The partial inverse minimum cut problem with L_1-norm is strongly NP-hard

Elisabeth Gassner*

Abstract

The partial inverse minimum cut problem is to minimally modify the capacities of a digraph such that there exists a minimum cut with respect to the new capacities that contains all arcs of a prespecified set. Orlin showed that the problem is strongly NP-hard if the amount of modification is measured by the weighted L_1-norm. We prove that the problem remains hard for the unweighted case and show that the NP-hardness proof of Yang [7] for this problem with additional bound constraints is not correct.

1 Introduction

A combinatorial optimization problem is usually given in the following form: Let M be a ground set, $\mathcal{F} \subseteq M$ be the set of feasible solutions and let $c(e) \in \mathbb{R}$ for $e \in M$ be cost coefficients of the elements in M. The task is to find a feasible solution $F \in \mathcal{F}$ that minimizes $\sum_{e \in F} c(e)$. Many classical problems, like the minimum spanning tree problem, the shortest path problem or the minimum cut problem, can be modelled in this way.

For an inverse combinatorial optimization problem, we are given an instance of a combinatorial optimization problem and in addition a prespecified feasible solution $\tilde{F} \in \mathcal{F}$. The task is to find new cost coefficients $\tilde{c}(e)$ $(e \in E)$ such that $\tilde{F}$ is an optimal solution with respect to $\tilde{c}$, i.e., $\sum_{e \in \tilde{F}} \tilde{c}(e) \leq \sum_{e \in F} \tilde{c}(e)$ holds for all $F \in \mathcal{F}$ and the distance between c and $\tilde{c}$ is as small as possible. The distance between these two cost coefficient vectors is usually measured by a weighted L_p-norm.

A partial inverse problem is a generalization of an inverse problem where instead of a feasible solution only a subset $J \subseteq M$ of elements that should be contained in an optimal solution is given, i.e., the task is to find a new cost coefficient vector $\tilde{c}$ such that there exists an optimal solution F^* with respect to $\tilde{c}$ with $J \subseteq F^*$ and the distance between $\tilde{c}$ and c is minimized.

Inverse and partial inverse approached have been applied to several classical combinatorial optimization problems, like the shortest path problem or the minimum spanning tree problem. For a comprehensive survey on inverse and partial inverse problems the interested reader is referred to the survey by Heuberger [4].

In this paper, the partial inverse minimum cut problem is investigated. We show that the partial inverse minimum cut problem is strongly NP-hard even if the (unweighted) L_1-norm is used to measure the amount of capacity modification. This result strengthens the result of Orlin [6] who proved strong NP-hardness of the partial inverse minimum cut problem with weighted L_1-norm. Moreover, we consider the investigations by Yang [7] for the partial inverse minimum cut problem with unweighted L_1-norm and bounds on the modified capacities. Yang claimed that the partial inverse minimum cut problem with unweighted L_1-norm and bounds on the modified capacities is weakly NP-hard. We show by a counter-example that the proof in [7] is not correct. However, we prove that

*gassner@opt.math.tugraz.at. Department of Optimization and Discrete Mathematics, Graz University of Technology, Steyrergasse 30, A-8010 Graz, Austria. This research has been supported by the Austrian Science Fund (FWF) Project P18918-N18.

strong NP-hardness of the problem considered by Yang follows from our NP-hardness result on the partial inverse minimum cut problem as well as from the investigations on the preprocessing minimum cut problem by Lai and Orlin [5].

2 The minimum cut problem and its (partial) inverse

Let $G = (V, E)$ be a digraph, $s \in V$ (source) and $t \in V$ (sink). An $(s-t)$-cut $(X, \bar{X})$ is a partition of V into two disjoint subsets X and $\bar{X}$ such that $s \in X$ and $t \in \bar{X}$. The set of all $(s-t)$-cuts is denoted by $\mathcal{S}$.

The minimum cut problem is to find an $(s-t)$-cut with minimum capacity with respect to the sum-composition, i.e., find a cut $(X, \bar{X}) \in \mathcal{S}$ that minimizes

$$c(X, \bar{X}) = \sum_{\substack{(i,j)\in E \\ i \in X \\ j \in \bar{X}}} c(e).$$

An arc $(i, j) \in E$ with $i \in X$ and $j \in \bar{X}$ is called cut-arc of $(X, \bar{X})$.

An instance of the inverse minimum cut problem is given by an instance of the minimum cut problem together with a cut $(X^*, \bar{X}^*) \in \mathcal{S}$ and a weighted L_p-norm. The task is to find new capacities $\tilde{c}(e)$ for $e \in E$ such that

- $(X^*, \bar{X}^*)$ is a minimum cut with respect to the capacities $\tilde{c}$, i.e., $\tilde{c}(X^*, \bar{X}^*) \leq \tilde{c}(X, \bar{X})$ for all $(X, \bar{X}) \in \mathcal{S}$, and
- the weighted L_p-norm of $c - \tilde{c}$ is minimum.

Most inverse approaches deal with a weighted L_1-norm or a weighted L_∞-norm. Let $w(e) \in \mathbb{R}_+$ (for $e \in E$) be some weights, then the weighted L_1-norm of $c - \tilde{c}$ is given by

$$\|c - \tilde{c}\|_{w,1} = \sum_{e \in E} w(e)|c(e) - \tilde{c}(e)|$$

while the weighted L_∞-norm of $c - \tilde{c}$ is equal to

$$\|c - \tilde{c}\|_{w,\infty} = \max_{e \in E} w(e)|c(e) - \tilde{c}(e)|.$$

If all weights are equal to 1 then we write $\|\cdot\|_1$ and $\|\cdot\|_\infty$, respectively.

Zhang and Cai [8] considered a generalized inverse minimum cut problem, where a set of cuts is given and there are bounds u_e^+ and u_e^- for the modified capacities. Their goal is to find new capacities $\tilde{c}$ such that $u_e^- \leq \tilde{c}_e \leq u_e^+$ holds for all $e \in E$, all prespecified cuts become minimum cuts with respect to $\tilde{c}$ and they use the weighted L_1-norm. They show that the unit-weight case can be transformed to a minimum cut problem while the nonunit-weight case can be modelled as minimum cost circulation problem. Hence, the inverse minimum cut problem with L_1-norm can be solved in polynomial time. Later, Ahuja and Orlin [1] came up with a simplified proof of the correctness of the algorithms of Zhang and Cai and showed that an analogue approach also works for the inverse shortest path and inverse assignment problem. Moreover, Ahuja and Orlin proved that if the underlying optimization problem for a linear cost function is solvable in polynomial time then its inverse problems under L_1- or L_∞-norms are also polynomially solvable. This result implies that the inverse minimum cut problem with L_∞-norm is solvable in polynomial time.

This paper deals with the partial inverse minimum cut problem (PIMC for short): Let $G = (V, E, c)$ be an arc-capacitated digraph with source $s \in V$ and sink $t \in V$. Moreover, let $Y, \bar{Y} \subset V$ be disjoint sets. Then the task of the partial inverse minimum cut problem is to find new capacities $\tilde{c}$ such that

- there exists a cut $(X^*, \bar{X}^*) \in \mathcal{S}$ that is a minimum cut with respect to $\tilde{c}$, $Y \subseteq X^*$, $\bar{Y} \subseteq \bar{X}^*$, and
- the weighted L_p-norm of $c - \tilde{c}$ is minimized.

An equivalent definition of the partial inverse minimum cut problem is to fix a partial cut by its cut-arcs, i.e., there is given a subset of arcs $J \subseteq E$. The goal is then to minimally modify the capacities such that there exists an optimal cut $(X^*, \bar{X}^*)$ with respect to the modified capacities such that $i \in X^*$ and $j \in \bar{X}^*$ holds for all $(i,j) \in J$. If all arcs in J are cut-arcs of $(X, \bar{X})$ then we say that $(X, \bar{X})$ covers J.

While the inverse minimum cut problem with L_1- and L_∞-norm are solvable in polynomial time, it turns out that the corresponding partial inverse minimum cut problems are substantially harder. Several NP-hardness results for the partial inverse minimum cut problem can be concluded from investigations on the so-called preprocessing problem.

The preprocessing problem is a special case of the partial inverse optimization problem where the partial solution contains exactly one edge and the (weighted) L_∞-norm is used. Lai and Orlin [5] considered the preprocessing minimum cut problem, i.e., PIMC with weighted L_∞-norm where the partial cut $(Y, \bar{Y})$ is given by one cut-arc e'. The goal is to change the capacities such that there exists a cut $(X^*, \bar{X}^*)$ that is minimal with respect to the modified capacities, $(X^*, \bar{X}^*)$ covers $\{e'\}$ and the (weighted) L_∞-distance between the original and the modified capacity vector is minimized. Lai and Orlin showed by a reduction from the restricted path problem that the (weighted) preprocessing minimum cut problem and henceforth PIMC with (weighted) L_∞-norm is strongly NP-hard. In a companion working paper, Orlin [6] also proved that PIMC with weighted L_1-norm is strongly NP-hard. However, they leave the complexity status of PIMC with unweighted L_1-norm as open question.

Yang [7] considered PIMC with additional bound constraints on the modified capacities and the unweighted L_1-norm and claims that this problem is weakly NP-hard. However, their proof is incorrect as can be seen by the following counter-example: Consider the instance $\{a_1 = 1, a_2 = 3, a_3 = 4, a_4 = 5, a_5 = 6, a_6 = 7\}$ of the partition problem. Then $I = \{1, 2, 3, 4\}$ is a partition and according to Yang's construction $(X^*, \bar{X}^*)$ with $X^* = \{s, u_1, u_2, u_3, v_1, v_2, w_1, w_2\}$ is a minimum cut with respect to the modified capacities $\tilde{c}(v_1, t) = 13 - \frac{1}{2}$, $\tilde{c}(w_1, t) = 13 - \frac{3}{2}$, $\tilde{c}(v_2, t) = 11$, $\tilde{c}(w_2, t) = 13 - \frac{5}{2}$ and $\tilde{c}(e) = c(e)$ otherwise. However, the cut $(X, \bar{X})$ with $X = X^* \setminus \{u_3\}$ satisfies $\tilde{c}(X, \bar{X}) < \tilde{c}(X^*, \bar{X}^*)$ which contradicts the minimality of $(X^*, \bar{X}^*)$.

Nevertheless, Yang's conjecture about the computational complexity of their problem is correct. According to Lai and Orlin [5], PIMC with L_∞-norm is strongly NP-hard which implies that PIMC with bound constraints and any arbitrary norm is also strongly NP-hard. On the other hand, our hardness result for PIMC with unweighted L_1-norm also implies NP-hardness of the associated problem with bounds.

3 The partial inverse minimum cut problem with unweighted L_1-norm

In this section, we consider the partial inverse minimum cut problem with unweighted L_1-norm which is called PIMC$_1$ for short. Throughout this section, we deal with the following representation of an instance: Let $G = (V, E)$ be a digraph, $c(e) \in \mathbb{R}_+$ for $e \in E$, $s \in V$, $t \in V$ and $J \subseteq E$. The task of PIMC$_1$ is to find new capacities $\tilde{c}$ such that there exists a cut $(X^*, \bar{X}^*)$ that is a minimum cut with respect to $\tilde{c}$, $(X^*, \bar{X}^*)$ covers J and $\|c - \tilde{c}\|_1 = \sum_{e \in E} |c(e) - \tilde{c}(e)|$ is minimum.

Let us first investigate a property of an optimal solution of PIMC$_1$:

Lemma 3.1. *There exists an optimal solution c^* of PIMC$_1$ such that $0 \leq c^*(e) \leq c(e)$ holds for all $e \in E$.*

Proof. Let $(X^*, \bar{X}^*) \in \mathcal{S}$ be a minimum cut with respect to c^* that covers J. Then every optimal solution of the inverse minimum cut problem where $(X^*, \bar{X}^*)$ is required to be become a minimum cut is also an optimal solution of the given instance of PIMC$_1$. Ahuja and Orlin [1] showed that the inverse minimum cut problem admits an optimal solution with the properties stated in the lemma. Hence, there also exists an optimal solution of PIMC$_1$ with these properties. □

Theorem 3.2. *The partial inverse minimum cut problem with unweighted L_1-norm is strongly* NP*-hard.*

Proof. The proof splits into the following three parts:
The construction: Consider the balanced minimum cut problem, BMCP for short: Let $\hat{G} = (\hat{V}, \hat{E})$ be a connected digraph with an even number n of vertices, m edges and let $1 \le K < m$ be an integer. The task of BMCP is to find a cut $(Y, \bar{Y})$ such that $|Y| = |\bar{Y}| = \frac{n}{2}$ holds and the number of cut-arcs of $(Y, \bar{Y})$ is at most K, i.e., $|\{(i,j) \in \hat{E} \mid i \in Y, j \in \bar{Y}\}| \le K$.

Observe that the undirected version of BMCP where the task is to find a balanced cut $(Y, \bar{Y})$ in an undirected graph where at most K edges have one endpoint in Y and the other endpoint in $\bar{Y}$ has been shown to be strongly NP-hard (Garey, Johnson and Stockmeyer [3]). Given an instance of the balanced minimum cut problem in an undirected graph and replace every edge $e = \{i, j\}$ by two directed arcs (i, j) and (j, i) then there is a one-to-one correspondence between balanced cuts in the undirected and its associated directed graph. Moreover, let $(Y, \bar{Y})$ be a balanced cut then the number of edges in the undirected graph with one endpoint in Y and the other endpoint in $\bar{Y}$ is equal to the number of cut-arcs of $(Y, \bar{Y})$ in the associated digraph. Hence, BMCP is strongly NP-hard.

Let $(\hat{G}, K)$ be an instance of BMCP with $\hat{V} = \{1, \dots, n\}$ where n is even and $|\hat{E}| = m > K$. We construct an instance (G, s, t, J, c) of PIMC_1 as follows: For each vertex $i \in \hat{V}$ there is a vertex gadget $G_i = (V_i, E_i)$ with

$$\begin{aligned} V_i &= \{s, a_i, b_i, c_i, t'\} \\ E_i &= \{(s, a_i), (s, b_i), (b_i, a_i), (b_i, c_i), (c_i, t')\} \end{aligned}$$

Moreover, there is the arc set $\tilde{E}$ with

$$\tilde{E} = \{(b_i, b_j) \mid (i, j) \in \hat{E}\}.$$

Finally, a vertex t and the arc (t', t) are added. Hence $G = (V, E)$ is of the form

$$\begin{aligned} V &= \left(\bigcup_{i=1}^{n} V_i \right) \cup \{t\} \\ E &= \left(\bigcup_{i=1}^{n} E_i \right) \cup \tilde{E} \cup \{(t', t)\} \end{aligned}$$

The set J is given by

$$J = \{(s, a_i), (c_i, t') \mid i = 1, \dots, n\}$$

and the capacities are equal to

$$c(e) = \begin{cases} 0 & e \in \{(s, a_i) \mid i = 1, \dots, n\} \\ 3m & e \in \{(b_i, a_i) \mid i = 1, \dots, n\} \\ 2m & e \in \{(s, b_i), (b_i, c_i), (c_i, t') \mid i = 1, \dots, n\} \\ 1 & e \in \tilde{E} \\ mn & e = (t', t) \end{cases}$$

See Figure 1 for an illustration.

We show that there exists a balanced $(s-t)$-cut $(Y, \bar{Y})$ in $\hat{G}$ with at most K cut-arcs if and only if there exists a feasible solution $\tilde{c}$ of the PIMC_1-instance with $\|c - \tilde{c}\|_1 \le \frac{7mn}{2} + K$.
The IF-direction: Assume that $\tilde{c}$ is a feasible solution of PIMC_1 with $\|c - \tilde{c}\|_1 \le \frac{7mn}{2} + K$ and let $(X^*, \bar{X}^*)$ be a minimum cut that covers J. According to Lemma 3.1, we may assume that $\tilde{c}$ satisfies $0 \le \tilde{c}(e) \le c(e)$ for all $e \in E$. Moreover, it is easy to see that we may assume that $\tilde{c}(e) = c(e)$ for every $e \in E$ that does not lie in the cut $(X^*, \bar{X}^*)$.

According to the famous max-flow min-cut theorem by Ford and Fulkerson [2] the capacity of a minimum cut is equal to the value of a maximum flow. If $(X^*, \bar{X}^*)$ is a

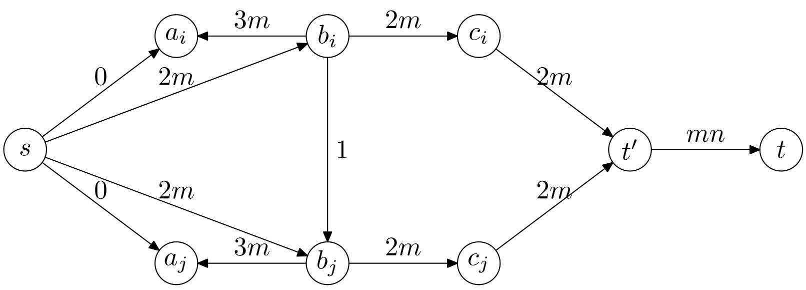

Figure 1: Constructed instance of PIMC_1 with $(i,j) \in \hat{E}$.

minimum cut with respect to $\tilde{c}$ and let f^* be a maximum flow with respect to $\tilde{c}$ then $\tilde{c}(i,j) = f^*(i,j)$ holds for all (forward-)arcs $(i,j) \in E$ with $i \in X^*$ and $j \in \bar{X}^*$ while $f^*(i,j) = 0$ holds for all (backward-)arcs $(i,j) \in E$ with $i \in \bar{X}^*$ and $j \in X^*$.

The fact that $(X^*, \bar{X}^*)$ covers J implies $\{s\} \cup \{c_i \mid i \in \hat{V}\} \subseteq X^*$ and $\{a_i \mid i \in \hat{V}\} \cup \{t', t\} \subseteq \bar{X}^*$. Hence, only vertices of type b_i may be in X^* or $\bar{X}^*$.

First consider $b_i \in X^*$: Then $(s, a_i), (b_i, a_i), (c_i, t')$ are cut-arcs and $(s, b_i), (b_i, c_i)$ are no cut-arcs. This implies that $\tilde{c}(s, b_i) = c(s, b_i) = 2m = c(b_i, c_i) = \tilde{c}(b_i, c_i)$ holds. Since there are no outgoing arcs of a_i there does not exist any $(s-t)$-flow f with $f(b_i, a_i) > 0$. Since (b_i, a_i) is a cut-arc, we have $\tilde{c}(b_i, a_i) = 0$. Finally, the modified capacity of (c_i, t') is of the form $\tilde{c}(c_i, t') = c(c_i, t') - x_i$ for some $x_i \geq 0$.

Next consider $b_i \in \bar{X}^*$: In this case, $(s, a_i), (s, b_i), (c_i, t')$ are cut-arcs and $(b_i, a_i), (b_i, c_i)$ and no cut-arcs. Hence, $\tilde{c}(b_i, a_i) = c(b_i, a_i) = 3m$ and $\tilde{c}(b_i, c_i) = c(b_i, c_i) = 2m$. Note that (b_i, c_i) is a backward-arc and hence $f^*(b_i, c_i) = 0$. This implies $0 = f^*(c_i, t') = \tilde{c}(c_i, t')$.

In order to find out the modified capacities of the remaining arcs, we consider the following flow f^*: $f^*(s, b_i) = f^*(b_i, c_i) = f^*(c_i, t') = 2m - x_i$ for all gadgets G_i with $b_i \in X^*$, $f^*(t', t) = 2m|X^*| - \sum_{b_i \in X^*} x_i$ and $f^*(i,j) = 0$ otherwise. Observe that $\tilde{c}(c_i, t') = 2m - x_i$ if $b_i \in X^*$ and $\tilde{c}(c_i, t') = 0$ if $b_i \in \bar{X}^*$. Therefore, f^* saturates all incoming arcs into t' and hence f^* is a maximum flow with respect to $\tilde{c}$. Observe that $f^*(s, b_i) = 0$ holds for all (s, b_i) with $b_i \in \bar{X}^*$ and since (s, b_i) is a forward-arc, we have $\tilde{c}(s, b_i) = 0$. Moreover, (t', t) is no cut-arc and hence $\tilde{c}(t', t) = c(t', t) = mn$. Finally, whenever $(b_i, b_j) \in \tilde{E}$ is a cut-arc, we have $\tilde{c}(b_i, b_j) = f^*(b_i, b_j) = 0$ and if $(b_i, b_j) \in \tilde{E}$ is no cut-arc then $\tilde{c}(b_i, b_j) = c(b_i, b_j) = 1$.

We are now able to compute $\|c - \tilde{c}\|_1$ and the capacity of cut $(X^*, \bar{X}^*)$: Let $k = |\{(b_i, b_j) \mid (i,j) \in \hat{E}, i \in X^*, j \in \bar{X}^*\}|$ and

$$Y = \{i \in \hat{V} \mid b_i \in X^*\}.$$

Then

$$\begin{aligned}
\|c - \tilde{c}\|_1 &= \sum_{b_i \in X^*} (c(b_i, a_i) + x_i) + \sum_{b_i \in \bar{X}^*} (c(s, b_i) + c(c_i, t')) + k \\
&= 3m|Y| + \sum_{i \in Y} x_i + 4m|\bar{Y}| + k \\
&= 4mn - m|Y| + \sum_{i \in Y} x_i + k
\end{aligned}$$

$$\tilde{c}(X^*, \bar{X}^*) = \sum_{b_i \in X} (2m - x_i) = 2m|Y| - \sum_{i \in Y} x_i$$

Since $\|c - \tilde{c}\|_1 \leq \frac{7mn}{2} + K$ holds, we get

$$0 \leq \sum_{i \in Y} x_i \leq m|Y| - \frac{mn}{2} + K - k. \tag{1}$$

Since arc (t', t) implies a cut with modified capacity mn and $(X^*, \bar{X}^*)$ is a minimum cut with respect to $\tilde{c}$, we have $\tilde{c}(X, \bar{X}) \leq mn$ which implies

$$2m|Y| - mn \leq \sum_{i \in Y} x_i \tag{2}$$

We show that $(Y, \bar{Y})$ with $\bar{Y} = \hat{V} \setminus Y$ is a balanced cut with at most K arcs in the cut. Observe that $K < m \leq m + k$ holds.

First assume that $|Y| \leq \frac{n}{2} - 1$: Then (1) implies

$$0 \leq m|Y| - \frac{mn}{2} + K - k < m\left(\frac{n}{2} - 1\right) - \frac{mn}{2} + m = 0$$

which is a contradiction.

If $|Y| \geq \frac{n}{2} + 1$ then $K - k < m$, (1) and (2) imply

$$2m|Y| - mn \leq \sum_{i \in Y} x_i < m|Y| - \frac{mn}{2} + m \leq m|Y| - \frac{mn}{2} + m(|Y| - \frac{n}{2}) = 2m|Y| - mn$$

which is again a contradiction.

We have shown that $(Y, \bar{Y})$ is a balanced cut, i.e., $|Y| = \frac{n}{2}$. Finally, we have to make sure that there are at most K cut-arcs in $(Y, \bar{Y})$. Observe that

$$k = |\{(b_i, b_j) \mid (i, j) \in \hat{E}, i \in X^*, j \in \bar{X}^*\}| = |\{(i, j) \in \hat{E} \mid i \in Y, j \in \bar{Y}\}|$$

holds, i.e., there are k cut-arcs. Inequality (1) for $|Y| = \frac{n}{2}$ implies

$$0 \leq m|Y| - \frac{mn}{2} + K - k = K - k$$

and hence $(Y, \bar{Y})$ is a balanced gut in $\hat{G}$ with at most K cut-arcs.

The ONLY IF-direction: Let us now assume that there exists a balanced cut $(Y, \bar{Y})$ in $\hat{G}$ with at most K cut arcs. Then let

$$\tilde{c}(e) = \begin{cases} 0 & \text{if } e = (b_i, a_i) \text{ and } i \in Y \\ 0 & \text{if } e \in \{(s, b_i), (c_i, t') \mid i \in \bar{Y}\} \\ 0 & \text{if } e = (b_i, b_j) \text{ and } i \in Y, j \in \bar{Y} \\ c(e) & \text{otherwise} \end{cases}.$$

Observe that the flow f^* such that there are $2m$ units of flow on every path (s, b_i, c_i, t') for $i \in Y$ is a maximum $(s - t')$-flow because all incoming arcs into t' are saturated. Consider the cut $(X^*, \bar{X}^*)$ with

$$X^* = \{s\} \cup \{b_i, c_i \mid i \in Y\} \cup \{c_i \mid i \in \bar{Y}\}.$$

This cut satisfies $\tilde{c}(X^*, \bar{X}^*) = 2m|Y| \leq mn$. Hence, $(X^*, \bar{X}^*)$ is a minimum $(s - t)$-cut in G and $(X^*, \bar{X}^*)$ covers J. Moreover,

$$\|c - \tilde{c})\|_1 = \sum_{i \in Y} 3m + \sum_{i \in \bar{Y}} 4m + |\{(b_i, b_j) \mid (i, j) \in \hat{E}, i \in Y, j \in \bar{Y}\}| \leq \frac{7mn}{2} + K.$$

Hence, the partial inverse minimum $(s - t)$-cut problem is strongly NP-hard even if all cost coefficients are equal to 1. □

References

[1] R. K. Ahuja and J. B. Orlin. Inverse optimization. *Oper. Res.*, 49(5):771–783, 2001.

[2] L. R. Ford, Jr. and D. R. Fulkerson. Maximal flow through a network. *Canad. J. Math.*, 8:399–404, 1956.

[3] M. R. Garey, D. S. Johnson, and L. Stockmeyer. Some simplified NP-complete graph problems. *Theoret. Comput. Sci.*, 1(3):237–267, 1976.

[4] C. Heuberger. Inverse combinatorial optimization: a survey on problems, methods, and results. *J. Comb. Optim.*, 8(3):329–361, 2004.

[5] T. Lai and J. Orlin. The complexity of preprocessing. Research Report of Sloan School of Management, MIT, 2003.

[6] J. Orlin. Partial inverse optimization problems. Working paper, Sloan School of Management, MIT.

[7] X. Yang. Complexity of partial inverse assignment problem and partial inverse cut problem. *RAIRO Oper. Res.*, 35(1):117–126, 2001.

[8] J. Zhang and M.-C. Cai. Inverse problem of minimum cuts. *Math. Methods Oper. Res.*, 47(1):51–58, 1998.