

# Reduced-order washout controllers stabilizing uncertain equilibrium points

メタデータ	言語: English 出版者: 公開日: 2017-10-03 キーワード (Ja): キーワード (En): 作成者: Takimoto, Takashi, Yamamoto, Shigeru, 山本, 茂 メールアドレス: 所属:
URL	<a href="https://doi.org/10.24517/00009667">https://doi.org/10.24517/00009667</a>

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 3.0 International License.



# Reduced-order Washout Controllers Stabilizing Uncertain Equilibrium Points

Takashi Takimoto and Shigeru Yamamoto

**Abstract**—We consider a local stabilization problem of an uncertain equilibrium point existed in a nonlinear continuous-time system by a finite-dimensional dynamical state feedback controller. In previous research, it is investigated that steady-state blocking zeros of the stabilizing controller play an important role. Such a controller is called a washout controller. In this paper, we develop a design method for reduced-order washout controllers whose order is less than the plant's order. Additionally, we also consider a local stabilization problem of an uncertain fixed point of a given discrete-time system.

## I. INTRODUCTION

This paper concerns a local stabilization problem of an uncertain equilibrium point of nonlinear dynamical systems by a dynamical state feedback controller. In a standard design procedure for feedback controllers to stabilize an equilibrium point, it is assumed that the equilibrium point is accurately known. However, the equilibrium point is generally uncertain in the real system. In stabilization of the uncertain equilibrium point, the uncertainty of the equilibrium point results in nonzero steady-state control input so that a different equilibrium point is stabilized.

It is well-known in chaos control community that delayed feedback control is a powerful control method for stabilizing equilibrium points without their exact information [1]. The delayed feedback controller eliminates the dependence on the steady-state to use its steady-state blocking zero (steady-state blocking zeros mean blocking zeros at zero frequency). In contrast to the simple structure of delayed feedback controllers, the design of feedback parameters is complicated, because the closed-loop system with delayed feedback is an infinite-dimensional system in continuous-time (see [2] and [3] for details).

An easier way is to adopt a finite-dimensional dynamical state feedback controller with steady-state blocking zeros. In [4] and [5], to stabilize the continuous-time system with the unknown equilibrium point, a certainty equivalence adaptive control scheme was proposed. In [6] and [7], a design method of a washout filter aided feedback controller for both continuous-time and discrete-time systems was discussed. The washout filter is a high-pass filter which can eliminate the steady-state input. Moreover, in [8] and [9], we proposed a finite-dimensional dynamic controller with steady-state blocking zeros, called a *washout controller*. The proposed

The first author is with the Department of Systems Innovation, Graduate School of Engineering Science, Osaka University, 1-3, Machikaneyama, Toyonaka, Osaka 560-8531, Japan [takimoto@hopf.sys.es.osaka-u.ac.jp](mailto:takimoto@hopf.sys.es.osaka-u.ac.jp)

The last author is with Graduate School of Natural Science and Technology, Kanazawa University, Kakuma, Kanazawa, Ishikawa 920-1192, Japan

washout controller is a generalization of washout filter aided feedback controllers proposed in [6] and [7].

In previous researches, the existence condition and parameterization of washout controllers were derived for the case where the order of the controller is the same as the plant's order [8], [9]. In this paper, as a special class of washout controllers, we propose a reduced-order washout controller whose order is the same as the control input vector. Then, we derive existence conditions of the reduced-order washout controller. Moreover, we show that such a reduced-order washout controller can be designed by solving a stabilization problem by constant state feedback.

## II. PROBLEM STATEMENT

We consider an  $n$ th-order nonlinear continuous-time system described by

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t)), \\ y(t) &= x(t),\end{aligned}\quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the input vector, and  $y(t) \in \mathbb{R}^n$  is the measured output (i.e., the state vector  $x(t)$  is assumed to be measured). We assume that  $f$  is differentiable. Let  $x_f$  be an equilibrium point of the system (1) with  $u(t) \equiv 0$ , that is,  $0 = f(x_f, 0)$ . Then, the linearized system around the equilibrium point  $x_f$  is given by

$$\begin{aligned}\delta\dot{x}(t) &= A\delta x(t) + Bu(t), \\ y(t) &= \delta x(t) + x_f,\end{aligned}\quad (2)$$

where

$$\delta x(t) = x(t) - x_f, \\ A = \left. \frac{\partial f(x, u)}{\partial x} \right|_{x=x_f, u=0}, \quad B = \left. \frac{\partial f(x, u)}{\partial u} \right|_{x=x_f, u=0}.$$

In this paper, we assume that  $(A, B)$  is stabilizable.

The control purpose is to stabilize the equilibrium point  $x_f$  of the system (1), that is, to design a feedback controller such that  $\lim_{t \rightarrow \infty} x(t) = x_f$  and  $\lim_{t \rightarrow \infty} u(t) = 0$ . To this end, we consider the local stabilization of the equilibrium point  $x_f$  of the system (1) without changing its equilibrium point  $x_f$ . In addition, it is assumed that the equilibrium point  $x_f$  of the system (1) is uncertain, because it is generally difficult to get the exact value of the equilibrium point  $x_f$  in the real system. Therefore, we consider the stabilization of the uncertain equilibrium point  $x_f$  by using only  $y(t) = x(t)$  as feedback. Of course, if the equilibrium point  $x_f$  is available, it can be directly used in state feedback as  $u(t) = G(y(t) - x_f) = G\delta x(t)$ , where  $G$  is a feedback gain.

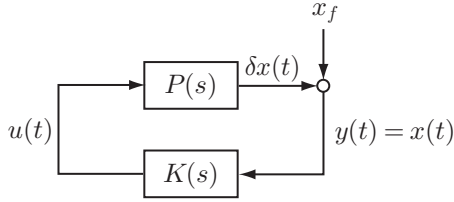


Fig. 1. Closed-loop system of the linearized system  $P(s)$  and the controller  $K(s)$ .

### III. WASHOUT CONTROL

We consider an  $\tilde{n}$ th-order continuous-time dynamic state feedback controller described by

$$K(s) : \begin{cases} \dot{w}(t) &= \hat{A}w(t) + \hat{B}y(t), \\ u(t) &= \hat{C}w(t) + \hat{D}y(t). \end{cases} \quad (3)$$

In the vicinity of the equilibrium point  $x_f$ , the closed-loop system with the dynamic controller (3) is given by

$$\begin{bmatrix} \delta \dot{x}(t) \\ \dot{w}(t) \end{bmatrix} = A_c \begin{bmatrix} \delta x(t) \\ w(t) \end{bmatrix} + B_c x_f, \quad (4)$$

where

$$A_c := \begin{bmatrix} A + B\hat{D} & B\hat{C} \\ \hat{B} & \hat{A} \end{bmatrix}, \quad B_c := \begin{bmatrix} B\hat{D} \\ \hat{B} \end{bmatrix}.$$

Moreover, by defining  $P(s) = (sI - A)^{-1}B$ , the closed-loop system is depicted in Fig. 1. For the closed-loop system, the equilibrium point  $x_f$  can be regarded as a steady-state disturbance. By this steady-state disturbance, in steady-state, the control input  $u(t)$  may be biased. Because, when the closed-loop system is stable, from the final value theorem, we have

$$\lim_{t \rightarrow \infty} u(t) = \{I - K(0)P(0)\}^{-1}K(0)x_f. \quad (5)$$

This suggests that another equilibrium point  $x'_f$  is stabilized so that  $0 = f(x'_f, u_f)$  where  $u_f \neq 0$  and  $x'_f \neq x_f$ . Hence, in the uncertain equilibrium stabilization, it is important that the dynamic controller (3) stabilizes the linearized system (2) and eliminates the influence of the steady-state disturbance  $x_f$ .

*Definition 1:* The linearized system (2) is said to be *washout controllable*, if there exists a finite-dimensional continuous-time controller (3) such that the closed-loop system (4) is asymptotically stable and  $\lim_{t \rightarrow \infty} u(t) = 0$  for any  $x_f \neq 0$ . Moreover, such a controller is called a *washout controller*.

Now, we consider a class of the controllers (3) having a steady-state blocking zero

$$\mathcal{K}_{\tilde{n}} = \left\{ (3) \mid \hat{A} \in \mathbb{R}^{\tilde{n} \times \tilde{n}}, \det \hat{A} \neq 0, \hat{D} - \hat{C}\hat{A}^{-1}\hat{B} = 0 \right\}, \quad (6)$$

where the subindex  $\tilde{n}$  indicates the order of the controller. It is obvious that for any  $\tilde{n}$  all members of  $\mathcal{K}_{\tilde{n}}$  satisfy  $K(0) = 0$ , that is,  $K(s) = 0$  at  $s = 0$ . If the closed-loop system by a controller  $K(s) \in \mathcal{K}_{\tilde{n}}$  is asymptotically stable, (2) is washout controllable. Because, from (5), we have  $\lim_{t \rightarrow \infty} u(t) = 0$ .

For any  $K(s) \in \mathcal{K}_{\tilde{n}}$ , there exists a vector  $w_f$  such that

$$\begin{cases} 0 &= \hat{A}w_f + \hat{B}x_f, \\ 0 &= \hat{C}w_f + \hat{D}x_f, \end{cases} \quad (7)$$

for any  $x_f$ . Then, from (7) and (4), the closed-loop system is given by

$$\begin{bmatrix} \delta \dot{x}(t) \\ \delta \dot{w}(t) \end{bmatrix} = A_c \begin{bmatrix} \delta x(t) \\ \delta w(t) \end{bmatrix}. \quad (8)$$

where  $\delta w(t) = w(t) - w_f$ . Therefore, the elimination of the dependence on steady-state disturbance  $x_f$  is reduced to the stabilization of the closed-loop system (8).

*Theorem 1:* If the linearized system (2) is washout controllable by a controller  $K(s) \in \mathcal{K}_{\tilde{n}}$ , then  $A$  is nonsingular.

*Proof:* When the closed-loop system (8) is asymptotically stable,  $A_c$  does not have any zero eigenvalues. Hence,  $\det A_c \neq 0$ . Under conditions  $\det \hat{A} \neq 0$  and  $\hat{D} - \hat{C}\hat{A}^{-1}\hat{B} = 0$ , since

$$\begin{aligned} \det A_c &= \det \hat{A} \det(A + B\hat{D} - B\hat{C}\hat{A}^{-1}\hat{B}) \\ &= \det \hat{A} \det A, \end{aligned}$$

we have  $\det A \neq 0$ . ■

*Theorem 2:* ([8]) If  $A$  is nonsingular, by using  $F$  and  $L$  such that  $A + BF$  and  $A + L$  are asymptotically stable, a class of  $n$ th-order continuous-time washout controllers  $\mathcal{K}_n$  is given by

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \mathcal{F}_l(J, Q), \quad Q = -FA^{-1}L, \quad (9)$$

$$J = \begin{bmatrix} \begin{bmatrix} A + BF + L & -L \\ F & 0 \end{bmatrix} & \begin{bmatrix} B \\ I \\ 0 \end{bmatrix} \\ \begin{bmatrix} -I & I \end{bmatrix} & \end{bmatrix},$$

where  $\mathcal{F}_l$  is the lower linear fractional transformation, that is,

$$\begin{aligned} \hat{A} &= (A + BF)A^{-1}(A + L), \\ \hat{B} &= -(A + BF)A^{-1}L, \\ \hat{C} &= FA^{-1}(A + L), \\ \hat{D} &= -FA^{-1}L. \end{aligned}$$

When the order of the controller is the same as the plant's order, the system (2) is washout controllable if and only if  $A$  is nonsingular.

*Remark 1:* In [6], [7], washout filter aided feedback controller have been proposed. Their controllers belong to

$$\mathcal{K}_n^c := \left\{ (3) \mid \hat{A} \in \mathbb{R}^{n \times n}, \det \hat{A} \neq 0, \hat{B} = -\hat{A}, \hat{D} = -\hat{C} \right\}.$$

In fact,  $\mathcal{K}_n^c \subset \mathcal{K}_n$ .

### IV. REDUCED-ORDER WASHOUT CONTROLLER

In Theorem 2, [6], and [7], the order of the dynamic controller (3) is the same as the plant's order. In this section, we derive a reduced-order dynamic controller and show that it can be designed by considering a subset of (6).

In the following, we assume that the order of the controller is the same as that of the input vector. Then, we consider a class of the dynamic controllers (3) which is given by

$$\mathcal{K}_m^r = \left\{ (3) \mid \hat{A} \in \mathbb{R}^{m \times m}, \det \hat{A} \neq 0, \hat{C} = \hat{A}, \hat{D} = \hat{B} \right\}. \quad (10)$$

Then, it is obvious that  $\mathcal{K}_m^r \subset \mathcal{K}_m$ . Therefore, all controllers of  $\mathcal{K}_m^r$  have the steady-state blocking zero.

Moreover, for any  $K(s) \in \mathcal{K}_m^r$ , there exists a  $w_f$  satisfying (7) for any  $x_f$ . Then, the closed-loop system by the controller  $K(s) \in \mathcal{K}_m^r$  is also given by (8). In addition, by using conditions  $\hat{C} = \hat{A}$  and  $\hat{D} = \hat{B}$ , the closed-loop system (8) can be rewritten as

$$\begin{aligned} \dot{X}(t) &= \bar{A}X(t) + \bar{B}\bar{u}(t), \\ \bar{u}(t) &= \bar{K}X(t), \end{aligned} \quad (11)$$

where

$$\begin{aligned} X(t) &= \begin{bmatrix} \delta x(t) \\ \delta w(t) \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ I_m \end{bmatrix}, \\ \bar{K} &= \begin{bmatrix} \hat{B} & \hat{A} \end{bmatrix}. \end{aligned}$$

Therefore, for  $K(s) \in \mathcal{K}_m^r$ , the design of feedback parameters  $\hat{A}$  and  $\hat{B}$  stabilizing the closed-loop system (8) can be cast into that of the state feedback gain  $\bar{K}$  stabilizing the closed-loop system (11). Moreover, when  $m < n$ ,  $\mathcal{K}_m^r$  gives the reduced-order controllers.

*Theorem 3:* If  $A$  is nonsingular, then there exists a  $m$ th-order continuous-time washout controller stabilizing the linearized system (2).

*Proof:* When  $A$  is nonsingular, we will show that there exists a controller  $K(s) \in \mathcal{K}_m^r$  stabilizing the linearized system (2), that is, there exists a feedback gain  $\bar{K} = \begin{bmatrix} \hat{B} & \hat{A} \end{bmatrix}$  such that the closed-loop system (11) is asymptotically stable and  $\det \hat{A} \neq 0$ . If  $(\bar{A}, \bar{B})$  is stabilizable, then there exists the feedback gain  $\bar{K}$  such that the closed-loop system (11) is asymptotically stable. In the following, it is shown that  $(\bar{A}, \bar{B})$  is stabilizable. Since  $(A, B)$  is stabilizable, for  $\forall \lambda \in \mathbb{C}^+ := \{\lambda \in \mathbb{C} \mid \operatorname{Re} \lambda \geq 0\}$ ,  $\operatorname{rank}[A - \lambda I_n \ B] = n$ . Then, for  $\forall \lambda \in \mathbb{C}^+ \setminus \{0\}$ , we have

$$\begin{aligned} &\operatorname{rank}[\bar{A} - \lambda I_{n+m} \ \bar{B}] \\ &= \operatorname{rank} \begin{bmatrix} A - \lambda I_n & 0 & B \\ 0 & -\lambda I_m & I_m \end{bmatrix} \\ &= \operatorname{rank}[A - \lambda I_n \ B] + \operatorname{rank}(-\lambda I_m) \\ &= n + m. \end{aligned}$$

Moreover, for  $\lambda = 0$ ,

$$\begin{aligned} \operatorname{rank}[\bar{A} - \lambda I_{n+m} \ \bar{B}] &= \operatorname{rank} \begin{bmatrix} A & 0 & B \\ 0 & 0 & I_m \end{bmatrix} \\ &= \operatorname{rank}(A) + m. \end{aligned}$$

Therefore,  $A$  is nonsingular if and only if  $(\bar{A}, \bar{B})$  is stabilizable. Next, we will show that  $\det \hat{A} \neq 0$ . When the closed-loop system (11) is asymptotically stable,  $A_c$  does

not have any zero eigenvalues, that is,  $\det A_c \neq 0$ . Under  $\det A \neq 0$ , since

$$\begin{aligned} \det A_c &= \det \begin{bmatrix} I & B \\ 0 & I \end{bmatrix} \begin{bmatrix} A & AB \\ \hat{B} & \hat{A} + \hat{B}B \end{bmatrix} \begin{bmatrix} I & -B \\ 0 & I \end{bmatrix} \\ &= \det A \det(\hat{A} + \hat{B}B - \hat{B}A^{-1}AB) \\ &= \det A \det \hat{A}, \end{aligned}$$

we have  $\det \hat{A} \neq 0$ . ■

From Theorem 1 and 3, it is concluded that the linearized system (2) is washout controllable by  $m$ th-order dynamic controller if and only if  $A$  is nonsingular.

## V. DISCRETE-TIME SYSTEM

We consider an  $n$ th-order nonlinear discrete-time system described by

$$\begin{aligned} x(k+1) &= f(x(k), u(k)), \\ y(k) &= x(k), \end{aligned} \quad (12)$$

where  $x(k) \in \mathbb{R}^n$ ,  $u(k) \in \mathbb{R}^m$ , and  $f$  is differentiable. It is assumed that there exists a fixed point  $x_f$  such that  $x_f = f(x_f, 0)$ . Then, the linearized system around the fixed point  $x_f$  is given by

$$\begin{aligned} \delta x(k+1) &= A\delta x(k) + Bu(k), \\ y(k) &= \delta x(k) + x_f, \end{aligned} \quad (13)$$

where  $\delta x(k) = x(k) - x_f$ . Moreover, we assume that  $(A, B)$  is stabilizable.

We consider a stabilization problem of an uncertain fixed point  $x_f$  by using an  $\tilde{n}$ th-order discrete-time dynamic state feedback controller described by

$$K(z) : \begin{aligned} w(k+1) &= \hat{A}w(k) + \hat{B}y(k), \\ u(k) &= \hat{C}w(k) + \hat{D}y(k). \end{aligned} \quad (14)$$

In the vicinity of the fixed point  $x_f$ , the closed-loop system by the dynamic controller (14) is given by

$$\begin{bmatrix} \delta x(k+1) \\ w(k+1) \end{bmatrix} = A_c \begin{bmatrix} \delta x(k) \\ w(k) \end{bmatrix} + B_c x_f. \quad (15)$$

Moreover, when the closed-loop system is stable, from the final value theorem, we have

$$\lim_{k \rightarrow \infty} u(k) = \{I - K(1)P(1)\}^{-1}K(1)x_f, \quad (16)$$

where  $P(z) := (zI - A)^{-1}B$ .

*Definition 2:* The linearized system (13) is said to be *washout controllable*, if there exists a finite-dimensional discrete-time controller (14) such that the closed-loop system (15) is asymptotically stable and  $\lim_{k \rightarrow \infty} u(k) = 0$  for any  $x_f \neq 0$ . Moreover, such a controller is called a *washout controller*.

Now, a transfer function matrix  $K(z)$  is said to have a steady-state blocking zero if  $K(z) = 0$  at  $z = 1$ . Then, we consider a class of the dynamic controllers (14) having a steady-state blocking zero

$$\begin{aligned} \mathcal{K}'_{\tilde{n}} &= \left\{ (14) \mid \hat{A} \in \mathbb{R}^{\tilde{n} \times \tilde{n}}, \right. \\ &\quad \left. \det(I - \hat{A}) \neq 0, \hat{D} + \hat{C}(I - \hat{A})^{-1}\hat{B} = 0 \right\}. \end{aligned} \quad (17)$$

Then, we have the following theorem.

**Theorem 4:** If the linearized system (13) is washout controllable by a controller  $K(z) \in \mathcal{K}'_n$ , then  $I - A$  is nonsingular.

*Proof:* We can prove Theorem 4 by using a procedure similar to the proof of Theorem 1. ■

**Theorem 5:** ([8]) If  $I - A$  is nonsingular, by using  $F$  and  $L$  such that  $A + BF$  and  $A + L$  are asymptotically stable, a class of  $n$ th-order discrete-time washout controllers  $\mathcal{K}'_n$  is given by

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \mathcal{F}_l(J, Q'), \quad Q' = -F(I - A)^{-1}L, \quad (18)$$

that is,

$$\begin{aligned} \hat{A} &= I - (I - A - BF)(I - A)^{-1}(I - A - L), \\ \hat{B} &= -(I - A - BF)(I - A)^{-1}L, \\ \hat{C} &= F(I - A)^{-1}(I - A - L), \\ \hat{D} &= F(I - A)^{-1}L. \end{aligned}$$

In the following, it is assumed that the order of the controller is same as that of the control vector. Then, we consider a class of the discrete-time dynamic controllers (14) which is given by

$$\mathcal{K}'_m = \left\{ (14) \mid \hat{A} \in \mathbb{R}^{m \times m}, \det \hat{A} \neq 0, \hat{C} = \hat{A} - I, \hat{D} = \hat{B} \right\}. \quad (19)$$

Then, it is obvious that  $\mathcal{K}'_m \subset \mathcal{K}'_m$ . Therefore, the controller  $K(z) \in \mathcal{K}'_m$  has the steady-state blocking zero.

We consider the closed-loop system by  $K(z) \in \mathcal{K}'_m$  in the vicinity of the fixed point  $x_f$ . Then, for any  $K(z) \in \mathcal{K}'_m$ , there exists a vector  $w_f$  such that

$$\begin{aligned} w_f &= \hat{A}w_f + \hat{B}x_f, \\ 0 &= \hat{C}w_f + \hat{D}x_f, \end{aligned} \quad (20)$$

for any  $x_f$ . By using such a vector  $w_f$ , the closed-loop system is given by

$$\begin{bmatrix} \delta x(k+1) \\ \delta w(k+1) \end{bmatrix} = A_c \begin{bmatrix} \delta x(k) \\ \delta w(k) \end{bmatrix}, \quad (21)$$

where  $\delta w(k) = w(k) - w_f$ . Moreover, under conditions  $\hat{C} = \hat{A} - I$  and  $\hat{D} = \hat{B}$ , the closed-loop system (21) can be rewritten as

$$\begin{aligned} X(k+1) &= \bar{A}X(k) + \bar{B}\bar{u}(k), \\ \bar{u}(k) &= \bar{K}X(k), \end{aligned} \quad (22)$$

where

$$\begin{aligned} X(k) &= \begin{bmatrix} \delta x(k) \\ \delta w(k) \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & -B \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ I_m \end{bmatrix}, \\ \bar{K} &= \begin{bmatrix} \hat{B} & \hat{A} \end{bmatrix}. \end{aligned}$$

Therefore, for  $K(z) \in \mathcal{K}'_m$ , the design of feedback parameters  $\hat{A}$  and  $\hat{B}$  stabilizing the closed-loop system (21) can be cast into that of the state feedback gain  $\bar{K}$  stabilizing the

closed-loop system (22). Moreover, when  $m < n$ ,  $\mathcal{K}'_m$  gives the reduced-order controllers.

**Theorem 6:** If  $I - A$  is nonsingular, then there exists a  $m$ th-order discrete-time washout controller stabilizing the linearized system (13).

*Proof:* The proof of this theorem is very similar to that of Theorem 3, and hence it is omitted. ■

From Theorem 4 and 6, it is concluded that the linearized system (13) is washout controllable by  $m$ th-order discrete-time dynamic controller if and only if  $I - A$  is nonsingular.

## VI. CONCLUSION

In this paper, we have considered the stabilization problem of the uncertain equilibrium point of the continuous-time system and the uncertain fixed point of the discrete-time system. We have proposed the reduced-order washout controller which is a finite-dimensional dynamical state feedback controller with steady-state blocking zeros. We have also shown that the reduced-order washout controller can be designed by solving a stabilization problem by constant state feedback.

## REFERENCES

- [1] K. Pyragas, "Continuous control of chaos by self-controlling feedback," *Physics Letters A*, vol. 170, pp. 421–428, 1992.
- [2] H. Kokame, K. Hirata, K. Konishi, and T. Mori, "State difference feedback for stabilizing uncertain steady states of non-linear systems," *International Journal of Control*, vol. 74, no. 6, pp. 537–546, 2001.
- [3] H. Kokame, K. Hirata, K. Konishi, and T. Mori, "Difference feedback can stabilize uncertain steady states," *IEEE Transaction on Automatic Control*, vol. 46, no. 12, pp. 1908–1913, 2001.
- [4] A. S. Bazanella, P. V. Kokotovic, and A. S. E. Silva, "A dynamic extension for  $L_gV$  controllers," *International Journal of Control*, vol. 73, no. 7, pp. 600–605, 2000.
- [5] A. S. Bazanella, P. V. Kokotovic, and A. S. E. Silva, "On the control of dynamic systems with unknown operating point," *International Journal of Control*, vol. 73, no. 7, pp. 600–605, 2000.
- [6] M. A. Hassouneh, H.-C. Lee, and E. H. Abed, "Washout filters in feedback control: Benefits, limitations and extensions," ISR, University of Maryland, College Park, Technical Research Report TR 2004-16, 2004.
- [7] M. A. Hassouneh, H.-C. Lee, and E. H. Abed, "Washout filters in feedback control: Benefits, limitations and extensions," *Proc. of the American Control Conference*, pp. 3950–3955, 2004.
- [8] T. Takimoto and S. Yamamoto, "An active control method for oscillatory motions in human behavior," *Proc. of SICE Annual Conference 2005*, pp. 3195–3198, 2005.
- [9] T. Takimoto, S. Yamamoto, and H. Oku, "Tuning of washout control for manual operations via closed-loop identification," *the 3rd International Symposium on Systems and Human Science (SSR2006)*.