Journal of Experimental & Theoretical Artificial Intelligence

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/teta20

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Published online: 10 Jun 2014.

Journal of Experimental & Theoretical Artificial Intelligence, 2014
http://dx.doi.org/10.1080/0952813X.2014.921733

A possibilistic defeasible logic programming approach to argumentation-based decision-making

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(*Received 12 June 2012; final version received 29 December 2013*)

The development of symbolic approaches to decision-making has become an ever-growing research line in artificial intelligence; *argumentation* has contributed to that with its unique strengths. Following this trend, this article proposes a general-purpose decision framework based on argumentation. Given a set of alternatives posed to the decision-maker, the framework represents the agent's preferences and knowledge by an epistemic component developed using *possibilistic defeasible logic programming*. The reasons by which a particular alternative is deemed better than another are explicitly considered in the argumentation process involved in warranting information from the epistemic component. The information warranted by the dialectical process is then used in decision rules that implement the agent's general decision-making policy. Essentially, decision rules establish patterns of behaviour of the agent specifying under which conditions a set of alternatives will be considered acceptable; moreover, a methodology for programming the agent's epistemic component is defined. It is demonstrated that programming the agent's epistemic component following this methodology exhibits some interesting properties with respect to the selected alternatives; also, when all the relevant information regarding the agent's preferences is specified, its choice behaviour coincides with respect to the optimum preference derived from a *rational preference relation*.

Keywords: non-monotonic reasoning; argumentation; possibilistic defeasible logic programming; decision making

1. Introduction

For a long time, *decision-making* has been a subject of active research in several areas of study such as Philosophy, Economy, Psychology and Computer Science, among others. It is clear that decision-making can be studied in as many ways as different research areas face the problems in this field of study using their particular tools and methodologies. In particular, in Computer Science, decision-making problems have been mainly tackled from the research field of artificial intelligence (AI).

Classical approaches to decision-making rely on *rationality* principles to model decision behaviour. Rationality has been a concept that many philosophers, sociologists, economists, psychologists and other interested parties have tried to define (Audi, 1999). Although it has been the focus of a great number of interesting debates, human rationality has escaped a precise

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characterisation, at least in such a way that could be implemented in computer science systems (Russell, 1997). Given that rationality is acting to the best interest in the situation at hand, as a matter of computational implementation, the concept of utility has taken a central stage, and for an agent to be rational it should strive to maximise the expected utility of its decisions in the context of its knowledge of the situation. The definition of the utility must aim at obtaining the best performance in a sense that it is defined by the creator of such an agent. Current research in AI has produced interesting utility functions that are oriented to particular domains in reduced scenarios, but there has been little advance regarding a general utility function that can be applied in arbitrary domains; thus, human rationality level has remained outside the reach of AI systems.

Despite the fact that it has been acknowledged that classical approaches to decision-making have solid theoretical foundations, they might not be appropriate to be applied in all settings (see e.g. Mellers, Schwartz, & Cooke, 1998; Zsambok, Beach, & Klein, 1992). In this context, Fox and Parsons (Parsons & Fox, 1996) were among the first to seriously pose the necessity of exploring symbolic approaches to decision-making based on Fox's former work in the 1980s. The results obtained by Fox suggested that numerical methods were not the only practical means of making decisions, and that knowledge-based qualitative models were much alike the way humans reason under uncertainty, than traditional statistical models. In this way, the idea of articulating decisions based on arguments became relevant to different approaches to decision-making, such as decision under uncertainty (Amgoud & Prade, 2006), multi-criteria decision (Ouerdane, Maudet, & Tsoukiàs, 2007), rule-based decisions (Kakas & Moraïtis, 2003) and case-based decisions (Bruninghaus & Ashley, 2003).

Argumentation systems are based on the construction and evaluation of interacting arguments that are intended to support, explain or attack statements that can be decisions, opinions, and so on. Argumentation has been applied to different domains such as non-monotonic reasoning, handling inconsistency in knowledge bases and modelling different kinds of dialogues, in particular persuasion and negotiation (see, Rahwan & Simari, 2009). Most of the proposals to qualitative decision-making in argumentation literature (e.g. Amgoud & Prade, 2004, 2006; Kakas & Moraïtis 2003; Parsons & Fox, 1996) share a common view with respect to decision-making, because they conceive it as a form of reasoning oriented towards action. Thus, all of them consider the goals of the agent or the expected values of the action to decide which action to accomplish. This is the main difference with our approach to decision-making which is orientated to the point of view of Marketing literature (Roberts & Lilien, 1993). In this view, each alternative is considered as a possible product to be bought by a consumer (decision-maker). In our view, this is an interesting approach with simple implementations, because only the properties of the alternatives are considered in the decision process and there is no need to explicitly represent the cognitive states of the agent, nor a particular agent architecture is needed to embed the framework into it.

The main contribution of this article is to propose a decision-making framework based on *possibilistic defeasible logic programming* (P-DeLP) (Alsinet, Chesñevar, Godo, & Simari, 2008); this is a concrete formalism which combines features from argumentation theory, logic programming and a unified treatment of possibilistic uncertainty and fuzziness. In this proposal, the reasons by which an alternative will be deemed better than the other are explicitly considered in the argumentation process involved in warranting information from a P-DeLP program. The information warranted by the dialectical process is then used in decision rules that implement the agent's general decision-making policy. Basically, decision rules establish the patterns of behaviour of the agent specifying under which conditions a set of alternatives will be considered acceptable. Moreover, a methodology for *programming* the components involved in the agent's decision process is included, and it is demonstrated that programming the knowledge base of an agent following this methodology guarantees that when all the relevant information about the agent's preferences is specified, its choice behaviour will coincide with the optimum preference derived from a *rational preference relation*. Also, we make explicit the connection between our proposal and the *choice rules* approach to decision-making (Samuelson, 1938), an approach that leaves room, in principle, for more general forms of individual behaviour than possible with the preference-based approach (PBA).

Our proposal consists of a general framework to qualitative decision-making that can be applied to different domains. Nonetheless, with explanatory purposes in mind, the principles stated in this work are exemplified in the well-known domain of apartments renting, which to the best of our knowledge was made popular in (Antoniou & van Harmelen, 2004).

The article is organised as follows. First, Section 2 introduces some basic concepts of decision-making from the point of view of the standard theory of individual rationality, which will be formally related with the argumentation-based approach proposed in this work. Second, Section 3 presents P-DeLP, the language used in our framework to perform knowledge and preferences representation, and reasoning. In particular, in this section, the formalisation of argument accrual presented (Gómez, Chesñevar, & Simari, 2009) in the context of P-DeLP is covered in more detail. Then, Section 4 presents the formal definition of our proposal, a general-purpose decision framework based on argumentation. In addition, it also lays down a formal comparison of the choice behaviour of the proposed framework with respect to Classical Decision Theory. Furthermore, a discussion considering how our proposal is related to other significant qualitative decision-making approaches is performed in Section 5. Finally, Section 6 offers the conclusions and future work.

2. Individual decision-making

As mentioned earlier, in classical approaches to decision-making, the objectives of a decision-maker are summarised in a *rational preference relation*, or in a *utility function* that represents it. Despite its many criticisms, these classical approaches have become 'the major paradigm in decision-making since the Second World War' (Schoemaker, 1982); this may be due to their solid theoretical underpinning. That is why, Parsons and Fox (1996) have stated that when developing decision-making models based on argumentation formalisms, a key issue relies on formally relate them to classical approaches to decision theory.

With this aim, this section introduces a brief overview of the theory of individual decision-making as presented in (Mas-Collel, Whinston, & Green, 1995), where two related approaches to model the agent's decision are considered. These approaches are described later in two separate sections. In Section 4.3, the choice behaviour of the argumentation-based decision framework proposed in this article is formally related with these approaches to model the agent's decisions.

2.1 Preference relations

The starting point for any individual decision problem is a set of possible (mutually exclusive) *alternatives* from which the decision-maker (an agent in our case) must choose. In the discussion that follows (as well as in the rest of the article), this set of alternatives will be denoted by X.

In classical decision-making domains, it is usually assumed that the agent's choice behaviour is modelled with a binary *preference relation* $\succsim$, where given $\{x, y\} \subseteq X$, $x \succsim y$ means that 'x is at least as good as y'. From $\succsim$ we can derive two other important relations:

- The *strict preference* relation $\succ$, defined as $x \succ y \Leftrightarrow x \succsim y$ but not $y \succsim x$ and read 'x is preferred to y'.
- The *indifference* relation $\sim$, defined as $x \sim y \Leftrightarrow x \succsim y$ and $y \succsim x$ and read 'x is indifferent to y'.

It is common to require the preference relation $\succsim$ to be *rational* (see Definition 2.1) and this is a necessary condition if $\succsim$ will be represented by a *utility* function (von Neumann & Morgenstern, 1953). The hypothesis of rationality is embodied in two basic assumptions about the preference relation $\succsim$, which is defined as follows:

Definition 2.1. A preference relation $\succsim$ is rational if it possesses the following two properties:[1]

(i) Completeness: for all $x, y \in X$, we have that $x \succsim y$ or $y \succsim x$ (or both).
(ii) Transitivity: for all $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

The assumption that $\succsim$ is complete states that the agent has a well-defined preference between any two possible alternatives. Besides, transitivity implies that it is impossible for the decision-maker to be faced with a sequence of pairwise choices in which her preferences appear to cycle. With respect to the choice behaviour of a decision-maker who has a rational preference relation $\succsim$ over X, when she faces a non-empty set of alternatives $B \subseteq X$, then her preference-maximising behaviour will choose any of the elements in the following set:

$$C^*(B, \succsim) = \{x \in B \mid x \succsim y \text{ for each } y \in B\}.$$

It is well known in decision theory community that completeness and transitivity assumptions are usually hard to satisfy in real-world problems, but the PBA is very relevant from a theoretical point of view. In fact, this approach is the most traditional way of modelling individual choice behaviour. Nonetheless, the *choice-based approach* (CBA) introduced in Section 2.2 is an interesting and more flexible formal model of theory of decision-making, since it is based on entirely behavioural foundations rather than considering individual decision-making as an introspection-based process.

2.2 Choice rules

The CBA (Samuelson, 1938) takes as primitive object the choice behaviour of the individual, which is represented by means of a *choice structure* ($\mathcal{B}\, C(\cdot)$), consisting of two elements:

- $\mathcal{B}$ is a set of subsets of X. Intuitively, each set $B \in \mathcal{B}$ represents a set of alternatives (or *choice experiment*) that can be conceivably posed to the decision-maker. In this way, if $X = \{x, y, z\}$ and $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$, we will assume that the sets $\{x, y\}$ and $\{x, y, z\}$ are valid choice experiments to be presented to the decision-maker.
- $C(\cdot)$ is a *choice rule* which basically assigns to each set of alternatives $B \in \mathcal{B}$ a non-empty set that represents the alternatives that the decision-maker *might* choose when presented the alternatives in B. Note that $C(B) \subseteq B$ for every $B \in \mathcal{B}$. When $C(B)$ contains a single element, this element represents the *individual's choice* among the alternatives in B. The set $C(B)$ might, however, contain more than one element and in this case they would represent the *acceptable alternatives* in B for the agent.

Similar to the rationality assumption of PBA (Definition 2.1), in the CBA there is a central assumption called the *weak axiom of revealed preference* (or WARP for short) (Samuelson, 1938). As it will be explained later, this axiom imposes an element of consistency on choice behaviour that is similar to the rationality assumptions of the PBA. The WARP axiom is recalled as follows:

Definition 2.2. A choice structure $(\mathcal{B}, C(\cdot))$ satisfies the WARP if the following property holds:

If for some $B \in \mathcal{B}$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \mathcal{B}$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$.

The weak axiom postulates that if there is some choice experiment $B \in \mathcal{B}$ such that x and y are presented as alternatives ($x, y \in B$) and 'x is *revealed at least as good as* y' (i.e. $x \in C(B)$) then there does not exist another choice experiment $B' \in \mathcal{B}$ where 'y is *revealed strictly preferred* to x' (i.e. $x, y \in B'$, $y \in C(B')$ and $x \notin C(B')$).

Intuitively, the WARP principle reflects the expectation that an individual's observed choices will display a certain amount of coherence. That is to say, if given $X = \{x, y, z\}$, $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$ and a choice rule C, $C(\{x, y\}) = \{x\}$, then the axiom states that it cannot be the case that $C(\{x, y, z\}) = \{y\}$. In fact, it states more: we must have $C(\{x, y, z\}) = \{x\}, = \{z\}$, or $= \{x, z\}$.

As mentioned earlier, the PBA and CBA have different perspectives to the theory of individual decision-making. The former considers it as a process of introspection while the latter makes assumptions about objects that are directly observable (choice behaviour) rather than things that are not (preferences). In spite of these differences, under certain conditions these two approaches are related. Later, we introduce a well-known (and very important) proposition, which states that if a decision-maker has a rational preference ordering $\succsim$, when faced with a choice experiment, her choices necessarily generate a choice structure that satisfies the WARP principle.

Proposition 2.3. Suppose that $\succsim$ is a rational preference relation. Then, the choice structure generated by $\succsim$, $(\mathcal{B}, C^*(\cdot, \succsim))$, satisfies the WARP.

The standard theory of individual rationality identifies individuals as a set of well-defined preferences, and treats an action as rational if it is the one most likely to satisfy these preferences. In this way, it provides the background against which it bounded rationality (Grüne-Yanoff, 2007) is discussed. As suggested in Simon (1972), individuals are limited to their rationality for at least these reasons:

(i) In order for someone to be rational, she has to fully know and understand the future consequences of her decision-making in the present.
(ii) Someone cannot know in the present, the future worth and the impact her actions will have in the future.
(iii) In order for someone to be rational, she has to know all of the alternatives. Usually in decision-making, the alternatives someone has in mind are limited and humans are restrained from making optimum decisions.

Taking into account that our approach to decision-making is orientated to the point of view of marketing literature, only point (iii) related to bounded rationality in the choice of alternatives is discussed in Section 4. Next, Section 3 introduces the formalism used to perform reasoning and knowledge representation in the decision framework proposed in Section 4.

3. P-DeLP: fundamentals

P-DeLP is a language which combines features from argumentation theory, logic programming and a unified treatment of possibilistic uncertainty and fuzziness. Originally proposed in

Chesñevar, Simari, Alsinet, and Godo (2004), it has been an object of active research concerning its formalisation. This section introduces the minima necessary concepts of P-DeLP which are used to define the argumentation-based decision framework presented in Section 4. For a more comprehensive view on any topic related to this formalism, refer to Alsinet et al. (2008) and Gómez et al. (2009).

3.1 P-DeLP language and arguments accrual

In P-DeLP language, a literal 'L' is a ground atom 'A' or a negated ground atom '$\sim A$', where '$\sim$' represents the *strong negation* and atom comes from the terminology of logic programming (Lifschitz, 1996). Hence, literals have no variables. Strong negation (García & Simari, 2004) allows for the representation of conflictive or contradictory information.

A *weighted* clause is a pair (φ, α), where φ is a rule $q \leftarrow p_1 \wedge \cdots \wedge p_k (k \geq 0)$ or a fact q (i.e. a rule with empty antecedent), where $q, p_1, \ldots, p_k$ are literals, and $\alpha \in [0, 1]$ expresses a lower bound for the necessity degree of φ.[2] A clause (φ, α) is referred as certain if $\alpha = 1$ and uncertain, otherwise. A set of P-DeLP clauses Γ is deemed *contradictory*, denoted as $\Gamma \vdash \perp$, if, for some atom a, $\Gamma \vdash (a, \alpha)$ and $\Gamma \vdash (\sim a, \beta)$, with $\alpha > 0$ and $\beta > 0$, where $\vdash$ stands for deduction by means of the following instance of the *generalised modus ponens* (*GMP*) *rule*:

$$\frac{\begin{array}{c}(q \leftarrow p_1 \wedge \cdots \wedge p_k, \alpha)\\ (p_1, \beta_1), \ldots, (p_k, \beta_k)\end{array}}{(q, \min(\alpha, \beta_1, \ldots, \beta_k))} \quad [GMP]$$

Moreover, a P-DeLP program (see Definition 3.1) is a set of P-DeLP clauses in which certain information is distinguished from uncertain information, with the additional requirement that certain knowledge is required to be *non-contradictory*.

Definition 3.1. A P-DeLP program $\mathcal{P}$ (or just program $\mathcal{P}$) is a pair (Π, Δ), where Π is a non-contradictory finite set of certain clauses, and Δ is a finite set of uncertain clauses.

Since literals are ground, certain and uncertain clauses are also ground. However, following the usual convention (Lifschitz, 1996), some examples will use schematic clauses with variables. Given a schematic clause R, Ground(R) stands for the set of all ground instances of R. In order to distinguish variables from other elements of a schematic rule, we will denote variables with an initial upper-case letter.

The notion of *argument* in P-DeLP (see Definition 3.2) refers to a tentative proof (as it relies to some extent on uncertain, possibilistic information) from a consistent set of clauses supporting a given conclusion Q with a necessity degree α.

Definition 3.2. Given a P-DeLP program $\mathcal{P} = (\Pi, \Delta)$, a set $\mathcal{A} \subseteq \Delta$ of uncertain clauses is an *argument* for a conclusion Q with necessity degree $\alpha > 0$, denoted $\langle \mathcal{A}, Q, \alpha \rangle$, iff:

- (i) $\Pi \cup \mathcal{A}$ is non contradictory;
- (ii) $\alpha = \max\{\beta \in [0, 1] | \Pi \cup \mathcal{A} \vdash (Q, \beta)\}$, i.e. α is the greatest degree of deduction of Q from $\Pi \cup \mathcal{A}$;
- (iii) $\mathcal{A}$ is minimal w.r.t. set inclusion, i.e. there is no $\mathcal{A}_1 \subset \mathcal{A}$ such that $\Pi \cup \mathcal{A}_1 \vdash (Q, \alpha)$.

It must be remarked that the three conditions in Definition 3.2 are inherited from similar definitions in argumentation literature (Chesñevar, Maguitman, & Loui, 2000). Moreover, notice that from the above-mentioned definition of argument, on the basis of a P-DeLP program $\mathcal{P}$, there may exist *different* arguments $\langle \mathcal{A}_1, Q, \alpha_1 \rangle$, $\langle \mathcal{A}_2, Q, \alpha_2 \rangle$, ..., $\langle \mathcal{A}_k, Q, \alpha_k \rangle$ supporting a given conclusion Q, with (possibly) different necessity degrees $\alpha_1, \alpha_2, \ldots, \alpha_k$.

As indicated earlier, the *GMP* inference rule allows to propagate necessity degrees; however, given different arguments supporting the same conclusion, it is not possible to accumulate their strength in terms of possibilistic values. To do this, in Gómez et al. (2009), the notion of *accrued structure*, which is recalled in Definition 3.3, was defined. An accrued structure accounts for several arguments supporting the same conclusion and whose necessity degree is defined in terms of two mutually recursive functions: $f^{+}_{\Phi}(\cdot)$ (the accruing function) and $f^{MP}_{\Phi}(\cdot)$ (which propagates necessity degrees as *GMP*).

Given that necessity degrees associated with accrued structures will be used to determine which attack constitutes a defeat, in order to avoid getting committed to a specific way of aggregating necessity degrees, it was assumed that $f^{+}_{\Phi}(\cdot)$ was parameterised w.r.t. a user-specified function, *ACC*, which must be defined according to the application domain. In addition, two properties were identified as reasonable to hold for any candidate instantiation of *ACC*:

Non-depreciation: $ACC(\alpha_1, \ldots, \alpha_n) \geq \max(\alpha_1, \ldots, \alpha_n)$ (i.e. accruing arguments result in a necessity degree not lower than any single argument involved in the accrual).

Maximality: $ACC(\alpha_1, \ldots, \alpha_n) = 1$ only if $\alpha_i = 1$ for some i, $1 \leq i \leq n$ (i.e. accrual means total certainty only if there is an argument with necessity degree 1).

Definition 3.3 (Accrued structure). Let $\mathcal{P}$ be a P-DeLP program, and Ω be a set of arguments in $\mathcal{P}$ supporting the same conclusion h, i.e. $\Omega = \{\langle \mathcal{A}_1, h, \alpha_1 \rangle, \ldots, \langle \mathcal{A}_n, h, \alpha_n \rangle\}$. The accrued structure for h (or just a-structure) from the set Ω (denoted *Accrual*(Ω)) is defined as a 3-tuple $[\Phi, h, \alpha]$, where $\Phi = \mathcal{A}_1 \cup \ldots \cup \mathcal{A}_n$ and α is obtained using two mutually recursive functions, $f^{+}_{\Phi}(\cdot)$ and $f^{MP}_{\Phi}(\cdot)$, defined as follows. Let q be a literal appearing in Φ and $(\varphi_1, \beta_1), \ldots, (\varphi_n, \beta_n)$ be all the weighted clauses in Φ with head q. Then

$$f^{+}_{\Phi}(q) =_{\text{def}} ACC(f^{MP}_{\Phi}(\varphi_1), \ldots, f^{MP}_{\Phi}(\varphi_n))$$

$$f^{MP}_{\Phi}(\varphi_i) =_{\text{def}} \begin{cases} \beta_{\text{i}} & \text{if } \varphi_{\text{i}} \text{ is a fact } q; \\ \min(f^{+}_{\Phi}(p_1), \ldots, f^{+}_{\Phi}(p_n), \beta_i) & \text{if } \varphi_i = q \leftarrow p_1, \ldots, p_n \end{cases}$$

Finally, $\alpha = f^{+}_{\Phi}(h)$. When $\Omega = \emptyset$, we get the special accrued structure $[\emptyset, \varepsilon, 0]$, representing the accrual of no argument.

Given an a-structure $[\Phi, h, \alpha]$, the set of *arguments in* $[\Phi, h, \alpha]$ is denoted as $Args([\Phi, h, \alpha])$ and it contains all arguments $\langle \mathcal{A}_i, h, \alpha_i \rangle$ s.t. $\mathcal{A}_i \subseteq \Phi$. It is worth noting that $Args([\emptyset, \epsilon, 0]) = \emptyset$. Later, three definitions concerning the properties of a-structures are introduced.

Definition 3.4 (Maximal a-structure). Let $\mathcal{P}$ be a P-DeLP program. An a-structure $[\Phi, h, \alpha]$ is *maximal* iff $Args([\Phi, h, \alpha])$ contains all arguments in $\mathcal{P}$ with conclusion h.

Definition 3.5 (Narrowing of an a-structure). Let $[\Phi, h, \alpha]$ and $[\Theta, h, \beta]$ be two a-structures. We say that $[\Theta, h, \beta]$ is a *narrowing* of $[\Phi, h, \alpha]$ iff $Args([\Theta, h, \beta]) \subseteq Args([\Phi, h, \alpha])$.

Definition 3.6 (a-substructure and complete a-substructure). Let $[\Phi, h, \alpha]$ and $[\Theta, k, \gamma]$ be two a-structures. Then, we say that $[\Theta, k, \gamma]$ is an *accrued substructure* (or just a-substructure) of $[\Phi, h, \alpha]$ iff $\Theta \subseteq \Phi$. We also say that $[\Theta, k, \gamma]$ is a *complete a-substructure* of $[\Phi, h, \alpha]$ iff for any other a-substructure $[\Theta', k, \gamma']$ of $[\Phi, h, \alpha]$ it holds that $\Theta' \subset \Theta$.

3.2 Modelling conflict and defeat among accrued structures

An a-structure $[\Phi, h, \alpha]$ stands for (possibly) several chains of reasoning (arguments) supporting the conclusion h, where some intermediate conclusions in $[\Phi, h, \alpha]$ could be shared by some, but not necessarily all the arguments in $[\Phi, h, \alpha]$. Thus, given two a-structures $[\Phi, h, \alpha]$ and $[\Psi, k, \beta]$, if the conclusion k of $[\Psi, k, \beta]$ contradicts some intermediate conclusion h' in $[\Phi, h, \alpha]$, then only those arguments in $Args([\Phi, h, \alpha])$ involving h' will be affected by the conflict. Next, the notion of *partial attack* is defined, where the attacking a-structure generally affects only a narrowing of the attacked a-structure (that structure containing exactly the arguments in the attacked a-structure affected by the conflict). This narrowing will be referred as the *attacked narrowing*.

Definition 3.7 (Partial attack and attacked narrowing). Let $[\Phi, h, \alpha]$ and $[\Psi, k, \beta]$ be two a-structures. $[\Psi, k, \beta]$ *partially attacks* $[\Phi, h, \alpha]$ at literal h', iff there exists a complete a-substructure $[\Phi', h', \alpha']$ of $[\Phi, h, \alpha]$ such that $k = \overline{h'}$. The a-substructure $[\Phi', h', \alpha']$ will be called the *disagreement a-substructure*. $[\Lambda, h, \gamma]$ is the *attacked narrowing* of $[\Phi, h, \alpha]$ associated with the attack iff $[\Lambda, h, \gamma]$ is the minimal narrowing of $[\Phi, h, \alpha]$ which has $[\Phi', h', \alpha']$ as an a-substructure.

Example 3.8. Let us consider the a-structures $[\Phi_3, x, 0.79]$ and $[\Psi_1, \sim z, 0.82]$ of Figure 1, where $\Phi_3 = \{(x \leftarrow z, 0.7), (z \leftarrow t, 0.6), (t, 1), (z \leftarrow v, 0.5), (v, 1), (x \leftarrow y, 1), (y \leftarrow u, 0.3), (u, 1)\}$.[3] Then, $[\Psi_1, \sim z, 0.82]$ partially attacks $[\Phi_3, x, 0.79]$ with disagreement a-substructure $[\Phi', z, 0.8] = [\{(z \leftarrow t, 0.6), (t, 1), (z \leftarrow v, 0.5), (v, 1)\}, z, 0.8]$. The attacked narrowing of $[\Phi_3, x, 0.79]$ is $[\{(x \leftarrow z, 0.7), (z \leftarrow t, 0.6), (t, 1), (z \leftarrow v, 0.5), (v, 1)\}, x, 0.7]$. Graphically, this attack relation is depicted with a dotted arrow (see Figure 1).

Analogous to P-DeLP arguments, the necessity degrees associated with a-structures will be used to decide whether a partial attack really succeeds and constitutes a defeat.

Definition 3.9 (Partial defeater). Let $[\Phi, h, \alpha]$ and $[\Psi, k, \beta]$ be two a-structures. Then, $[\Psi, k, \beta]$ is a *partial defeater* of $[\Phi, h, \alpha]$ (or equivalently that $[\Psi, k, \beta]$ is a successful attack on $[\Phi, h, \alpha]$) iff (1) $[\Psi, k, \beta]$ attacks $[\Phi, h, \alpha]$ at literal h', where $[\Phi', h', \alpha']$ is the disagreement a-substructure, and (2) $\beta \geq \alpha'$.

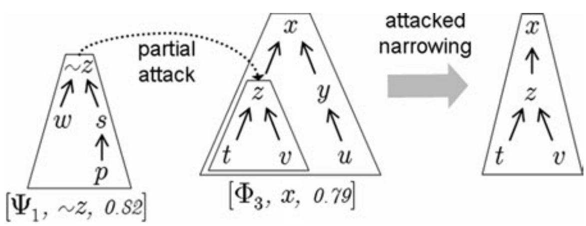

Figure 1. Partial attack.

Example 3.10. Let us consider the attack from $[\Psi_1, \sim z, 0.82]$ against $[\Phi_3, x, 0.79]$ with disagreement a-substructure $[\Phi', z, 0.8]$ in Example 3.8 (Figure 1). As the necessity degree associated with the attacking a-structure (0.82) is greater than the necessity degree associated with the disagreement a-substructure (0.8), then the attack succeeds, constituting a defeat. Graphically, this defeat relation is depicted with a continuous arrow, as shown in Figure 2.

Given an attack relation, we will identify two complementary narrowings associated with the attacked a-structure: the narrowing that becomes defeated as a consequence of the attack, and the narrowing that remains undefeated. For example, Figure 2 illustrates a successful attack from $[\Psi_1, \sim z, 0.82]$ against $[\Phi_3, x, 0.79]$, as well as the associated defeated and undefeated narrowings of $[\Phi_3, x, 0.79]$.

Definition 3.11 (Undefeated and defeated narrowings). Let $[\Phi, h, \alpha]$ and $[\Psi, k, \beta]$ be two a-structures such that $[\Psi, k, \beta]$ attacks $[\Phi, h, \alpha]$. Let $[\Lambda, h, \gamma]$ be the attacked narrowing of $[\Phi, h, \alpha]$. Then, the *defeated narrowing* of $[\Phi, h, \alpha]$ associated with the attack, denoted as $\mathrm{N}_{\mathrm{w}}^{\mathrm{D}}([\Phi, h, \alpha], [\Psi, k, \beta])$, is defined by cases as follows:

(i) $N_w^{\mathrm{D}}([\Phi, \mathrm{h}, \alpha], [\Psi, \mathrm{k}, \beta]) =_{def} [\Lambda, h, \gamma]$, if $[\Psi, k, \beta]$ is a partial defeater of $[\Phi, h, \alpha]$, or
(ii) $N_{\mathrm{w}}^{\mathrm{D}}([\Phi, \mathrm{h}, \alpha], [\Psi, \mathrm{k}, \beta]) =_{def} [\emptyset, \varepsilon, 0]$, otherwise.

The *undefeated narrowing* of $[\Phi, h, \alpha]$, denoted as $N_{\mathrm{w}}^{\mathrm{U}}([\Phi, \mathrm{h}, \alpha], [\Psi, \mathrm{k}, \beta])$, is the a-structure $Accrual(Args([\Phi, h, \alpha]) \backslash Args(N_{\mathrm{w}}^{\mathrm{D}}([\Phi, \mathrm{h}, \alpha], [\Psi, \mathrm{k}, \beta])))$.

Until now, we have considered only *single* attacks. When a single attack succeeds, a non-empty narrowing of the attacked a-structure becomes defeated. But two or more a-structures could simultaneously attack another (combined attack), possibly affecting different narrowings of the target a-structure, and thus causing a bigger narrowing to become defeated (compared with the defeated narrowings associated with the individual attacks). Even though *combined attacks* have been defined in Gómez et al. (2009), this notion will not be used in the argumentation-based framework proposed in Section 4 and in spite of making reference to this concept in Definition 3.13, is not formally introduced in this overview of P-DeLP fundamentals.

3.3 Dialectical analysis for accrued structures

Given a program $\mathcal{P}$ and a literal h, we are interested in determining whether h is ultimately accepted (or *warranted*), and if so, with which necessity degree. In order to determine this, the maximal a-structure $[\Phi, h, \alpha]$ supporting h will be considered, as well as which is the final undefeated narrowing of $[\Phi, h, \alpha]$ after considering all possible a-structures attacking it. As those attacking a-structures may also have other a-structures attacking them, this strategy prompts a recursive dialectical analysis formalised as discussed later.

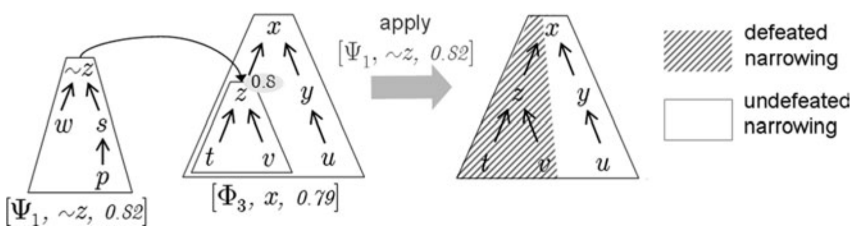

Figure 2. Defeated and undefeated narrowings.

Definition 3.12 (Accrued dialectical tree). Let $\mathcal{P}$ be a P-DeLP program and h be a literal. Let $[\Phi, h, \alpha]$ be the maximal a-structure for h in $\mathcal{P}$. The *accrued dialectical tree* for h, denoted as $\mathcal{T}_h$, is defined as follows:

(i) The root of the tree is labelled with $[\Phi, h, \alpha]$.
(ii) Let N be an internal node labelled with $[\Theta, k, \beta]$. Let Σ be the set of all disagreement a-substructures associated with the attacks in the path from the root to N. Let $[\Theta_i, k_i, \beta_i]$ be a maximal a-structure attacking $[\Theta, k, \beta]$ s.t. $[\Theta_i, k_i, \beta_i]$ has no a-substructures in Σ. Then, the node N has a child node N_i labelled with $[\Theta_i, k_i, \beta_i]$. If there is no a-structure attacking $[\Theta, k, \beta]$ satisfying the above-mentioned condition, then N is a leaf.

The condition involving the set Σ avoids the introduction of a new a-structure as a child of a node N if it is already present in the path from the root to N (resulting in a circularity). This requirement is needed in order to avoid *fallacious* reasoning, as discussed in Chesñevar et al. (2004). Once the dialectical tree has been constructed, each combined attack is analysed, from the deepest attacks to the attacks against the root, in order to determine the undefeated narrowing of each node in the tree.

Definition 3.13 (Undefeated narrowing of a node). Let $\mathcal{T}_h$ be the accrued dialectical tree for a given literal h. Let N be a node of $\mathcal{T}_h$ labelled with $[\Theta, k, \beta]$. Then, the *undefeated narrowing* of N is defined as follows:

(i) If N is a leaf, then the undefeated narrowing of N is its own label $[\Theta, k, \beta]$.
(ii) Otherwise, let $M_1, \ldots, M_n$ be the children of N and let $[\Lambda_i, k, \gamma_i]$ be the undefeated narrowing of the a-structure labelling the child node M_i, $1 \leq i \leq n$. Then, the undefeated narrowing of N is the undefeated narrowing of $[\Theta, k, \beta]$ associated with the combined attack involving all the $[\Lambda_i, k, \gamma_i]$, $1 \leq i \leq n$.

Definition 3.14 (Warrant). Let $\mathcal{P}$ be a P-DeLP program and let h be a literal. Let $[\Phi, h, \alpha]$ be the maximal a-structure for h such that its undefeated narrowing in $\mathcal{T}_h$ is a non-empty a-structure $[\Phi', h, \alpha']$. Then, we say that h is *warranted* w.r.t. $\mathcal{P}$ with necessity α' and that $[\Phi', h, \alpha']$ is a *warranted a-structure*.

The concept of warrant stated earlier is a key issue to define the notion of warranted literal from the epistemic component presented in Definition 4.5 in Section 4. As is shown in Section 4.3, this notion together with a methodology to develop an epistemic component allows to guarantee that the defined epistemic component implements a rational preference relation $\succsim$.

4. The argumentation-based decision framework

The argumentation-based decision framework defined in this section is conceptually composed by three components. The first component, the set X of all the available alternatives that could be presented to the decision-maker, is the same component referred in Section 2. The second component, which represents the agent's knowledge and preferences, is introduced in Section 4.1; while the third, the decision component, is described in Section 4.2. To conclude the definition of the argumentation-based decision framework proposed in this work, Section 4.3 formalises its choice behaviour according to the general theory of choice of Classical Decision Theory.

4.1 Epistemic component

The agent's epistemic component allows it to reason and compare alternatives among each other, but to determine which is/are the best alternative/s to be chosen, a device called *decision rule* will be introduced in Section 4.2. In this way, given the set X of all the possible candidate alternatives, when a choice experiment $B \subseteq X$ is presented to the decision-maker, decision rules will be used to select an alternative from B.

As expected, the agent should be provided with a set $\mathbf{P} = \{p_1, \ldots, p_n\}$ $(n > 0)$ of preference criteria that will be used to compare the elements in X. An important issue related to this set is that preference criteria do not generate cyclic preferences, and thus is necessary to define an ordering of the preference criteria (*L-order*) to indicate the priority that exists among them, but before, the definition of *comparison literal* is conveniently introduced.

Definition 4.1 (Comparison literal). Let X be the set of alternatives provided to the agent and $\mathbf{P} = \{p_1, \ldots, p_n\}$ $(n > 0)$ be the set of preference criteria that will be used to compare the elements in X. Given a preference criterion p_i, its associated *comparison literal*, is a binary literal $c_{p_i}(\cdot, \cdot)$ that states the preference between two alternatives of X, based on their attribute values. The set $\{c_{p_1}(\cdot, \cdot), \ldots, c_{p_n}(\cdot, \cdot)\}$ of all the comparison literals associated with the preference criteria in $\mathbf{P}$ will be denoted as $\mathcal{C}$.

*Definition 4.2 (*L-order*).* Let $\mathcal{C}$ be a set of comparison literals, an *L-order* '$>_{\mathcal{C}}$' is a strict total order over the elements of $\mathcal{C}$.

In terms of the aforesaid *L-order*, the transitivity property guarantees that cyclic preferences do not occur. The comparison literals defined earlier represent in a symbolic way the agent's preferences, and its use in the body of the clauses of a P-DeLP program will allow us to compare alternatives among each other and determine which is/are the best with respect to the preferences they represent. Later, we introduce the definition of a special kind of P-DeLP program which includes comparison literals in the body of its clauses and which also satisfies other features.

Definition 4.3 (Conformant program). Let X be the set of all the alternatives provided to the agent, $\mathcal{P} = (\Pi, \Delta)$ be a P-DeLP program, $\mathbf{P}$ be a set of preference criteria, $\mathcal{C}$ be a set of comparison literals associated to $\mathbf{P}$ and '$>_{\mathcal{C}}$' be an L-order over the elements of $\mathcal{C}$. The P-DeLP program $\mathcal{P}$ is *conformant* with respect to $>_{\mathcal{C}}$ if it satisfies the following requirements:

(i) For all pair of alternatives $x, y \in X$ which have different attribute values with respect to preference criterion $p_i \in \mathbf{P}$, there exists a $(Q, \beta) \in \Delta$ such that $Q = c_{p_i}(x, y)$ or $Q = c_{p_i}(y, x)$.

(ii) For all pair of alternatives $x, y \in X$ which have the same attribute values with respect to all the preference criteria in $\mathbf{P}$, it holds that $(sp(x, y), 1) \in \Pi$.

(iii) For each comparison literal $c_{p_i}(\cdot, \cdot) \in \mathcal{C}$, it holds that $\{(better(W, Y) \leftarrow c_{p_i}(W, Y), \alpha_i), (\sim better(W, Y) \leftarrow c_{p_i}(Y, W), \alpha_i)\} \subseteq \Delta$.

(iv) $\{(\sim better(W, Y) \leftarrow sp(W, Y), 1), (\sim better(W, Y) \leftarrow sp(Y, W), 1)\} \subseteq \Pi$.

(v) Given two alternatives $x, y \in X$ and two preference criteria $p_i, p_j \in \mathbf{P}$ such that 'x is *preferred* to y with respect to p_i' and 'y is *preferred* to x with respect to p_j', if $(c_{p_i}, c_{p_j}) \in >_{\mathcal{C}}$ then for all the arguments $\langle \mathcal{A}_{i_1}, better(x, y), \alpha_i \rangle$, $\langle \mathcal{A}_{i_2}, \sim better(y, x), \alpha_i \rangle$, $\langle \mathcal{E}_{j_1}, better(y, x), \delta_j \rangle$, $\langle \mathcal{E}_{j_2}, \sim better(x, y), \delta_j \rangle$ built from (Π, Δ) it holds that $\alpha_i > \delta_j$ $(\mathcal{A}_{i_1}, \mathcal{E}_{i_2}, \mathcal{A}_{j_1}, \mathcal{A}_{j_2} \subset \Delta$ *and* $i, j \in \{1, \ldots, n\}$ *with* $|\mathbf{P}| = n)$.

As it can be observed later in the methodology proposed in Definition 4.8, to build a conformant P-DeLP program is a key issue to get the epistemic component (Definition 4.5) used in the decision framework (Definition 4.13). The restrictions on the kind of clauses that compose this kind of program allow to compare alternatives among each other, with respect to the preference criteria provided to the decision-maker.

For instance, item (i) of Definition 4.3 states that uncertain factual clauses belonging to Δ compare two alternatives *x, y* with respect to a particular preference criterion, represented by its associated comparison literal. Indeed, for each criterion p_i in which two alternatives *x, y* differ, either $c_{p_i}(x, y)$ or $c_{p_i}(y, x)$ will belong to Δ. Similarly, if two alternatives *x, y* have the same properties, i.e. their attribute values coincide for all the preference criteria, then item (ii) guarantees that a certain clause $(sp(x, y), 1)$ will belong to Π. As expressed in items (iii) and (iv) of Definition 4.3, clauses which include rules have as head a literal $better(X, Y)$ or $\sim better(X, Y)$ stating that either X is better than Y or X is not better than Y, based on the evidence provided by the body of the rules. Finally, item (v) states that the force of an argument supporting that an alternative x is (not) better than alternative y is directly related to the ordering of the comparison literal in the *L-order* $>_{\mathcal{C}}$. To illustrate a concrete P-DeLP conformant program, in Example 4.4 it is considered an excerpt of the example proposed in Antoniou and van Harmelen (2004) concerning apartments renting.

Example 4.4 (Apartments renting). Carlos is looking for an apartment of at least 45 m^2 with at least two bedrooms. If it is on the third floor or higher, the house must have an elevator. Also, pet animals must be allowed. Carlos is willing to pay \$300 for a centrally located 45 m^2 apartment, and \$250 for a similar flat in the suburbs. In addition, he is willing to pay an extra \$5 per square meter for a larger apartment, and \$2 per square meter for a garden. He is unable to pay more than \$400 in total. If given the choice, he would go for the cheapest option. His second priority is the presence of a garden and his lowest priority is additional space.

Each available apartment is given a unique name and its properties are represented as facts. The description of the available apartments are summarised in Table 1.

If we match Carlos' requirements and the available apartments, we see that

- flat a_1 is not acceptable because it has only one bedroom;
- flats a_4 and a_6 are unacceptable because pets are not allowed;
- for a_2, Carlos is willing to pay \$300, but price is higher and
- flats a_3, a_5 and a_7 are acceptable.

Although the definition of conformant P-DeLP program does not concern how X is composed, or even more which alternatives of X are acceptable, without loss of generality in this example only the acceptable alternatives are considered.

Table 1. Available apartments.

Flat	Bedrooms	Size	Central	Floor	Lift	Pets	Garden	Price
a_1	1	50	Yes	1	No	Yes	0	300
a_2	2	45	Yes	0	No	Yes	0	335
a_3	2	65	No	2	No	Yes	0	350
a_4	2	55	No	1	Yes	No	15	330
a_5	3	55	Yes	0	No	Yes	15	350
a_6	2	60	Yes	3	No	No	0	370
a_7	3	65	Yes	1	No	Yes	12	375

Given that Carlos' preferences are based on price, garden size and size, in that order, their associated comparison literals are $price(\cdot,\cdot)$, $garden\_size(\cdot,\cdot)$ and $size(\cdot,\cdot)$, respectively, and $>_{\mathcal{C}}=\{(price, garden\_size), (price, size), (garden\_size, size)\}$. Thus, a conformant P-DeLP program would be:

$$\Delta = \left\{ \begin{array}{ll} (price(a_3,a_7), 0.69) & (better(W,Y) \leftarrow size(W,Y), 0.33) \\ (price(a_5,a_7), 0.69) & (\sim better(W,Y) \leftarrow size(Y,W), 0.33) \\ (garden\_size(a_5,a_3), 0.67) & (better(W,Y) \leftarrow garden\_size(W,Y), 0.66) \\ (garden\_size(a_5,a_7), 0.35) & (\sim better(W,Y) \leftarrow garden\_size(Y,W), 0.66) \\ (garden\_size(a_7,a_3), 0.6) & (better(W,Y) \leftarrow price(W,Y), 0.99) \\ (size(a_3,a_5), 0.05) & (\sim better(W,Y) \leftarrow price(Y,W), 0.99) \\ (size(a_7,a_5), 0.05) & \end{array} \right\}$$

$$\Pi = \{ (\sim better(W,Y) \leftarrow sp(W,Y), 1) \quad (\sim better(W,Y) \leftarrow sp(Y,W), 1) \}.$$

It can be noted that the arguments which can be generated from the above-mentioned P-DeLP program satisfy point (v) of Definition 4.3. For instance, a_5 is preferred to a_7 with respect to the criteria *price* and *garden_size*, while the opposite occurs if the criterion *size* is considered. In this way, since $\{(price, size), (garden\_size, size)\} \subset >_{\mathcal{C}}$ it holds that arguments $\langle \mathcal{A}_1, better(a_5,a_7), 0.35\rangle$ and $\langle \mathcal{A}_2, better(a_5,a_7), 0.69\rangle$ are proper defeaters for $\langle \mathcal{E}_1, \sim better(a_5,a_7), 0.05\rangle$ and the same occurs with $\langle \mathcal{A}_3, \sim better(a_7,a_5), 0.35\rangle$ and $\langle \mathcal{A}_4, \sim better(a_7,a_5), 0.69\rangle$ with respect to $\langle \mathcal{E}_2, better(a_7,a_5), 0.05\rangle$:

$$\begin{aligned} \mathcal{A}_1 &= \{(better(a_5,a_7) \leftarrow garden\_size(a_5,a_7), 0.66), (garden\_size(a_5,a_7), 0.35)\} \\ \mathcal{A}_2 &= \{(better(a_5,a_7) \leftarrow price(a_5,a_7), 0.99), (price(a_5,a_7), 0.69)\} \\ \mathcal{A}_3 &= \{(\sim better(a_7,a_5) \leftarrow garden\_size(a_5,a_7), 0.66), (garden\_size(a_5,a_7), 0.35)\} \\ \mathcal{A}_4 &= \{(\sim better(a_7,a_5) \leftarrow price(a_5,a_7), 0.99), (price(a_5,a_7), 0.69)\} \\ \mathcal{E}_1 &= \{(\sim better(a_5,a_7) \leftarrow size(a_7,a_5), 0.33), (size(a_7,a_5), 0.05)\} \\ \mathcal{E}_2 &= \{(better(a_7,a_5) \leftarrow size(a_7,a_5), 0.33), (size(a_7,a_5), 0.05)\}. \end{aligned}$$

Definition 4.3 states nothing about how to compute the necessity degrees of the clauses belonging to (Π, Δ), and hence the necessity degrees of the arguments built from (Π, Δ), but imposes restrictions on them that must be satisfied. In the particular program presented here, these values were calculated as follows:

(i) Normalise the alternatives' attribute values to interval [0, 1] for all of the preference criteria (see Table 2).
(ii) Compare the alternatives among each other with respect to the normalised preference criteria (see first column of Table 3). The alternative which appears as first argument of the comparison literal has a better attribute value (with respect to its associated preference criterion) than the alternative that appears as second argument. The necessity degree of the clause is calculated as the absolute value of the remainder of their normalised attribute values. If two alternatives have the same attribute value with respect to a particular preference criterion, then no clause is included in the program since its associated necessity degree would be zero. For instance, alternatives a_3 and a_7

Table 2. Normalised attribute values for the acceptable apartments.

Flat	Size	Garden	Price	Size ([0, 1])	Garden ([0, 1])	Price ([0, 1])
a_3	65	0	350	1	0	0.93
a_5	55	15	350	0.85	1	0.93
a_7	65	12	375	1	0.8	1
Max	**65**	**15**	**375**	**1**	**1**	**1**

have the same price, thus neither $(price(a_3, a_7), 0)$ nor $(price(a_7, a_3), 0)$ is included in (Π, Δ).

(iii) Divide the necessity degrees obtained in previous step by the number of preference criteria provided to the decision-maker, i.e. by 3 in this case (see second column of Table 3).

(iv) Maps the necessity degrees obtained in previous step to subintervals of $(0, 1)$ depending on the comparison literal in the clause (see third column of Table 3). This step is necessary to fulfil point (v) of Definition 4.3. Thus, the necessity degree of each clause is mapped to a subinterval proportionally ranked with the ordering in $>_{\mathcal{C}}$ of the comparison literal present in that clause. In this case, the values of the clauses having the comparison literal *size* were mapped to subinterval $[0, 0.34)$, while the subintervals $[0.34, 0.67)$ and $[0.67, 1)$ were assigned to the comparison literals *garden_size* and *price*, respectively.

(v) For each clause (φ, α) such that φ is a rule of the kind $better(W, Y) \leftarrow c_{p_i}(W, Y)$ or $\sim better(W, Y) \leftarrow c_{p_i}(Y, W)$, set α to the upper bound value of the subinterval assigned to $c_{p_i}(\cdot, \cdot)$.

In the original example of (Antoniou & van Harmelen, 2004), the main idea was to illustrate how a non-monotonic rule system, in particular Defeasible Logic (Nute, 1994), can be used in an electronic commerce application, as a brokered trade. That is why much of the emphasis of the example concerns describing the formalisation of Carlos' requirements and thus determining which alternatives are acceptable. Then, from the acceptable alternatives how to define the rules to select an apartment for Carlos is also explained. Conversely, in Example 4.4 the primary focus was to define a conformant P-DeLP program to clarify the aspects involved in Definition 4.3. Indeed, as we shown in Definitions 4.13 and 4.14, in our approach the acceptable alternatives belong to the choice experiment posed to the decision-maker, and from this set the primary focus is to make a rational choice. This means that how the elements of $\mathcal{B}$ will be determined is not of concern. This view is based on the approach of (Mas-Collel, Whinston, & Green, 1995) where it is stated that $\mathcal{B}$ should be thought of as an exhaustive listing of all the choice experiments that the institutionally, physically or otherwise restricted social situation can conceivably pose to the decision-maker.

Table 3. Alternatives comparison.

$(price(a_3, a_7), 0.07)$	$(price(a_3, a_7), 0.02)$	$(price(a_3, a_7), 0.69)$
$(price(a_5, a_7), 0.07)$	$(price(a_5, a_7), 0.02)$	$(price(a_5, a_7), 0.69)$
$(garden\_size(a_5, a_3), 1)$	$(garden\_size(a_5, a_3), 0.33)$	$(garden\_size(a_5, a_3), 0.67)$
$(garden\_size(a_5, a_7), 0.2)$	$(garden\_size(a_5, a_7), 0.01)$	$(garden\_size(a_5, a_7), 0.35)$
$(garden\_size(a_7, a_3), 0.8)$	$(garden\_size(a_7, a_3), 0.26)$	$(garden\_size(a_7, a_3), 0.6)$
$(size(a_3, a_5), 0.15)$	$(size(a_3, a_5), 0.05)$	$(size(a_3, a_5), 0.05)$
$(size(a_7, a_5), 0.15)$	$(size(a_7, a_5), 0.05)$	$(size(a_7, a_5), 0.05)$

In theory, it could be the case that $\mathcal{B} = 2^X - \emptyset$, but in practice the choice experiments presented to the agent will compose a much more compact set. In this way, in the context of bounded rationality stated at the end of Section 2, the rationality of our decision-maker is naturally limited to the available alternatives it has.

Definition 4.5 (Epistemic component). An epistemic component $(\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$ is a 5-tuple, where $\mathcal{C}$ is a set of comparison literals representing the agent's preferences, $>_{\mathcal{C}}$ is an *L-order* defined over $\mathcal{C}$, ACC is a user-specified function to aggregate necessity degrees, Π is a set of certain clauses and Δ is a set of uncertain clauses.

As shown later, in order to get an epistemic component with certain features, a methodology (Definition 4.8) is proposed to develop it. The use of this methodology allows to define in a direct way the concept of warranted literal from the epistemic component based on the concept of warranted literal from a conformant P-DeLP program, as stated in Definition 4.9. But before, it is conveniently introduced the definition of conformant accrual function and a related property that this function should satisfy.

Property 4.6. Let $\mathcal{P} = (\Pi, \Delta)$ be a conformant P-DeLP program, X be the set of all the available alternatives, $\mathbf{P}$ be the set of preference criteria that will be used to compare the elements in X and $\mathbf{A}_{\mathcal{P}}$ be the set of all the arguments which can be generated from $\mathcal{P}$. Any user-specified function ACC should satisfy the following requirement:

Given any two alternatives $x, y \in X$ with different attribute values, and the accrued structures $[\Phi_x, better(x, y), \alpha]$, $[\Phi_y, \sim better(y, x), \alpha]$, $[\Phi_{x'}, \sim better(x, y), \beta]$ and $[\Phi_{y'}, better(y, x), \beta]$ built from $\mathcal{P}$, it holds that $\alpha \neq \beta$. In particular, in an overall evaluation if x is preferred to y with respect to $\mathbf{P}$, it will hold that $\alpha > \beta$.

Definition 4.7. Let $\mathcal{P} = (\Pi, \Delta)$ be a conformant P-DeLP program. The accruing function $f^{+}_{\Phi}(\cdot)$ is conformant with respect to $\mathcal{P}$, if ACC[4] besides satisfying the *non-depreciation* and *maximality* properties also satisfies Property 4.6.

Property 4.6. is fundamental since it would be counterproductive if a function ACC would yield as result $ACC(\alpha_1, \ldots, \alpha_i) = \alpha = ACC(\beta_1, \ldots, \beta_j) = \beta$, given two alternatives $x, y \in X$ with different properties, and the a-structures $[\Phi_x, better(x, y), \alpha]$, $[\Phi_y, \sim better(y, x), \alpha]$, $[\Phi_{x'}, \sim better(x, y), \beta]$ and $[\Phi_{y'}, better(y, x), \beta]$ built from $\mathcal{P}$. This would imply that the answer to the queries $better(x, y)$, $\sim better(x, y)$, $better(y, x)$ and $\sim better(y, x)$ would be undecided, a fact clearly undesirable, considering that alternatives x and y are different, and hence, a warrant to support that one is better than the other should exist.

Definition 4.8. (Methodology $\mathcal{M}$ for building the epistemic component)

Consider an agent which has a set X of alternatives and a set $\mathbf{P} = \{p_1, \ldots, p_n\}$ $(n > 0)$ of preference criteria that it wants to use for comparing the elements in X.

Step 1: Define a set of comparison literals $\mathcal{C}$, as indicated in Definition 4.1.
Step 2: Define '$>_{\mathcal{C}}$', as stated in Definition 4.2.
Step 3: Build a conformant P-DeLP program $\mathcal{P} = (\Pi, \Delta)$, as outlined in Definition 4.3.
Step 4: Specify a function ACC satisfying the *non-depreciation* and *maximality* properties

so as to define the accruing function $f^{+}_{\Phi}(\cdot)$ (see Definition 3.3). Furthermore, *ACC* should satisfy Property 4.6 to preserve distinctness among the accrued structures supporting complementary literals.

Definition 4.9 (Warranted literal from the epistemic component). Let $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$ be the agent's epistemic component built following methodology $\mathcal{M}$. A literal L is warranted from $\mathcal{K}$, if and only if L is warranted from the conformant P-DeLP program (Π, Δ).

As it can be observed in Definition 4.8, in addition to the *non-depreciation* and *maximality* properties, function *ACC* should also satisfy Property 4.6 to preserve distinctness among the accrued structures supporting complementary literals. This is a key issue to state the following proposition and its related corollary.[5]

Proposition 4.10. Let $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$ be the agent's epistemic component built following methodology $\mathcal{M}$, X be the set of all the possible alternatives and $x, y \in X$ be two alternatives with different properties. Then, either the query *better* (x, y) or $\sim$ *better* (x, y) is warranted from $\mathcal{K}$.

Corollary 4.11. Let $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$ be the agent's epistemic component built following methodology $\mathcal{M}$, X be the set of all the possible alternatives and $x, y \in X$ be two alternatives with different properties. If either *better* (x, y) or $\sim$ *better* (x, y) is warranted from $\mathcal{K}$, then either $\sim$ *better* (y, x) or *better* (y, x) is also warranted from $\mathcal{K}$, respectively.

Proposition 4.10 and Corollary 4.11 state that given an epistemic component $\mathcal{K}$ built following methodology $\mathcal{M}$, then either the query *better* (x, y) or $\sim$ *better* (x, y) is warranted from $\mathcal{K}$, for alternatives $x, y \in X$ with different properties. Similarly, if alternatives $x, y \in X$ have the same properties,[6] then the queries $\sim$ *better* (x, y) and $\sim$ *better* (y, x) will be warranted from $\mathcal{K}$, as stated in Proposition 4.12.

Proposition 4.12. Let $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$ be the agent's epistemic component built following methodology $\mathcal{M}$, and X be the set of all possible alternatives. If $x, y \in X$ are two alternatives with the same properties then queries $\sim$ *better* (x, y) and $\sim$ *better* (y, x) are both warranted from $\mathcal{K}$.

So far, in this section only the epistemic component of the proposed decision framework has been described. As shown in Definition 4.13, this framework is a triple and only remains to define its third component which is introduced in Section 4.2.

Definition 4.13 (Decision framework). A decision framework is a triple $\langle X, \mathcal{K}, \Gamma \rangle$ where X is the set of all the possible alternatives the agent has, $\mathcal{K}$ is an epistemic component built following methodology $\mathcal{M}$ and Γ is the agent's set of decision rules.

4.2 Decision component

This component, denoted as Γ in Definition 4.13, is the component which effectively implements the agent's decision-making policy based on the device called *decision rule*, whose formal definition is stated as follows.

Definition 4.14 (Decision rule). Let $B \in \mathcal{B}$ be a choice experiment. A decision rule is denoted $(D \overset{B}{\Leftarrow} P, notT)$, where $D \subseteq B$, P is a set of ground literals representing preconditions and T is a set of ground literals representing constraints.

A decision rule $(D \overset{B}{\Leftarrow} P, not\, T)$ can be read as 'if all the preconditions of P are warranted and no constraint of T is warranted from the agent's epistemic component, then D is a subset of alternatives from B to be selected'. Hence, D will represent those alternatives that this rule decides to adopt from the choice experiment B posed to the decision-maker. This idea is formalised in the Definition 4.15.

Definition 4.15 (Applicable decision rule). Let $B \in \mathcal{B}$ be a choice experiment and $\mathcal{K}$ be an epistemic component. A decision rule $(D \overset{B}{\Leftarrow} P, not\, T)$ is applicable with respect to $\mathcal{K}$, if every precondition $p_i \in P$ is warranted from $\mathcal{K}$ and every constraint $t_j \in T$ fails to be warranted from $\mathcal{K}$.

Decision rules are ground, however, following the common convention (Lifschitz, 1996), a decision rule with variables is a 'schematic decision rule' that represents a set of (ground) decision rules. Although the agent could have any set of decision rules, in our framework it is proposed to use a particular set of decision rules that are represented by the two schematic decision rules DR1 and DR2. Rule DR1 states that an alternative $W \in B$ will be chosen, if W is better than another alternative Y and there is not a better alternative Z than W. Besides, rule DR2 states that two alternatives W, $Y \in B$ with the same properties will be chosen if there is not a better alternative Z than W and Y. In our proposed framework, the set of all the available decision rules to the agent is denoted by Γ, and the formal results presented in Section 4.3 refer to this set of decision rules.

$$\{W\} \overset{B}{\Leftarrow} \{better\,(W, Y)\},\ not\,\{better\,(Z, W)\} \quad \text{(DR1)}$$

$$\{W, Y\} \overset{B}{\Leftarrow} \{same\_prop(W, Y)\},\ not\,\{better\,(Z, W)\} \quad \text{(DR2)}$$

Definition 4.16 (Acceptable alternatives). Let $B \in \mathcal{B}$ be a set of alternatives posed to the agent and $\langle X, \mathcal{K}, \Gamma \rangle$ be the agent's decision framework. Let $\{D_i \overset{B}{\Leftarrow} P_i,\ not\, T_i\}_{i=1\ldots n} \subseteq \Gamma$ be the set of applicable decision rules with respect to $\mathcal{K}$. The set of acceptable alternatives of the agent will be $\Omega_B = \cup_{i=1}^{n} D_i$.

The set Ω_B is a subset of B and if Ω_B contains a single element, that element is the agent's individual choice from among the alternatives in B. However, if Ω_B contains more than one element, then they represent acceptable alternatives that the agent might choose.

In algorithm 1, a general algorithm to compute the set of acceptable alternatives is presented. As it can be observed that function μ has as input parameter a choice experiment (B). A choice experiment is a set containing at least one element, hence, this function returns *failure* if receives as argument an empty set (step (1)). If the choice experiment has one element, then it is thus returned as solution since there is only one trivial choice to be made (step (2)). Then, if a non-empty set was received as parameter, the resulting set *sol* is initialised (step (3)) and a local copy (*ch*) of the original choice experiment is made (step (4)).

The computing process to determine the set of acceptable alternatives ends when *ch* becomes empty (step (6)), thus exiting the main loop (step (5)) returning the computed set of acceptable alternatives *sol* (step (13)). While there are alternatives in *ch*, an alternative is removed from this set and is assigned to *h* (step (7)). If there is not a better alternative than *h* in the choice experiment (step (9)) and *h* is better than any other alternative in the choice experiment (step (8)), then *h* is added to the resulting set *sol* (step (10)), otherwise is discarded (step (9)). Besides, if *h* is not better than any other alternative in the choice experiment (step (8)), but there is no other alternative (let us denoted it as h') in the choice experiment better than *h* (step (11)), then it holds that *h* and h' have the same properties, and they are the best, therefore *h* is added to the resulting set *sol* (step (12)). It is worth mentioning that in turn (when selected in step (7)) h' will also be added to *sol*.

Algorithm 1. Compute acceptable alternatives

```
      function μ(choice-experiment) returns non-empty-set-of-alternatives, or failure
(1)     if EMPTY?(choice-experiment) then return failure
(2)     if SINGLETON?(choice-experiment) then return choice-experiment
(3)     sol ← Ø
(4)     ch← choice-experiment
(5)     loop do
(6)       if EMPTY?(ch) then exit
(7)       h← REMOVE-ELEMENT(ch)
(8)       if IS-h-BETTER-THAN-ANY-OTHER?(choice-experiment) then
(9)         if ANY-BETTER-THAN-h?(choice-experiment) then discard h
(10)          else ADD-ELEMENT(sol,h)
        else
(11)          if ANY-BETTER-THAN-h?(choice-experiment) then discard h
(12)          else ADD-ELEMENT(sol,h)
(13)      return sol
```

This algorithm is based on the assumption that it is always possible for the decision-maker to compare alternatives among each other (steps (8), (9) and (11)) and determine which is/are the best. In Proposition 4.17, it is stated that this algorithm is correct. The acquainted reader should have noticed that function $\mu(\cdot)$ implements a choice rule $C(\cdot)$.

Proposition 4.17. A*lgorithm 1 is correct*.

To conclude this section, we present an example that illustrates the choice behaviour of a decision-maker equipped with the decision framework proposed in Definition 4.13.

Example 4.18 (Apartments renting example continued). Let us suppose that Carlos has another preference criterion, if the neighbourhood is a noisy place. Let us also suppose that neighbourhood noise may be judged as *very quiet*, *quiet*, *moderated*, *noisy* and *very noisy*, and that Carlos' preference on this criterion are given in the same order these five linguistic expressions were stated. In this way, if given the choice, Carlos would go for the cheapest option first, then his second and third priorities would be the presence of a garden and additional space, respectively, letting the neighbourhood noise as his lowest priority. Therefore, the *L-order* would be: $>_C = \{(price, size), (price, nbhd\_noise), (price, garden\_size), (garden\_size, size), (garden\_size, nbhd\_noise), (size, nbhd\_noise)\}$.

Let us also suppose that though a_5 and a_7 are both centrally located, a_5 is near to the trade centre and thus is in a very noisy zone. Conversely, a_7 is located in a quiet residential area, while a_3 is in a neighbourhood with moderated noise given that is placed on the suburbs. Table 4

Table 4. Acceptable apartments.

Flat	Bedrooms	Size	Central	Floor	Lift	Pets	Garden	Price	NBHD noise
a_3	2	65	No	2	No	Yes	0	350	Moderated
a_5	3	55	Yes	0	No	Yes	15	350	Very noisy
a_7	3	65	Yes	1	No	Yes	12	375	Quiet

summarises the properties of the acceptable alternatives that will be presented to the broker agent.

Taking into account the approach used in Example 4.4 to calculate the necessity degrees of the clauses in (Π, Δ), the symbolic values of the criterion *neighbourhood noise* should be mapped to a quantitative scale. In consequence, the symbolic values *very quiet*, *quiet*, *moderated*, *noisy* and *very noisy*, will be quantitatively valued as 15, 13, 9, 5 and 2, respectively.[7] The P-DeLP conformant program modelling this new situation is as follows:

$$\Delta = \left\{ \begin{array}{ll} (price(a_3, a_7), 0.77) & (better\,(W, Y) \leftarrow nbhd\_noise(W, Y), 0.25) \\ (price(a_5, a_7), 0.77) & (\sim better\,(W, Y) \leftarrow nbhd\_noise(Y, W), 0.25) \\ (garden\_size(a_5, a_3), 0.75) & (better\,(W, Y) \leftarrow size(W, Y), 0.5) \\ (garden\_size(a_5, a_7), 0.55) & (\sim better\,(W, Y) \leftarrow size(Y, W), 0.5) \\ (garden\_size(a_7, a_3), 0.7) & (better\,(W, Y) \leftarrow garden\_size(W, Y), 0.75) \\ (size(a_3, a_5), 0.29) & (\sim better\,(W, Y) \leftarrow garden\_size(Y, W), 0.75) \\ (size(a_7, a_5), 0.29) & (better\,(W, Y) \leftarrow price(W, Y), 0.99) \\ (nbhd\_noise(a_3, a_5), 0.21) & (\sim better\,(W, Y) \leftarrow price(Y, W), 0.99) \\ (nbhd\_noise(a_7, a_3), 0.08) & \\ (nbhd\_noise(a_7, a_5), 0.14) & \end{array} \right\}$$

$$\Pi = \{ (\sim better\,(W, Y) \leftarrow sp(W, Y), 1) \qquad (\sim better\,(W, Y) \leftarrow sp(Y, W), 1) \}$$

Given $X = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ and $\mathcal{B} = \{\{a_3\}, \{a_5\}, \{a_7\}, \{a_3, a_5\}, \{a_3, a_7\}, \{a_5, a_7\}, \{a_3, a_5, a_7\}\}$, if the choice experiment to be presented to the agent is $B = \{a_3, a_5, a_7\}$, then the following arguments will be built from the above-defined P-DeLP conformant program:[8]

$$\mathcal{A}_1 = \{(better\,(a_3, a_7) \leftarrow price(a_3, a_7), 0.99), (price(a_3, a_7), 0.77)\}$$

$$\mathcal{A}_2 = \{(\sim better\,(a_7, a_3) \leftarrow price(a_3, a_7), 0.99), (price(a_3, a_7), 0.77)\}$$

$$\mathcal{A}_3 = \{(better\,(a_5, a_7) \leftarrow price(a_5, a_7), 0.99), (price(a_5, a_7), 0.77)\}$$

$$\mathcal{A}_4 = \{(\sim better\,(a_7, a_5) \leftarrow price(a_5, a_7), 0.99), (price(a_5, a_7), 0.77)\}$$

$$\mathcal{A}_5 = \{(better\,(a_5, a_3) \leftarrow garden\_size(a_5, a_3), 0.75), (garden\_size(a_5, a_3), 0.75)\}$$

$$\mathcal{A}_6 = \{(\sim better\,(a_3, a_5) \leftarrow garden\_size(a_5, a_3), 0.75), (garden\_size(a_5, a_3), 0.75)\}$$

$$\mathcal{A}_7 = \{(better\,(a_5, a_7) \leftarrow garden\_size(a_5, a_7), 0.75), (garden\_size(a_5, a_7), 0.55)\}$$

$$\mathcal{A}_8 = \{(\sim better\,(a_7, a_5) \leftarrow garden\_size(a_5, a_7), 0.75), (garden\_size(a_5, a_7), 0.55)\}$$
$$\mathcal{A}_9 = \{(better\,(a_7, a_3) \leftarrow garden\_size(a_7, a_3), 0.75), (garden\_size(a_7, a_3), 0.7)\}$$
$$\mathcal{A}_{10} = \{(\sim better\,(a_3, a_7) \leftarrow garden\_size(a_7, a_3), 0.75), (garden\_size(a_7, a_3), 0.7)\}$$
$$\mathcal{A}_{11} = \{(better\,(a_3, a_5) \leftarrow size(a_3, a_5), 0.5), (size(a_3, a_5), 0.29)\}$$
$$\mathcal{A}_{12} = \{(\sim better\,(a_5, a_3) \leftarrow size(a_3, a_5), 0.5), (size(a_3, a_5), 0.29)\}$$
$$\mathcal{A}_{13} = \{(better\,(a_7, a_5) \leftarrow size(a_7, a_5), 0.5), (size(a_7, a_5), 0.29)\}$$
$$\mathcal{A}_{14} = \{(\sim better\,(a_5, a_7) \leftarrow size(a_7, a_5), 0.5), (size(a_7, a_5), 0.29)\}$$
$$\mathcal{A}_{15} = \{(better\,(a_3, a_5) \leftarrow nbhd\_noise(a_3, a_5), 0.25), (nbhd\_noise(a_3, a_5), 0.21)\}$$
$$\mathcal{A}_{16} = \{(\sim better\,(a_5, a_3) \leftarrow nbhd\_noise(a_3, a_5), 0.25), (nbhd\_noise(a_3, a_5), 0.21)\}$$
$$\mathcal{A}_{17} = \{(better\,(a_7, a_3) \leftarrow nbhd\_noise(a_7, a_3), 0.25), (nbhd\_noise(a_7, a_3), 0.08)\}$$
$$\mathcal{A}_{18} = \{(\sim better\,(a_3, a_7) \leftarrow nbhd\_noise(a_7, a_3), 0.25), (nbhd\_noise(a_7, a_3), 0.08)\}$$
$$\mathcal{A}_{19} = \{(better\,(a_7, a_5) \leftarrow nbhd\_noise(a_7, a_5), 0.25), (nbhd\_noise(a_7, a_5), 0.14)\}$$
$$\mathcal{A}_{20} = \{(\sim better\,(a_5, a_7) \leftarrow nbhd\_noise(a_7, a_5), 0.25), (nbhd\_noise(a_7, a_5), 0.14)\}.$$

To calculate the accrued structures for these arguments, the *ACC* function defined later, with $K = 0.1$, will be used:[9]

$$ACC(\alpha_1, \ldots, \alpha_n) = \left[1 - \prod_{i=1}^{n}(1 - \alpha_i)\right] + K \max(\alpha_1, \ldots, \alpha_n)\prod_{i=1}^{n}(1 - \alpha_i) \text{ with } K \in (0, 1).$$

As it can be observed, 12 a-structures can be built to support the reasons by which an alternative should be deemed better than the other.

$[\Phi_1, better\,(a_3, a_5), 0.46]$,	$[\boldsymbol{\Phi'_1}, \sim \mathbf{better}\,(\boldsymbol{a_3}, \boldsymbol{a_5}), \mathbf{0.77}]$,	$\Phi_1 = \mathcal{A}_{11} \cup \mathcal{A}_{15}$,	$\Phi'_1 = \mathcal{A}_6$;
$[\Phi_2, \sim better\,(a_5, a_3), 0.46]$,	$[\boldsymbol{\Phi'_2}, \mathbf{better}(\boldsymbol{a_5}, \boldsymbol{a_3}), \mathbf{0.77}]$	$\Phi_2 = \mathcal{A}_{12} \cup \mathcal{A}_{16}$,	$\Phi'_2 = \mathcal{A}_5$;
$[\boldsymbol{\Phi_3}, \mathbf{better}(\boldsymbol{a_3}, \boldsymbol{a_7}), \mathbf{0.79}]$,	$[\Phi'_3, \sim better\,(a_3, a_7), 0.74]$,	$\Phi_3 = \mathcal{A}_1$,	$\Phi'_3 = \mathcal{A}_{10} \cup \mathcal{A}_{18}$;
$[\boldsymbol{\Phi_4}, \sim \mathbf{better}(\boldsymbol{a_7}, \boldsymbol{a_3}), \mathbf{0.79}]$,	$[\Phi'_4, better\,(a_7, a_3), 0.74]$,	$\Phi_4 = \mathcal{A}_2$,	$\Phi'_4 = \mathcal{A}_9 \cup \mathcal{A}_{17}$;
$[\boldsymbol{\Phi_5}, \mathbf{better}(\boldsymbol{a_5}, \boldsymbol{a_7}), \mathbf{0.9}]$,	$[\Phi'_5, \sim better\,(a_5, a_7), 0.41]$,	$\Phi_5 = \mathcal{A}_3 \cup \mathcal{A}_7$,	$\Phi'_5 = \mathcal{A}_{14} \cup \mathcal{A}_{20}$;
$[\boldsymbol{\Phi_6}, \sim \mathbf{better}(\boldsymbol{a_7}, \boldsymbol{a_5}), \mathbf{0.9}]$,	$[\Phi'_6, better\,(a_7, a_5), 0.41]$,	$\Phi_6 = \mathcal{A}_4 \cup \mathcal{A}_8$,	$\Phi'_6 = \mathcal{A}_{13} \cup \mathcal{A}_{19}$;

Those a-structures warranted from the dialectical process (shown in bold), will be used by algorithm 1 to compute the set of acceptable alternatives. In this particular case, only DR1 can be applied. Despite the fact that its precondition can be warranted by a-structure $[\Phi_3, better\,(a_3, a_7), 0.79]$, for alternative a_3 the restriction of decision rule DR1 can also be warranted by $[\Phi'_2, better\,(a_5, a_3), 0.77]$ and hence, a_3 becomes unacceptable. Then, with either $[\Phi'_2, better\,(a_5, a_3), 0.77]$ or $[\Phi_5, better\,(a_5, a_7), 0.9]$ DR1's precondition can be warranted, and as there is no warranted a-structure supporting a conclusion of the kind $better\,(Z, a_5)$ to warrant DR1's restriction, a_5 becomes the first acceptable alternative. Finally, as it is not possible for a_7 to warrant DR1's precondition (since there are two warranted a-structures supporting $\sim better\,(a_7, Y)$), then $\Omega_B = \{a_5\}$.

Despite the fact that in this example a new preference criterion was added with respect to the setting presented in Example 4.4, the alternative selected was the same as the one chosen in the original example of (Antoniou & van Harmelen, 2004). Indeed, the interested reader can prove that accordingly, Example 4.4 will yield the same result.

To choose alternative a_5 is quite obvious for the so-called human common sense, since it has the best attribute values in the two most important preference criteria, in spite of being very noisy in the scenario depicted in Example 4.18. If the neighbourhood noise had been assigned a highest priority in the *L-order*, the result almost certainly would be different.

From Example 4.18, it is clear that with this setting, the same set of acceptable alternatives will be obtained for choice experiments $\{a_3, a_5\}$ and $\{a_5, a_7\}$, respectively. When choice experiment $\{a_3, a_7\}$ is considered, arguments $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_9$, $\mathcal{A}_{10}$, $\mathcal{A}_{17}$ and $\mathcal{A}_{18}$ will also be generated, and hence a-structures $[\Phi_3, \mathit{better}(a_3, a_7), 0.79]$, $[\Phi_3^{'}, \sim \mathit{better}(a_3, a_7), 0.74]$, $[\Phi_4, \sim \mathit{better}(a_7, a_3), 0.79]$ and $[\Phi_4^{'}, \mathit{better}(a_7, a_3), 0.74]$ will interact in the dialectical process to yield the conclusions $\mathit{better}(a_3, a_7)$ and $\sim \mathit{better}(a_7, a_3)$ as warranted. In this way, decision rule DR1 will choose a_3 as acceptable alternative.

The above-mentioned discussion shows that the choice behaviour of the broker agent satisfies the WARP principle, as stated in Theorem 4.21 in Section 4.3. In Section 4.3, the choice behaviour of the proposed decision framework is formalised according to the general theory of choice of Classical Decision Theory.

4.3 Formal comparison with respect to Classical Decision Theory

As stated in Section 1, Fox's earlier work on exploring symbolic approaches to decision-making produced noteworthy findings, such as that knowledge-based qualitative models usually offered a complement to numerical methods, because in general they lacked the sound theoretical foundation of numerical approaches. Thus, in this section, the choice behaviour of the decision framework formulated in Definition 4.13, with respect to the optimum choice of Classical Decision Theory, is formalized. To start with, Lemma 4.19 defines how a rational preference relation is obtained from the epistemic component of the decision framework.

Lemma 4.19. Let X be the set of alternatives provided to the agent and $\mathbf{P} = \{p_1, \ldots, p_n\}$ $(n > 0)$ be the set of preference criteria that will be used to compare the elements in X. Let $\succsim_{\mathbf{P}}$ be a rational preference relation over the elements of X, based on $\mathbf{P}$. Let $\mathcal{C}$ be the set of comparison literals associated to $\mathbf{P}$. Let $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, \mathrm{ACC}, \Pi, \Delta)$ be an epistemic component built following methodology $\mathcal{M}$. Then, the following statements hold:

(i) The literals $\mathit{better}(x, y)$ and $\sim\mathit{better}(y, x)$ are warranted from $\mathcal{K}$, iff $(x, y) \in \succsim_{\mathbf{P}}$ and $(y, x) \notin \succsim_{\mathbf{P}}$.

(ii) The literals $\sim\mathit{better}(x, y)$ and $\sim\mathit{better}(y, x)$ are warranted from $\mathcal{K}$, iff $(x, y), (y, x) \in \succsim_{\mathbf{P}}$.

In the same way, Theorems 4.20 and 4.21 formalise the choice behaviour of a decision-maker which uses the decision framework defined in Definition 4.13, with respect to the classical approaches presented in Sections 2.1 and 2.2, respectively.

Theorem 4.20. Let X be the set of all the possible alternatives the agent has, and $B \subseteq X$ be a choice experiment presented to the agent. Let $\langle X, \mathcal{K}, \Gamma \rangle$ be the agent's decision framework where $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, \mathrm{ACC}, \Pi, \Delta)$. Then, the set of acceptable alternatives of the agent is $\Omega_B = C^*(B, \succsim)$.

Theorem 4.21. Let $\langle X, \mathcal{K}, \Gamma \rangle$ be the agent's decision framework where $\mathcal{K}$ $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, \text{ACC}, \Pi, \Delta)$. Given the set $\mathcal{B}$ of possible choice experiments, and given function $\mu(\cdot)$ described in algorithm 1, then the choice structure $(\mathcal{B}, \mu(\cdot))$ satisfies the WARP.

In Section 5, we present a discussion of several argumentation-based approaches to decision-making, as well as its relation to classical approaches of decision theory.

5. Related work

To the best of our knowledge, the first work on symbolic decision-making following the point of view of Marketing literature to decision-making was (Governatori, ter Hofstede, & Oaks, 2000), where the application of Defeasible Logic for automated negotiation was investigated. In this work, two cases of study were faced: one concerning a brokered trade and another considering a simple case of negotiation. For the purpose of our work, only the brokered trade is analysed, where Defeasible Logic was used to select goods against a set of constraints and then to choose the most appropriate goods.

The scenario consisted of a broker who had five yachts to sell (from sellers with different requirements) and a buyer contacted him to buy a yacht with particular features. The trade was performed in a two-stage process formalised in terms of two correlated defeasible theories: the first for filtering and the second for choosing. In an isomorphic form, the same approach was used in Antoniou and van Harmelen (2004) and Antoniou, Skylogiannis, Bikakis, Doerr, and Bassiliades (2007) for the apartments renting example, where given the buyer's requirements, the alternatives (apartments/yachts) were filtered to obtain the set of acceptable alternatives. Then, from this set, the second theory was used to derive the decision of choosing a particular alternative (buy a yacht or rent an apartment).

The first theory mainly consists of rules of the kind $\mathtt{r}_{\mathtt{i}}$: `¬pets(X) ⇒¬acceptable(X)`, where the head of the rules support whether an alternative `X` is acceptable or not, depending on the buyer's requirements which are given by the predicates in the body of the rules. Then, the priority among the rules will determine which alternatives remain acceptable. For instance, given that any apartment is a priori acceptable ($\mathtt{r}_1$: `⇒acceptable(X)`) since $\mathtt{r}_{\mathtt{i}} > \mathtt{r}_1$, those in which pets are not allowed, they will not be acceptable. Similarly, the second theory contains rules such as $\mathtt{r}_{\mathtt{j}}$: `cheapest(X), largestGarden(X) ⇒ rent(X)` and r_k: cheapest(X) ⇒ rent(X) with their priorities set accordingly, e.g. $\mathtt{r}_{\mathtt{j}} > \mathtt{r}_{\mathtt{k}}$, in this case. Besides, since at most one apartment can be rented, literals `rent(x)` are conflicting and this is represented by using conflict sets: `C(rent(x))` = { `¬rent(x)`} ∪{ `rent(y)` | `y` ≠`x`}.

As stated in Prakken (2005), this approach, called *knowledge representation* approach consists of encoding the accrual of reasons by hand, as a conditional with a conjunction of the accruing reasons in the antecedent. Conversely, the approach followed in this article corresponds to the approach defined as the *inference* approach. In this approach, accrual is regarded as a step in the inference process, where after all relevant arguments (based on individual reasons) have been constructed, they are aggregated and a weighting mechanism decides the conflict between the two conflicting sets of reasons. As indicated in Prakken (2005), both approaches may have their pros and cons, and the choice of method will depend on the nature of the application domain.

In our proposal, the decision-maker is provided with a choice experiment which resembles the set of acceptable alternatives built by the first defeasible theory described earlier. Hence, despite the fact that we could build this set with a P-DeLP program following the same approach, we concentrate on choosing what we have called the acceptable alternatives, which would correspond to the chosen alternatives by the second defeasible theory referred earlier. Besides, a key issue of the decision framework proposed in our work is that its choice behaviour has been formalised with respect to the general theory of choice of Classical Decision Theory.

The decision framework presented in Ferretti, Errecalde, García, and Simari (2008) is analogous to the framework proposed in this work in that taking as basis an existing argumentation-based formalism (Defeasible Logic Programming; García and Simari (2004)), a methodology is proposed to develop such decision framework based on the particular features of the formalism. Besides, by using the decision rules device, a connection with Classical Decision Theory, regarding the agent's choice behaviour, was also achieved. Nonetheless, a limitation that this approach has is that no explicit multi-criteria aggregation is used within the framework.

As far as we know, the aforesaid approaches based on Defeasible Logic and Defeasible Logic Programming are the only approaches in the literature on argumentation-based decision-making which adopt the point of view of Marketing. In fact, most of the proposals to qualitative decision-making in argumentation literature share a common view with respect to decision-making, because they conceive it as a form of reasoning oriented towards action. Thus, all of them consider the agent's goals or the expected values of the action, to decide which action to accomplish. This is the main difference with respect to our proposal, which has led us to formalise it with respect to Classical Decision Theory, in a different way. However, not all the proposal described later are related to Classical Decision Theory, and those related to it, have faced the relationship from different perspectives.

For example, in Parsons and Fox (1996) some ideas were exposed to support why argumentation should be considered as a symbolic model of decision-making. Towards this aim, Fox and Parsons (1997, 1998) used the non-standard logic LA (Krause, Ambler, Elvang-Gøransson, & Fox, 1995) as the basis to develop an argumentation system to make decisions about the expected values of actions. They proposed an approach analogous to the decision theoretic notion of expected value. In this approach, compound arguments are built, based on three steps of constructing and combining belief arguments and value arguments. In the first step, an argument in LA is built supporting that the state associated with a proposition C will occur if action A is taken. In the second step, a mechanism AV simply assigns a confidence value to C. Finally, in step three a mechanism LEV derives arguments over sentences in LA and AV to conclude an expected value for A, consistent with the value assigned to C. To choose between alternative actions, they used the expected value to construct a preference ordering over a set of alternative actions. Thus, given sets of arguments supporting alternative actions, the action with highest aggregated value (which indicates the force of the set of arguments supporting it) is chosen.

The core idea of the proposal formalised in Section 4.3 is similar to that of Fox and Parsons' proposal. Both approaches take as basis an existing argumentation system for handling belief, to develop a system to decide between competing alternatives. However, the way this extension is accomplished is very different. Fox and Parsons propose a combined system LA/AV/LEV where compound arguments exist supporting alternative actions, and then, the action selected is the one which has a supporting set of arguments with highest force (aggregated value). In our proposal, the underlying argumentation system (P-DeLP) is neither modified nor extended. In fact, a methodology to developed the decision-maker's epistemic component (knowledge and preferences) is proposed, so that the arguments generated from this component are used as support information to the framework's decision component.

Finally, both approaches are related to Classical Decision Theory but they differ in accomplishing this relation. Fox and Parsons conceive argumentation as a symbolic model of decision-making and use as underlying argumentation formalism the logic LA, whose theoretic proof method to reason under uncertainty is coherent with the semantics of Category Theory and Dempster–Shafer theory. In our case, the whole design of the framework contributes to get a choice behaviour consistent with the general theory of choice of Classical Decision Theory.

The work presented by Kakas and Moraïtis (2003) is an ambitious proposal. They defined an argumentation-based framework to support the decision-making of an agent within a modular architecture. This work can conceptually be divided in three pieces. First, the argumentation-based decision framework is presented, which extends the already existing framework Logic Programming without Negation as Failure (*LPwNF*) (Dimopoulos & Kakas, 1995), to allow the priority relation on the sentences of the theory not to be simply a static relation, but a dynamic relation that captures the non-static preferences associated with roles and context. Thus, arguments and their strengths depend on the particular context that the agent finds itself. Second, it is shown how the integration of abduction within this framework enables the agent to operate in environments where there may be incomplete information. Thirdly, motivated by works in Cognitive Psychology, an argumentation-based personality theory for agents has been integrated within the framework. Although there is no formal connection between Decision Theory and their proposal, they contextualise their work within the general field of qualitative decision theory.

Our proposal coincides with Kakas and Moraïtis', in that both approaches are modular to specify the preferences of the agent. In our case, preferences are given by the *L-order* defined over the set of comparison literals, which represent the preference criteria of the agent. In this way, modifying elements of this *L-order* allow us to change the agent's preferences in a flexible way. The same occurs with the way preferences among rules are represented in *LPwNF*, since the predicate $h\_p(r_1, r_2)$ is used to state that rule r_1 has a higher priority than rule r_2. Nonetheless, in our proposal preferences are static once the agent's knowledge base has been built; while in the extended version of *LPwNF* presented in (Kakas & Moraïtis 2003), the agent may change his preferences as result of an argumentation process.

In Amgoud and Prade, (2009), Amgoud and Prade present a very general and abstract argument-based framework for decision-making. The decision process within the framework follows two main steps. First, arguments for beliefs and arguments for options are built and evaluated using classical acceptability semantics. Second, pairs of options are compared using decision principles. These principles are based on the accepted arguments supporting the decisions and they are classified into three categories, whether they consider only arguments in favour or against a decision, both types of arguments, or an aggregation of them into a meta-argument. This work remains close to the classical view of decision in that it leaves aside aspects of practical reasoning, such as goal generation, feasibility and planning, to concentrate on the issue of justifying (based on argumentation) the best decision to make in a given situation. Besides, it has a logical view of decision that unifies the treatment of multiple criteria decision and decision under uncertainty.

As indicated by Amgoud and Prade, in multiple criteria decision-making, each candidate decision $d \in D$ is evaluated from a set C of m different points of view ($i = 1 \ldots m$), called criteria. Then, two families of approaches can be distinguished. On one hand, we have those based on a global aggregation of value criteria-based functions, where the obtained global absolute evaluations are of the form $g(f_1(C_1(d), \ldots, f_m(C_m(d)))$ and the mappings f_i map the original evaluations on a unique scale, which assumes commensurability. On the other hand, we have the ones that aggregate the preference indices $R_i(d, d')$ into a global preference $R(d, d')$.

Amgoud and Prade follow the former approach while we follow the latter. In our framework, we consider the decision-maker having many preference criteria to evaluate each candidate decision, and by building accrued structures we explicitly compute an aggregation of criteria evaluation or preference indices. Then, using decision rules (see Definition 4.14) a full ranking of the elements can be obtained. In this respect, our approach corresponds to that referred in (Amgoud & Vesic, 2012) as the *cumulative way* of defining argument-based decisions in a multiple criteria decision context. As stated in this work, the accrual of arguments is a key issue to capture the gist of decision-making in an argumentative framework. Besides, given that accrued arguments in our decision framework have as conclusions whether an alternative has been deemed better than another (with respect to a set of criteria), the framework does not suffer the problem stated in (Amgoud & Vesic, 2012) about determining when two arguments are in conflict with respect to the same option. Moreover, this notion of compromise referred in (Amgoud & Vesic, 2012) as indispensable in decision-making to choose the best option is satisfied in our proposed framework by the way how a-structures are built from conformant programs and how they interact with decision rules.

The last proposal contemplated in this section has been made by Bench-Capon, Atkinson, and McBurney (2012) where they demonstrate how a qualitative framework for decision-making can be used to model scenarios from experimental economic studies. In fact, the argumentation-based approach they use is based on the general argumentation approach to practical reasoning developed in (Atkinson, Bench-Capon, & McBurney, 2006), formulating the problem to be modelled as an Action-based Alternating Transition System (Atkinson & Bench-Capon, 2007). Given this model, arguments can then be generated to propose and attack particular actions based on the values promoted or demoted through its execution. Instantiating the argument scheme and the critical questions give rise to a set of conflicting arguments and to determine their acceptability a Value-based Argumentation Framework (VAF) was used (Bench-Capon, 2003). A VAF is an extension of Dung's Argumentation Framework (Dung, 1995), where each argument in the graph is associated with the value promoted by that argument. The purpose of this extension is to distinguish attack from defeat, relative to the audience's preference ordering on the values.

In Bench-Capon et al. (2012), the connection with Decision Theory is achieved by modelling the *Dictator* and the *Ultimatum Game* and explaining by means of the proposed approach the results that have been reported in the literature in different human cultures.

To sum up, it is worth mentioning that similar to the proposals of Fox and Parsons, and Amgoud and Prade, our model is not developed considering an agent with a particular architecture, as Kakas and Moraïtis' proposal. Moreover, when considering the traditional approaches to Classical Decision Theory, our work mainly differs in that the analysis is directly addressed on the agent's preference relation and not on a utility function that represents this relation, as usual in these cases. This is a very important feature since in our proposal modifying preference criteria can be easily accomplished by conveniently modifying the defined *L-order*. In opposition, in the traditional cases that use a utility function, this cannot be performed in a direct way or even the whole recalculation of the utility function might be needed. Besides, considering the agent's preference relation allowed us to establish a direct connection between our argumentation-based decision-making approach and more essential approaches for modelling individual choice behaviour, such as the CBA.

6. Conclusions and future work

Classical Decision Theory is based on the probabilistic view of uncertainty when reasoning about actions. The costs and benefits of actions' possible outcomes are weighted all together with their respective probabilities to obtain a preference ordering on the expected utility of the alternative actions. Many researchers from different research fields have pointed out that to completely specify the utilities and probabilities required by Classical Decision Theory, makes this approach awkward to be applied in complex tasks involving common-sense knowledge representation (Doyle & Thomason, 1999). This fact emphasised the study of new qualitative approaches to decision-making with the aim of decreasing the amount of numerical information required. Among the multiple qualitative approaches proposed within the field of AI, the idea of making decisions based on arguments has become a general approach to support different decision approaches, such as decision under uncertainty, multi-criteria decision, rule-based decision and case-based decision-making.

To the best of our knowledge, the decision framework proposed in this work together with the symbolic decision-making models presented in Governatori et al. (2000) and Ferretti et al. (2008) are the only state-of-the-art proposals following the point of view of Marketing literature in argumentation-based decision-making. Moreover, regarding the choice behaviour exhibited by agents endowed with these decision frameworks, our proposal and the proposal in Ferretti et al. (2008) are the only proposals consistent with the general theory of choice of Classical Decision Theory. In opposition to the proposal in (Ferretti et al., 2008), the present approach allows explicit multi-criteria aggregation. In consequence, this work presents the first structured argumentation-based decision framework that will make effective use of the options as considered in the Marketing literature, that is also formally related to Classical Decision Theory, and which allows explicit multi-criteria aggregation.

This framework combines the use of argument accrual and decision rules to make decisions. A methodology to define the agent's epistemic component is defined, so that the choice behaviour of the proposed decision framework coincides with respect to the optimum choice of a rational preference relation. Our proposal consists of a general framework to qualitative decision-making that can be applied to different domains, nonetheless, with explanatory purposes the principles stated in this work were exemplified in the well-known domain regarding apartments renting. When modelling this domain, a particular *ACC* function, which satisfies all the requirements stated by the methodology, was proposed. Similarly, a particular way of computing the necessity degrees of the clauses belonging to the underlying P-DeLP program of the epistemic component was introduced. Considering this particular issue, as future work, the impact on the framework's decision behaviour when the preference criteria are not uniformly mapped to the interval $(0, 1)$ will be studied.

At present, in our proposal, in the development of the decision framework we have only considered a setting with perfectly certain outcomes. As future work, we plan to develop a new methodology that will take advantage of the structure of uncertain alternatives to restrict the preferences that a rational decision-maker may hold.

Acknowledgements

This work is partially supported by Universidad Nacional de San Luis (PROICO 30312) and Secretaría General de Ciencia y Tecnología – Universidad Nacional del Sur. We also want to thank Mauro Gómez Lucero for useful discussions on the topic of this article.

Notes

1. In Order Theory, a binary relation satisfying these properties is called a *total pre-order*.
2. The necessity degree of φ refers to the degree of possibilistic entailment of q.
3. This example has been taken from (Gómez et al., 2009), please refer to this source to see how the necessity degree values were calculated using One-Complement accrual function.
4. Remember that $f^{+}_{\Phi}(\cdot)$ is parameterised with respect to a user-specified *ACC* function to aggregate necessity degrees (see Definition 3.3).

5. The proof of this proposition as well as the proofs of the remaining propositions, lemmas and theorems in this article, can be found in Appendix A.
6. Two alternatives have the same properties if their attribute values coincide for all the preference criteria used to compare them.
7. These values have been arbitrarily assigned to reflex the priority among the attribute values of the *neighbourhood noise* preference criterion.
8. To simplify notation, given an argument $\langle \mathcal{A}, h, \alpha \rangle$, only the set of uncertain clauses $\mathcal{A}$ will be given since the conclusion h and its associated necessity degree α can be obtained from it.
9. This function is a variant of the One-Complement accrual function used in (Gómez et al. 2009) where K aims at weighting the importance given to the highest priority preference criterion.

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Appendix A: Proofs of theorems, lemmas and propositions

Proposition 4.10. Let $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$ *be the agent's epistemic component built following methodology* $\mathcal{M}$*, X be the set of all the possible alternatives and* $x, y \in X$ *be two alternatives with different properties. Then, either the query* $better(x, y)$ *or* $\sim better(x, y)$ *is warranted from* $\mathcal{K}$*.*

Proof.

Let $\mathbf{P}_x$ be the set of all the preference criteria p_i such that alternative x has a better attribute value than alternative y with respect to p_i. Similarly, let $\mathbf{P}_y$ be the set of all the preference criteria p_j such that y has a better attribute value than x with respect to p_j.

Given methodology $\mathcal{M}$, it can be stated that two a-structures $[\Phi_1, better(x, y), \alpha]$ and $[\Phi_2, \sim better(x, y), \beta]$ can be built such that

$$Args([\Phi_1, better(x, y), \alpha]) = \{\langle \mathcal{A}_1, better(x, y), \alpha_1\rangle, \ldots, \langle \mathcal{A}_n, better(x, y), \alpha_n\rangle\}, \text{ where}$$
$$\mathcal{A}_i = \{(better(x, y) \leftarrow cp_i(x, y), \alpha_{ri}), (cp_i(x, y), \alpha_{fi})\}, \quad \alpha_i = \min\{\alpha_{ri}, \alpha_{fi}\},$$
$$cp_i \in \mathbf{P}_x, |\mathbf{P}_x| = n \text{ and } \alpha = ACC(\alpha_1, \ldots, \alpha_n).$$

$$Args([\Phi_2, \sim better(x, y), \beta]) = \{\langle \mathcal{A}_1, \sim better(x, y), \beta_1\rangle, \ldots, \langle \mathcal{A}_m, \sim better(x, y), \beta_m\rangle\}, \text{ where}$$
$$\mathcal{A}_j = \{(\sim better(x, y) \leftarrow cp_j(y, x), \beta_{rj}), (cp_j(y, x), \beta_{fj})\}, \quad \beta_j = \min\{\beta_{rj}, \beta_{fj}\}, cp_j \in \mathbf{P}_y,$$
$$|\mathbf{P}_y| = m \text{ and } \beta = ACC(\beta_1, \ldots, \beta_m).$$

Besides, by Property 4.6, it holds that $ACC(\alpha_1, \ldots, \alpha_n) \neq ACC(\beta_1, \ldots, \beta_m)$ and depending on which alternative is preferred in an overall evaluation with respect to $\mathbf{P}$, then it will also hold that either $\alpha > \beta$ or $\beta > \alpha$. If $\alpha > \beta$, then a-structure $[\Phi_1, better(x, y), \alpha]$ will be warranted. Conversely, $[\Phi_2, \sim better(x, y), \beta]$ will be warranted when $\beta > \alpha$ holds. In consequence, either the query $better(x, y)$ or $\sim better(x, y)$ will be warranted from $\mathcal{K}$.

Proposition 4.12. Let $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$ *be the agent's epistemic component built following methodology* $\mathcal{M}$ *and X be the set of all possible alternatives. If* $x, y \in X$ *are two alternatives with the same properties then queries* $\sim better(x, y)$ *and* $\sim better(y, x)$ *are both warranted from* $\mathcal{K}$*.*

Proof.

By hypothesis, alternatives x and y have the same properties, and therefore, by point 2 of Definition 4.3 it holds that $(sp(x, y), 1) \in \Pi$. In this way, using the two rules that belong to Π by point 4 of Definition 4.3, two empty arguments $\mathcal{E}_1 = \langle \emptyset, \sim better(x, y), 1\rangle$ and $\mathcal{E}_2 = \langle \emptyset, \sim better(y, x), 1\rangle$ can be built from $\mathcal{K}$. In addition, two maximal a-structures $[\Phi_1, h_1, \alpha_1]$ and $[\Phi_2, h_2, \alpha_2]$ will also be built such that

$$Args([\Phi_1, h_1, \alpha_1]) = \{\mathcal{E}_1\}, \quad h_1 = \sim better(x, y),$$

$$Args([\Phi_2, h_2, \alpha_2]) = \{\mathcal{E}_2\}, \quad h_2 = \sim better(y, x) \text{ and } \alpha_1 = \alpha_2 = 1.$$

Furthermore, as there is no other clause in (Π, Δ) which allows to build a-structures supporting the conclusions $better(x, y)$ or $better(y, x)$, two accrued dialectical trees $\mathcal{T}_{\sim better(x,y)}$ and $\mathcal{T}_{\sim better(y,x)}$ will be built having the maximal a-structures $[\Phi_1, h_1, \alpha_1]$ and $[\Phi_2, h_2, \alpha_2]$ as root nodes, respectively. In fact, these trees will have only one node each, yielding the a-structures $[\Phi_1, h_1, \alpha_1]$ and $[\Phi_2, h_2, \alpha_2]$ themselves as undefeated narrowings in $\mathcal{T}_{\sim better(x,y)}$ and $\mathcal{T}_{\sim better(y,x)}$, respectively. Then, both a-structures will be warranted supporting the conclusions $\sim better(x,y)$ and $\sim better(y,x)$.

Proposition 4.17. Algorithm 1 is correct.

Proof.

By Propositions 4.10 and 4.12, it is guaranteed that it is always possible for the decision-maker to compare two alternatives among each other and determine whether there is one alternative which is the best, or whether they have the same properties.

Owing to this fact, if the resulting set '*sol*' contains only one alternative then this is the best among all. Conversely, if '*sol*' contains more than one alternative, they will have the same properties and they will be the best alternatives among those belonging to the choice experiment. Similarly, each alternative is compared against the remaining alternatives belonging to the choice experiment (steps (8), (9) and (11)); thus, those alternatives being the best will certainly belong to the resulting set '*sol*'. Hence, this algorithm is correct.

Lemma 4.19. Let X be the set of alternatives provided to the agent and $\mathbf{P} = \{p_1, \ldots, p_n\}$ $(n > 0)$ be the set of preference criteria that will be used to compare the elements in X. Let $\succsim_{\mathbf{P}}$ be a rational preference relation over the elements of X, based on $\mathbf{P}$. Let $\mathcal{C}$ be the set of comparison literals associated to $\mathbf{P}$. Let $\mathcal{K} = (\mathcal{C}, >_C, ACC, \Pi, \Delta)$ be an epistemic component built following methodology $\mathcal{M}$. Then, the following statements hold:

(i) The literals $better(x,y)$ and $\sim better(y,x)$ are warranted from $\mathcal{K}$, iff $(x,y) \in \succsim_{\mathbf{P}}$ and $(y,x) \notin \succsim_{\mathbf{P}}$.
(ii) The literals $\sim better(x,y)$ and $\sim better(y,x)$ are warranted from $\mathcal{K}$, iff $(x,y), (y,x) \in \succsim_{\mathbf{P}}$.

Proof.

(1)

Let $\mathbf{P}_x$ be the set of all the preference criteria p_i such that alternative x has a better attribute value than alternative y with respect to p_i. Similarly, let $\mathbf{P}_y$ be the set of all the preference criteria p_j such that y has a better attribute value than x with respect to p_j.

$\Rightarrow$)

By hypothesis, $better(x,y)$ and $\sim better(y,x)$ are warranted from $\mathcal{K}$, in this way, given methodology $\mathcal{M}$, it can be stated that:

(i) There exist two warranted a-structures $[\Phi_1, better(x,y), \alpha]$ and $[\Phi_2, \sim better(y,x), \alpha]$ which support these literals as conclusions.
(ii) There are two other a-structures, $[\Phi_3, better(y,x), \beta]$ and $[\Phi_4, \sim better(x,y), \beta]$, which are defeated by $[\Phi_2, \sim better(y,x), \alpha]$ and $[\Phi_1, better(x,y), \alpha]$, respectively.
(iii) $Args([\Phi_1, better(x,y), \alpha]) = \{\langle \mathcal{A}_1, better(x,y), \alpha_1\rangle, \ldots, \langle \mathcal{A}_n, better(x,y), \alpha_n\rangle\}$ such that $\mathcal{A}_i = \{(better(x,y) \leftarrow cp_i(x,y), \alpha_{ri}), (cp_i(x,y), \alpha_{fi})\}$, $\alpha_i = \min\{\alpha_{ri}, \alpha_{fi}\}$, $cp_i \in \mathbf{P}_x$ and $|\mathbf{P}_x| = n$.
(iv) $Args([\Phi_2, \sim better(y,x), \alpha]) = \{\langle \mathcal{A}_1, \sim better(y,x), \alpha_1\rangle, \ldots, \langle \mathcal{A}_n, \sim better(y,x), \alpha_n\rangle\}$ such that $\mathcal{A}_i = \{(\sim better(y,x) \leftarrow cp_i(x,y), \alpha_{ri}), (cp_i(x,y), \alpha_{fi})\}$, $\alpha_i = \min\{\alpha_{ri}, \alpha_{fi}\}$, $cp_i \in \mathbf{P}_x$ and $|\mathbf{P}_x| = n$.
(v) $Args([\Phi_3, better(y,x), \beta]) = \{\langle \mathcal{A}_1, better(y,x), \beta_1\rangle, \ldots, \langle \mathcal{A}_m, better(y,x), \beta_m\rangle\}$ such that $\mathcal{A}_j = \{(better(y,x) \leftarrow cp_j(y,x), \beta_{rj}), (cp_j(y,x), \beta_{fj})\}$, $\beta_j = \min\{\beta_{rj}, \beta_{fj}\}$, $cp_j \in \mathbf{P}_y$ and $|\mathbf{P}_y| = m$.
(vi) $Args([\Phi_4, \sim better(x,y), \beta]) = \{\langle \mathcal{A}_1, \sim better(x,y), \beta_1\rangle, \ldots, \langle \mathcal{A}_m, \sim better(x,y), \beta_m\rangle\}$ such that $\mathcal{E}_j = \{(\sim better(x,y) \leftarrow cp_j(y,x), \beta_{rj}), (cp_j(y,x), \beta_{fj})\}$, $\beta_j = \min\{\beta_{rj}, \beta_{fj}\}$, $cp_j \in \mathbf{P}_y$ and $|\mathbf{P}_y| = m$.
(vii) Since $ACC(\alpha_1, \ldots, \alpha_n) > ACC(\beta_1, \ldots, \beta_m)$, it turns out that the aggregated preference of the criteria in $\mathbf{P}_x$ is higher than the aggregated preference of the criteria in $\mathbf{P}_y$, and thus $(x,y) \in \succsim_{\mathbf{P}}$ and $(y,x) \notin \succsim_{\mathbf{P}}$ hold.

$\Leftarrow$)

By hypothesis, it holds that $(x,y) \in \succsim_{\mathbf{P}}$ and $(y,x) \notin \succsim_{\mathbf{P}}$; thus, it turns out that the aggregated preference of the criteria in $\mathbf{P}_x$ is higher than the aggregated preference of the criteria in $\mathbf{P}_y$. In this way, given methodology $\mathcal{M}$, it can be stated that:

(i) For each $p_i \in \mathbf{P}_x$, there exists a clause $(Q, \alpha_i) \in \Delta$, such that $Q = c_{p_i}(x, y)$. Let Q_x be the set of all such clauses.
(ii) For each $p_j \in \mathbf{P}_y$, there exists a clause $(Q, \alpha_j) \in \Delta$, such that $Q = c_{p_j}(y, x)$. Let Q_y be the set of all such clauses.
(iii) Each clause in Q_x will provide the evidence to build two arguments $\langle \mathcal{A}_1, \mathit{better}(x, y), \alpha_1 \rangle$ and $\langle \mathcal{A}_2, \sim \mathit{better}(y, x), \alpha_2 \rangle$ from the epistemic component:

$$\mathcal{A}_1 = \{(\mathit{better}(x, y) \leftarrow c_{p_i}(x, y), \alpha_r), (c_{p_i}(x, y), \alpha_i)\}$$

$$\mathcal{A}_2 = \{(\sim \mathit{better}(y, x) \leftarrow c_{p_i}(x, y), \alpha_r), (c_{p_i}(x, y), \alpha_i)\}(\alpha_1 = \alpha_2 = \min\{\alpha_r, \alpha_i\}).$$

(iv) Each clause in Q_y will provide the evidence to build two arguments $\langle \mathcal{E}_1, \mathit{better}(y, x), \delta_1 \rangle$, $\langle \mathcal{E}_2, \sim \mathit{better}(x, y), \delta_2 \rangle$ from the epistemic component:

$$\mathcal{E}_1 = \{(\mathit{better}(y, x) \leftarrow c_{p_j}(y, x), \delta_r), (c_{p_j}(y, x), \delta_j)\}$$

$$\mathcal{E}_2 = \{(\sim \mathit{better}(x, y) \leftarrow c_{p_j}(y, x), \delta_r), (c_{p_j}(y, x), \delta_j)\}(\delta_1 = \delta_2 = \min\{\delta_r, \delta_j\}).$$

(v) For all the arguments of the kind $\langle \mathcal{A}_i, Q, \alpha_i \rangle$, $\langle \mathcal{E}_j, \bar{Q}, \delta_j \rangle$ referred earlier, it turns out that either $\alpha_i > \delta_j$ or $\delta_j > \alpha_i$ hold, depending on whether $(c_{p_i}, c_{p_j}) in >_{\mathcal{C}}$ or $(c_{p_j}, c_{p_i}) \in >_{\mathcal{C}}$ hold.
(vi) All the above-mentioned arguments (points 3 and 4) will be accrued in a-structures $[\Phi_1, \mathit{better}(x, y), \alpha]$, $[\Phi_2, \sim \mathit{better}(y, x), \alpha]$, $[\Phi_3, \mathit{better}(y, x), \delta]$ and $[\Phi_4, \sim \mathit{better}(x, y), \delta]$, respectively, as specified in Definition 3.3.
(vii) Four accrued dialectical trees $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_3$ and $\mathcal{T}_4$ will be built having $[\Phi_1, \mathit{better}(x, y), \alpha]$, $[\Phi_2, \sim \mathit{better}(y, x), \alpha]$, $[\Phi_3, \mathit{better}(y, x), \delta]$ and $[\Phi_4, \sim \mathit{better}(x, y), \delta]$ as root nodes, respectively.
(viii) Since *ACC* satisfies Property 4.6 and the non-depreciation and maximality properties, $\mathcal{T}_1$ and $\mathcal{T}_2$ will be one-node trees and hence, undefeated conclusions. Conversely, $\mathcal{T}_3$ and $\mathcal{T}_4$ will be two-node trees having as only leaves $[\Phi_1, \mathit{better}(x, y), \alpha]$ and $[\Phi_2, \sim \mathit{better}(y, x), \alpha]$, respectively, thus yielding the conclusions $\mathit{better}(y, x)$ and $\sim \mathit{better}(x, y)$ as defeated conclusions.

(2)
$\Rightarrow$)
By hypothesis, $\sim\mathit{better}(x, y)$ and $\sim\mathit{better}(y, x)$ are warranted from $\mathcal{K}$, in this way, given methodology $\mathcal{M}$, it can be stated that:

(i) There exist two warranted a-structures, $[\Phi_1, \sim \mathit{better}(x, y), \alpha_1]$ and $[\Phi_2, \sim \mathit{better}(y, x), \beta_1]$, which support these literals as conclusions.
(ii) $\mathit{Args}([\Phi_1, \sim \mathit{better}(x, y), \alpha_1]) = \{\langle \mathcal{A}_1, \sim \mathit{better}(x, y), 1 \rangle\}$ such that $\langle \mathcal{A}_1, \sim \mathit{better}(x, y), 1 \rangle = \langle \{(\sim \mathit{better}(x, y) \leftarrow \mathit{sp}(x, y), 1), (\mathit{sp}(x, y), 1)\}, \sim \mathit{better}(x, y), 1 \rangle$, hence, yielding $\alpha_1 = 1$.
(iii) $\mathit{Args}([\Phi_2, \sim \mathit{better}(y, x), \beta_1]) = \{\langle \mathcal{A}_2, \sim \mathit{better}(y, x), 1 \rangle\}$ such that $\langle \mathcal{A}_2, \sim \mathit{better}(y, x), 1 \rangle = \langle \{(\sim \mathit{better}(y, x) \leftarrow \mathit{sp}(x, y), 1), (\mathit{sp}(x, y), 1)\}, \sim \mathit{better}(y, x), 1 \rangle$, also yielding $\beta_1 = 1$.
(iv) Since $(\mathit{sp}(x, y), 1) \in \Pi$, alternatives x and y have the same attribute values with respect to all the preference criteria in $\mathbf{P}$, and therefore $(x, y), (y, x) \in \succsim_{\mathbf{P}}$.

$\Leftarrow$)
By hypothesis, $(x, y), (y, x) \in \succsim_{\mathbf{P}}$, that is to say that alternative x is indifferent to y with respect to preference criteria in $\mathbf{P}$. This implies that both alternatives have the same attribute values for all the preference criteria in $\mathbf{P}$. In this way, given methodology $\mathcal{M}$, it can be stated that:

(i) A clause $(\mathit{sp}(x, y), 1) \in \Pi$.
(ii) Two arguments $\mathcal{A}_1$ and $\mathcal{A}_2$ will be built to support the conclusions $\sim \mathit{better}(x, y)$ and $\sim \mathit{better}(y, x)$, respectively:

$$\langle \mathcal{A}_1, \sim \mathit{better}(x, y), 1 \rangle = \langle \{(\sim \mathit{better}(x, y) \leftarrow \mathit{sp}(x, y), 1), (\mathit{sp}(x, y), 1)\}, \sim \mathit{better}(x, y), 1 \rangle,$$

$$\langle \mathcal{A}_2, \sim \mathit{better}(y, x), 1 \rangle = \langle \{(\sim \mathit{better}(y, x) \leftarrow \mathit{sp}(x, y), 1), (\mathit{sp}(x, y), 1)\}, \sim \mathit{better}(y, x), 1 \rangle.$$

(iii) Two maximal a-structures $[\mathcal{A}_1, \sim \mathit{better}(x, y), 1]$ and $[\mathcal{A}_2, \sim \mathit{better}(y, x), 1]$ will be built to support the conclusions $\sim \mathit{better}(x, y)$ and $\sim \mathit{better}(y, x)$, respectively.

(iv) Two accrued dialectical trees $\mathcal{T}_{\sim better\,(x,y)}$ and $\mathcal{T}_{\sim better\,(y,x)}$ will be built having the maximal a-structures $[\mathcal{A}_1, \sim better\,(x,y), 1]$ and $[\mathcal{A}_2, \sim better\,(y,x), 1]$ as root nodes, respectively. In fact, these trees will have only one node each, yielding the a-structures $[\mathcal{A}_1, \sim better\,(x,y), 1]$ and $[\mathcal{A}_2, \sim better\,(y,x), 1]$ themselves as undefeated narrowings in $\mathcal{T}_{\sim better\,(x,y)}$ and $\mathcal{T}_{\sim better\,(y,x)}$, respectively. Then, both a-structures will be warranted supporting the conclusions $\sim better\,(x,y)$ and $\sim better\,(y,x)$.

Theorem 4.20. Let X be the set of all the possible alternatives the agent has, and $B \subseteq X$ be a choice experiment presented to the agent. Let $\langle X, \mathcal{K}, \Gamma \rangle$ be the agent's decision framework where $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$. Then, the set of acceptable alternatives of the agent is $\Omega_B = C^*(B, \succsim)$.

Proof.

By Definition 4.16, $\Omega_B = \cup_{i=1}^{n} D_i$, where each set D_i contains the alternatives selected by an applicable decision rule. By Definition 4.15, a decision rule is applicable if its preconditions are warranted from the epistemic component $\mathcal{K}$, and its restrictions are not. Decision rules in Γ have as restriction that an alternative W will belong to Ω_B if *better* (Z, W) cannot be warranted from $\mathcal{K}$, namely no better alternative Z $(\in B)$ than W exists. The precondition of decision rule $DR1$ requires that *better* (W, Y) be warranted from $\mathcal{K}$, namely that W must be strongly preferred to other alternative $Y \in B$. Besides, the precondition of decision rule $DR2$ requires that $sp(W, Y)$ be warranted from $\mathcal{K}$, that is to say that W must be indifferent to other alternative $Y \in B$. Moreover, by Propositions 4.10, 4.12 and Corollary 4.11, it is guaranteed that it will always be possible to warrant the preconditions and/or restrictions of decision rules in Γ. In this way, in Ω_B there will only be alternatives $x \in B$ that are strongly preferred or indifferent to any other alternative $y \in B$; i.e. $x \succsim y$. Therefore, $\Omega_B = \{x \in B \mid x \succsim y \text{ for each } y \in B\} = C^*(B, \succsim)$.

Theorem 4.21. Let $\langle X, \mathcal{K}, \Gamma \rangle$ be the agent's decision framework where $\mathcal{K} = (\mathcal{C}, >_{\mathcal{C}}, ACC, \Pi, \Delta)$. Given the set $\mathcal{B}$ of possible choice experiments, and given function $\mu(\cdot)$ described in algorithm 1, then the choice structure $(\mathcal{B}, \mu(\cdot))$ satisfies the WARP.

Proof.

Function $\mu(\cdot)$ described in algorithm 1 implements a choice rule. Then, it remains to check that $(\mathcal{B}, \mu(\cdot))$ satisfies the choice behaviour restrictions imposed by WARP.

As stated in Lemma 4.19, $\mathcal{K}$ implements a rational preference relation $\succsim$, and by Theorem 4.20 it holds that $\Omega_B = C^*(B, \succsim)$.

Suppose that for some $B \in \mathcal{B}$, we have $x, y \in B$ and $x \in C^*(B, \succsim)$. By definition of $C^*(B, \succsim)$, this implies $x \succsim y$. To check whether the weak axiom holds, suppose that for some $B' \in \mathcal{B}$ with $x, y \in B'$, we have $y \in C^*(B, \succsim)$. This implies that $y \succsim z$ for all $z \in B'$. But we already know that $x \succsim y$. Hence, by transitivity, $x \succsim z$ for all $z \in B'$, and so $x \in C^*(B', \succsim)$. This is precisely the conclusion that the WARP demands.