approximation). By using these LNC techniques as references, we also revealed the interplay between throughput and APDD approximation, including a relation between strong throughput β -approximation and strong APDD 2β -approximation, as well as the independence between weak throughput and APDD approximation. Besides, we negated the strong and weak throughput approximation and the strong APDD approximation of memoryless LNC techniques. Our results could inspire new approaches to design and evaluate LNC techniques.

TABLE I
THE APPROXIMATION PERFORMANCE OF THREE CLASSES OF LNC.

	Strong Throughput	Strong APDD	Weak Throughput	Weak APDD
RLNC	yes	yes	yes	yes
Partition-based	open	open	yes	may not be
Memoryless LNC	no	no	no	open

As future work, we wish to tackle the interplay between strong APDD approximation and weak throughput approximation, the strong throughput and APDD approximation performance of partition-based LNC techniques, and the weak APDD approximation performance of memoryless LNC techniques. We are also interested in extending our research to index coding [14], as well as and other applications of LNC, such as cooperative data exchange [18].

APPENDIX A PROOF OF THEOREM 6

Our proof involves two types of SFMs:

- $\mathbf{A}_1(K)$: every pair of data packets is wanted by a different receiver. There are $N = \frac{K(K-1)}{2}$ receivers;
- $\mathbf{A}_2(K, m)$: every data packet is wanted by m different receivers. Every pair of data packets is wanted by a different receiver. There are $N = mK + \frac{K(K-1)}{2}$ receivers.

Note that $U_{\min} = 2$ for $\mathbf{A}_1(K)$. We prove the theorem by proving that $U_{\min}(\text{memoryless}) = \lceil \log_2 K \rceil + 1$ for $\mathbf{A}_1(K)$.

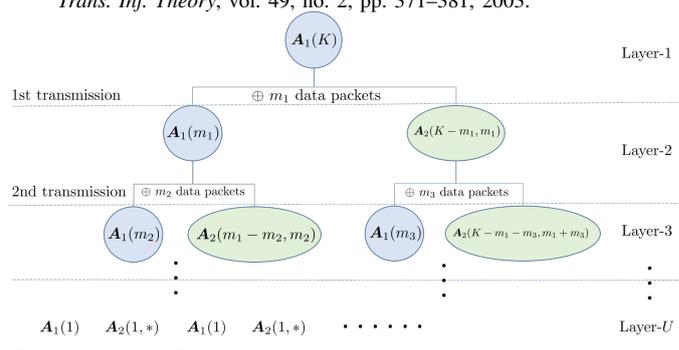
The transmission starts by sending as \mathbf{c}_1 the XOR of any $m_1 \geq 1$ data packets in $\mathbf{A}_1(K)$. The resulted SFM consists of two sub-SFMs: 1) an $\mathbf{A}_1(K - m_1)$, which contains the m_1 data packets and the receivers who want 2 data packets from \mathbf{c}_1 and thus discard \mathbf{c}_1 ; 2) an $\mathbf{A}_2(K - m_1, m - 1)$, which contains the remaining $K - m_1$ data packets and the remaining receivers, which either has decoded one wanted data packet from \mathbf{c}_1 and still want one data packet from $K - m_1$, or want 2 data packets from $K - m_1$. These two sub-SFMs are independent in the sense that a coded packet of $\mathbf{A}_1(m_1)$ and a coded packet of $\mathbf{A}_2(K - m_1, m_1)$ can be XOR-ed and sent without affecting their decodability for receivers.

Similarly, we can show that after sending the XOR of any arbitrary m_3 data packets from $\mathbf{A}_2(K - m_1, m_1)$, the resulted SFM consists of two independent sub-SFMs: an $\mathbf{A}_1(m_3)$ and an $\mathbf{A}_2(K - m_1 - m_3, m_1 + m_3)$.

Continuing the logic, after the u -th transmission ($u \geq 1$), $\mathbf{A}_1(K)$ is split into 2^{u-1} type-1 sub-SFMs and 2^{u-1} type-2 sub-SFMs. Only sub-SFMs that consists of a single data packet can be completed in one coded transmission and be removed. The evolution of $\mathbf{A}_1(K)$ is demonstrated in a layered graph in figure 2. The u -th layer corresponds to the SFM before the u -th coded transmission. The total number of coded transmissions is thus the number of layers plus one. It is clear that the minimum number of layers is $\lceil \log_2 K \rceil$, which is achieved by XOR-ing half of the data packets from each sub-SFM. Thus, $U_{\min}(\text{memoryless}) = \lceil \log_2 K \rceil + 1$.

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The $(U + 1)$ -th transmission will finish the broadcast.

Fig. 2. The decoding evolution of $\mathbf{A}_1(K)$. (Note that '*'s are arbitrary positive integers whose values depend on the coded packets.)